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Navier—Stokes Millennium Prize problem is solved

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NAVIER–STOKES MILLENNIUM PRIZE PROBLEM IS SOLVED

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Abstract. I have written a solution to Navier–Stokes Millennium Prize problem.
MSC Class: 35Q30.

1. Introduction

To cite an Encyclopedia of 2023 AD: "*Since understanding the Navier–Stokes equations is considered the first step to understanding the elusive phenomenon of turbulence, the Clay Mathematics Institute in May 2000 made this problem one of its seven Millennium Prize problems in mathematics. It offered a prize to the first person providing a solution for a specific statement of the problem: Prove or give a counter-example of the following statement: In three space dimensions and time, given an initial velocity field, there exists a vector velocity and a scalar pressure field, which are both smooth and globally defined, that solve the NavierStokes equations.*"

The above formulation of Navier–Stokes problem has terms from Physics: velocity (in the following, $\vec{u}$), space (in the following, coordinate vector $\vec{r}$), time t, and pressure (in the following, the influence of pressure is hidden within $\vec{f}$). The density field is ρ.

Therefore, having contradictions with the Physical picture, I have found countless counter-examples against these equations; namely, equations are not invariant under coordinate transformations (in particular, "boosts"). And these equations are not correctly derived (or even not derived, but they appear to be bluntly postulated by Dr. Stokes [1]), so there are counter-examples for Mathematicians as well.

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2. Example of Mathematical Contradiction

The derivation begins with ansatz:

$$\rho(\vec{r},t)\,\frac{\partial\vec{u}(\vec{r},t)}{\partial t}=\vec{f}(\vec{r},t)\,, \tag{1}$$

where ρ is density of the moving medium.

Let me insert $\vec{r}=\vec{r}(t)$,

$$\rho(\vec{r}(t),t)\,\frac{d\,\vec{u}(\vec{r}(t),t)}{d\,t}=\vec{f}(\vec{r}(t),t)\,. \tag{2}$$

Then,

$$\rho(\vec{r}(t),t)\left(\left.\frac{\partial\vec{u}(\vec{r},t)}{\partial t}\right|_{r=r(t)}+\frac{d\vec{r}(t)}{d\,t}\left.\frac{\partial\vec{u}(\vec{r},t)}{\partial\,\vec{r}}\right|_{r=r(t)}\right)=\vec{f}(\vec{r}(t),t)\,. \tag{3}$$

Let me remove the seemingly unnecassary notation (t) by replacing $\vec{r}(t)$ with $\vec{r}$, and I get the Navier-Stokes equations

$$\rho(\vec{r},t)\left(\frac{\partial\vec{u}(\vec{r},t)}{\partial t}+\vec{u}\,\nabla\vec{u}\right)=\vec{f}(\vec{r},t)\,, \tag{4}$$

where

$$\nabla\vec{u}\equiv\frac{\partial\vec{u}(\vec{r},t)}{\partial\,\vec{r}}\,. \tag{5}$$

With all respect to Dr. Stokes, equation (4) contradicts equation (1).

3. Example of Physical Contradiction

Let the velocity pattern of the fluid is $\vec{u}$. Let me run toward the fluid with velocity $\vec{w}$. Then according to me, the fluid approaches me with velocity $\vec{V}=\vec{u}-\vec{w}$, where $\vec{w}$ is a constant vector (it means, its components are just a numbers, not a functions of coordinates x,y,z,t). All other inner parameters of the fluid (ρ, $\vec{f}$) remain the same. If Eq.(1) is the description of this fluid, then

$$\rho\,\frac{\partial\vec{V}}{\partial t}=\vec{f}\,. \tag{6}$$

As you see, the Eq.(6) is the same as Eq.(1), but with velocity $\vec{V}$ in the role of $\vec{u}$. If Eq.(4) is the description of the fluid, then

$$\rho\left(\frac{\partial\vec{V}}{\partial t}+\vec{V}\,\nabla\vec{V}+\vec{w}\,\nabla\vec{V}\right)=\vec{f}\,. \tag{7}$$

As you see, the Eq.(7) is not the same as Eq.(4), with velocity $\vec{V}$ in the role of $\vec{u}$. In other words, Eq.(4) is not invariant under Galilean

Coordinate Transformation. All non-relativistic Physical systems (low-velocity systems) satisfy the Galilean Coordinate Transformation.

References

[1] Navier. Mémoire sur les lois du mouvement des fluides. Mémoires de l'Académie des sciences de l'Institut de France. 1822. Vol. 6; Stokes. On the theories of internal friction of fluids in motion, and of the equilibrium and motion of elastic solids. Transactions of the Cambridge Philosophical Society. 1845. Vol. 8.