Review of: "A Convergence Not Metrizable"

Reynaldo Rojas-Hernández[1]

1 Universidad Michoacana de San Nicolás de Hidalgo, Mexico

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The result is interesting, but can be proved in a more general and direct way.

\textbf{Proposition.} \textit{For a topological space M and a space N containing at least two points, if the space of functions (N^M,τ) in the pointwise convergence (product) topology admits a metric d such that (N^M,τ) and (N^M,d) have the same convergent sequences, then M is countable}.

To prove it, we can assume that $0,1 \in N$ and consider the compact space $K \subset (N^M,\tau)$ consisting of all characteristic functions of points in M together with the zero constant function. Observe that K is the one-point compactification of the discrete space of cardinality M, and so K is metrizable if and only if M is countable. Besides, K is a Fréchet space. But K as a subspace of (N^M,d) also is Fréchet. Since (N^M,τ) and (N^M,d) have the same convergent sequences, It follows that the topology on K inherited from (N^M,τ) coincides with the topology on K inherited from (N^M,d), and hence K is metrizable. Therefore M is countable.