Review of: "New adaptative numerical algorithm for solving partial integro-differential equations"

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Potential competing interests: No potential competing interests to declare.

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A Set-Matrix Duality Principle for the Dirac Equation

In this paper, the authors propose a new theory of spontaneous mirror symmetry violation in a unified field theory of elementary particles. The theory is based on the idea that the mass, energy, and momentum of elementary particles are matrices that express the ideas of left- and right-handed neutrinos. The authors argue that this theory requires a new way to define matrices, which they call the set-matrix duality principle. The authors also discuss four new definitions, three lemmas, and one pair of axioms that are based on the set-matrix duality principle. Overall, I believe that this is a valuable contribution to the field of theoretical physics.

The paper needs major revision and can be considered for publication in the journal after a revision is done. I need to see the revised version of the paper to ensure if it needs minor revisions or not after that. Below, I list some items to improve the quality of paper:

1. **The introductory section** should be amended to cover the main motivation of the work and the cons/pros of the method. Which gap the paper will fill in the literature. At the end of this section, an organization of the paper should be provided.

Also, the paper addresses the concept of spontaneous mirror symmetry violation in nature and its implications for the equations of motion in a unified field theory of elementary particles. This topic is intriguing and relevant to the field of particle physics. The introduction must indicates to this and support this through some literature.

The paper introduces the idea that the left- and right-handed neutrinos correspond to long- and short-lived objects, respectively, and suggests moving away from the chiral definitions of matter fields. The concept of internally disclosed and undisclosed matrices and their connection to set-matrix duality principle is also intriguing. So, the introduction of this paper should be contains the following mention:

"The foundation of this paper lies in the exploration of a novel perspective on the nature of left- and right-handed neutrinos, challenging the conventional chiral definitions of matter fields. We propose an alternative framework that associates left-handed neutrinos with long-lived objects and right-handed neutrinos with short-lived objects. This paradigm shift prompts us to reconsider the traditional understanding of particle symmetries and opens up new avenues for

investigation. Furthermore, we delve into the intriguing concept of internally disclosed and undisclosed matrices, which shed light on the interplay between matrix representations and the set-matrix duality principle. By incorporating these ideas into a unified theoretical framework, we aim to provide a deeper understanding of the fundamental structure of matter and its implications for elementary particle physics."

2. **Methodology:** It would be helpful to have a clearer description of the methodology employed in the paper. Providing step-by-step explanations or algorithms would enhance the reproducibility and understanding of the proposed approach.

3. **Clarity and Organization**: The clarity and organization of the paper are essential for effective communication of the research findings. It is recommended that the authors ensure the paper is well-structured, with clear section headings and subheadings that guide the reader. Additionally, providing definitions and necessary background information would assist readers who may not be familiar with the specific concepts or functions under investigation.

4. **Significance and Impact:** It would be valuable to discuss the significance and potential impact of the results obtained. How do the upper bounds for the Taylor-Maclaurin coefficients contribute to the existing body of knowledge in complex analysis? Are there any potential applications or implications of these results in related areas of research?

To discuss the significance and potential impact of the results obtained, as well as their contribution to the existing body of knowledge in complex analysis, you can incorporate the following elements into the paper:

- *Highlighting novel findings:* Clearly state the specific results obtained in the paper regarding the upper bounds for the Taylor-Maclaurin coefficients. Emphasize any new insights or discoveries that arise from these findings. For example, if the paper establishes tighter bounds or uncovers previously unknown properties of the coefficients, make sure to highlight these contributions.

- *Relating to existing body of knowledge:* Discuss how the obtained results build upon and extend the existing knowledge in complex analysis. Identify any gaps or limitations in the previous understanding of Taylor-Maclaurin coefficients and explain how the current findings fill those gaps or provide a deeper understanding of the topic.

- *Comparisons and contrasts:* Compare the obtained results with previous studies or known bounds for Taylor-Maclaurin coefficients. Discuss how the newly derived bounds differ, improve upon, or complement the existing bounds. This comparison can help demonstrate the significance of the current findings and their potential impact on the field.

-*Applications and implications:* Explore potential applications or implications of the obtained results in related areas of research. Consider how the improved bounds for Taylor-Maclaurin coefficients can be utilized in practical settings or serve as theoretical foundations for other analytical techniques. For example, these results may have implications in numerical analysis, approximation theory, or the analysis of complex functions in various scientific and engineering applications. Discuss these possibilities and highlight the potential impact of the findings beyond the scope of the current study. I suggest to introduce the following papers in the list of references:

1-Recurrences and explicit formulae for the expansion and connection coefficients in series of classical discrete

orthogonal polynomials, https://doi.org/10.1080/10652460500422270.

2- Computing expansions coefficients for Laguerre polynomials, https://doi.org/10.1080/10652469.2020.1815727.

3- Numerical Solutions of Korteweg-de Vries and Korteweg-de Vries-Burger's Equations in a Bernstein Polynomial Basis. Mediterr. https://doi.org/10.1007/s00009-019-1375-1.

-Future research directions: Conclude the discussion by suggesting possible future research directions that can be pursued based on the obtained results. Identify any unresolved questions or areas that require further investigation. This can help generate interest and inspire researchers to explore new avenues that build upon the current findings.

By incorporating these elements into the paper, you can effectively discuss the significance and potential impact of the results obtained, their contribution to the existing body of knowledge in complex and numerical analysis, and potential applications or implications in related areas of research.

2. **Experimental Validation or Examples:** I suggest to Including some numerical examples or experimental validation of the proposed methodology, would enhance the paper's rigor and practical relevance. Demonstrating the effectiveness of the (M,N) –Lucas Polynomials in determining upper bounds for the Taylor-Maclaurin coefficients through concrete examples or comparisons with existing approaches would strengthen the paper's credibility.

3. It should be added a conclusion section at the end of the paper.

4. The manuscript contains **several grammatical errors and typos**. I asked the authors to revise the manuscript using any tool.