

Review of: "A Random Journey Through the Math of Gambling"

Xiaowei Tan¹

¹ Columbia University

Potential competing interests: No potential competing interests to declare.

This manuscript explores random walks with reflecting and absorbing barriers. The analysis effectively tackles both unbiased and biased cases, providing valuable insights into how walk behavior changes with probabilities of moving up or down. Propositions with clear formulas and insightful remarks strengthen the analysis, while the introduction of transient vs. recurrent events and the connection to gambling scenarios demonstrates the practical implications of the research. Overall, the manuscript offers a well-structured and insightful analysis, but some minor adjustments could enhance its clarity and impact.

Strengths:

- **Clear Structure:** The manuscript presents a well-organized exploration of random walks with reflecting and absorbing barriers. Theorems, propositions, and implementations are clearly presented.
- **Comprehensiveness:** The work effectively tackles both unbiased and biased cases, providing valuable insights into how walk behavior changes with probabilities of moving up or down.
- **Rigorous Analysis:** Propositions with clear formulas and insightful remarks on their behavior under different parameter settings strengthen the analysis.
- **Meanful Applications:** The introduction of transient vs. recurrent events and the connection to gambling scenarios demonstrate the practical implications of the research.
- **Focus on Long-Term Behavior:** Exploring the limiting probability of returning to zero for infinitely many steps provides valuable insights.
- **Pedagogical Approach:** The manuscript is clearly written and aims to be accessible to a broad audience, offering interpretations, numerical examples, and original nuances.
- **Appropriate Language:** The manuscript uses language appropriate for the target audience.

Areas for Improvement:

- **Jargon Consideration:** Depending on the target audience of the academic journal, referencing the Borel-Cantelli Lemma by name might require justification or a brief explanation.
- **Conciseness:** The manuscript might benefit from length reduction by moving some content (detailed proofs, extensive derivations) to an appendix.
- **Literature Review:** Including a more comprehensive literature review could strengthen the background and

demonstrate awareness of current research.

- **Appendix Consistency:** Ensure consistent numbering between proof statements in the appendix and those in the main body.
- **Appendix Clarity:** Briefly explain the concept of F_n as the history of the martingale up to time n within the appendix or a relevant section.
- **Crediting Existing Work:** The paper should acknowledge and compare its contribution to existing educational resources and research on ballot problems, random walks (e.g., arcsine law), and ruin problems.
- **Proofreading:** The manuscript contains a number of typographical errors and formatting inconsistencies.
- **Tailoring for Audience:** Consider if the target journal is suitable for a more textbook-like presentation.
- **Originality for Target Audience:** The classical content might be suitable for beginners, but consider incorporating elements to enhance depth, such as:
 - Discussing the connection between random walks and Brownian motion using Donsker's Invariance Principle.
 - Briefly explaining how Donsker's Invariance Principle allows for solving problems with the normal distribution (Brownian motion) instead of the binomial distribution (random walk).
 - Mentioning the Banker's Ruin problem and the arcsine law, highlighting a derivation using Donsker's Invariance Principle and the Feynman-Kac formula (acknowledging this as an advanced topic).

Overall Assessment:

This manuscript offers a well-structured and insightful analysis of random walks with reflecting and absorbing barriers. The exploration of both unbiased and biased cases, combined with the use of propositions and insightful remarks, demonstrates a strong understanding of the subject matter. The introduction of transient vs. recurrent events and the connection to gambling scenarios adds depth and practical relevance. With minor adjustments to address potential jargon issues, conciseness, literature review, appendix details, crediting existing work, and potentially tailoring the style for the target audience, this manuscript has the potential for publication.