Research Article

Bridging Theory and Practice in Implementing EL-MIATTs: Logic-Driven Algorithms for MIATTs Generation and Assessment

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Based on the assumption that the true target for a given machine learning task is not assumed to exist as a well-defined object in the real world, the concept of Multiple Inaccurate True Targets (MIATTs) has been proposed to provide a data foundation for the paradigm of Evaluation and Learning with MIATTs (EL-MIATTs), particularly in scenarios where obtaining the true target is difficult, costly, or infeasible. Scientifically generating MIATTs and assessing their quality are pivotal steps in bridging theory and practice to realize the EL-MIATTs framework in real-world applications. In this paper, starting from first principles and leveraging the driving force of logic with necessarily provided resources, we propose two complementary algorithms: an abductive reasoning–driven algorithm for MIATTs generation and a Boolean algebra–driven algorithm for MIATTs assessment. The former enables the construction of MIATTs sets for individual instances by logically abducing plausible approximations of the underlying true target, while the latter provides a principled approach to quantitatively assessing the quality of MIATTs sets through Boolean operations on semantic facts. To further mitigate the multidisciplinary difficulties of implementing these logic-driven algorithms in practice, we also propose two simplified solutions for MIATTs generation and assessment that rely on retrievable real-world resources. Additionally, physical interpretations are presented for MIATTs and their assessment indicators. Together, these contributions to MIATTs generation and assessment strengthen the feasibility of EL-MIATTs in domains where the true target remains uncertain or inaccessible.

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1. Introduction

In many tasks of machine learning (ML), the true target often eludes precise definition in the real world, posing significant challenges for traditional supervision paradigms. To address this—based on the assumption that the true target for a given ML task is not assumed to exist as a well-defined object—the concept of Multiple Inaccurate True Targets (MIATTs) has been proposed as a foundational data mechanism for the Evaluation and Learning with MIATTs (EL-MIATTs) framework [1]. An MIATTs set consists of individual inaccurate true targets, each encoding partial semantic facts. Taken together, they approximate the underlying true target with wider coverage. This reframes supervision as distributional and multi-perspective rather than single-target, enabling robust assessment and training in uncertain or resource-constrained settings. The scientific generation and assessment of MIATTs are vital to bridging theoretical principles with practical implementations, ensuring applicability in real-world scenarios where accurate true target is inaccessible. This paper focuses on these processes to connect theory and practice in advancing the EL-MIATTs framework toward future applications.

Grounded in first principles and harnessing the rigorous deductive and inferential power of logic [2][3] alongside essential resources, we propose two complementary logic-driven algorithms: one powered by abductive reasoning [4] for generating MIATTs, enabling the creative hypothesis of plausible approximations in ambiguous scenarios, and the other rooted in Boolean algebra [5] for assessing MIATTs, facilitating precise evaluation through set-theoretic operations like unions and intersections to quantify semantic coverage and quality. The former enables the construction of MIATT sets for individual instances, while the latter provides a principled framework for quantitatively evaluating their quality. A comprehensive formalization and analysis of the two logic-driven algorithms for MIATTs generation and assessment are presented in Sections 3 and 4.

Nonetheless, both logic-driven algorithms demand expertise across multiple domains: preparing domain-specific resources such as knowledge bases, labels, or semantic descriptions of the underlying true target; logically abducing plausible approximations of that target; and performing Boolean operations on its associated semantic facts. These multidisciplinary requirements make their implementation from scratch in real-world scenarios challenging. Fortunately, for specific tasks, real-world resources can be conveniently retrieved—particularly with the previously accumulated task-specific data and model resources [6][7][8][9][10][11] and the emergence of large AI models [12], which are fundamentally transforming knowledge acquisition, extending the boundaries of productivity, and reshaping paradigms of human–machine collaboration. Building on such resources, we further propose two simplified solutions corresponding to the two logic-driven algorithms for MIATTs generation and assessment. The use of retrievable real-world resources provides practical and simplified alternatives to both the abductive reasoning–driven and Boolean algebra–driven approaches, lowering implementation barriers while preserving their essential logical foundations for MIATTs generation and assessment. The formalization and analysis of the two simplified solutions, together with their summarization through comparative analysis of the original versus simplified approaches and geometric visualization of commonly generated MIATTs and their assessment indicators, are presented in Section 5.

At the fundamental physical level [13][14][15][16], MIATTs can be interpreted as a noisy upper approximation to the underlying true target: they ensure broad semantic coverage of the underlying semantic facts while also admitting a degree of redundancy (overlap or noise). The MIATTs assessment indicators, such as partial representation and redundancy, directly capture this trade-off by quantifying how much useful information is preserved versus how much overlap or noise is introduced within an MIATTs set. The physical meaning of MIATTs lies in providing practical, approximate surrogates for the underly true target when it is uncertain, inaccessible, or inherently ambiguous. In parallel, the assessment indicators translate abstract Boolean-algebraic measures into interpretable dimensions of quality. Taken together, these interpretations highlight the dual role of MIATTs and their assessment indicators: (1) to ground weak supervision in a physically meaningful approximation of the truth, and (2) to operationalize quality control in assessing such approximations. This duality underscores their practical value as a bridge between theory and application in domains where accurate true target is unattainable. The physical interpretations of MIATTs and their assessment indicators are presented in Section 6.

Collectively, these advances for scientifically generating and assessing MIATTs aim to guarantee the practical value of MIATTs, which lies in their ability to transform the challenges of imperfect supervision into opportunities for constructing more robust predictive models. By aggregating multiple approximate true targets, they safeguard against noise and biases while enabling distributional supervision over partial signals. This makes MIATTs particularly valuable in domains where the true target is uncertain or inaccessible, thereby laying the foundation for the EL-MIATTs paradigm. Further discussion on these advances and the related practical value of MIATTs is provided in Section 7.

In summary, the contributions of this paper are as follows:

- We bridge theory and practice in realizing the EL-MIATTs paradigm by exploring logic-driven algorithms for MIATTs generation and assessment.
- We propose two complementary algorithms: an abductive reasoning–driven algorithm for generating MIATTs and a Boolean algebra–driven algorithm for assessing their quality.
- To reduce implementation barriers, we further propose two simplified solutions that leverage retrievable real-world resources while preserving the essential logical foundations of the original two logic-driven approaches.
- We provide physical interpretations of MIATTs and their assessment indicators, elucidating their dual role as both approximations of the true target and tools for quality control.
- We establish a structured yet flexible framework for generating and assessing MIATTs, transforming the challenge of an undefinable true target into an opportunity for robust, distributional evaluation and learning in support of the EL-MIATTs paradigm.

The remainder of this paper is structured as follows: Section 2 introduces the definition and essence of MIATTs. Sections 3 and 4 respectively present the two proposed logic-driven algorithms for MIATTs generation and assessment. Section 5 describes the simplified alternatives to these logic-driven approaches. Section 6 provides the physical interpretations of MIATTs and their assessment indicators. Section 7 discusses the advances achieved in this work for scientifically generating and assessing MIATTs, emphasizing their practical value. Finally, Section 8 concludes the paper with limitations and directions for future research.

2. MIATTs: Multiple Inaccurate True Targets

Building upon the fundamental assumption that the true target for a given machine learning task is not assumed to exist as a well-defined object in the real world, the definition of MIATTs [1] is described as:

Let t^* be the underlying (possibly undefinable) true target and $SF(t^*)$ its set of semantic facts. A MIATTs set is $MIATTs = \{t_n{}^* | n \in \{1, \cdots, N\}, N \geq 2\}$, where each $t_n{}^*$ satisfies $SF(t_n{}^*) \subset SF(t^*)$ and $\bigcup_{n=1}^{N} SF(t_n{}^*) \subseteq SF(t^*)$. That is each $t_n{}^*$ encodes part of t^*, and together the MIATTs set approximates it.

In essence, MIATTs formalize the idea that supervision in real-world machine learning is often fragmented and noisy. Each individual inaccurate true target encodes only a partial subset of semantic facts of the underlying truth, yet collectively the MIATTs set can approximate its structure with broader coverage. This formulation transforms supervision from a rigid single-target paradigm into a distributional and multi-perspective one, thereby offering a principled foundation for the EL-MIATTs framework to realize robust evaluation and learning under uncertainty.

3. Abductive Reasoning-Driven MIATTs Generation

Abductive reasoning [4], which extends the principles of logic [3] to be a form of logical inference that seeks the best possible explanation for a given set of observations, provides the foundation for deriving an algorithm for MIATT generation. Section 3.1 offers a brief review of abductive reasoning. Section 3.2 introduces the abductive reasoning–driven algorithm for MIATT generation with respect to the definition of MIATTs. Section 3.3 analyzes its implementation and the associated challenges in practice.

3.1. Abductive reasoning

Abductive reasoning was first systematically conceptualized by Charles Sanders Peirce [4]. Unlike deduction, which derives logically necessary conclusions from general rules, and induction, which generalizes from repeated observations, abduction generates plausible hypotheses that can explain surprising or incomplete evidence.

Formally, abductive reasoning can be expressed as:

- Rule: If H were true, then O would be expected.
- Observation: O is observed.
- Abduction: Therefore, H is a plausible explanation.

Abduction is inherently inference to the best explanation [17], which means that multiple competing hypotheses may exist, but the most coherent, parsimonious, or contextually adequate one is provisionally adopted. This non-monotonic nature—where new evidence may invalidate prior abductive conclusions—distinguishes it from classical logical inference [18].

In artificial intelligence and cognitive science, abductive reasoning has been studied as a core mechanism for diagnosis (e.g., medical reasoning, fault detection), commonsense reasoning, and narrative understanding [19][20]. More recently, it has gained traction in natural language understanding, where systems must hypothesize hidden causal or intentional structures underlying text [21].

Thus, abductive reasoning provides a crucial theoretical lens for modeling how agents—human or artificial—bridge the gap between data and explanatory hypotheses, enabling reasoning under

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uncertainty and incomplete knowledge.

3.2. Algorithm: Abductive reasoning-driven MIATTs generation

Regarding the definition of MIATTs, previous studies [6][7][8][9] have shown that suitable algorithms for MIATT generation can be developed, driven by abductive reasoning theory. In particular, given a knowledge base and instances with limited and inaccurate labels, these algorithms are constructed through a series of abstract abduction steps. The stepwise procedures of these algorithms are described as follows.

Input:

- Raw data instances I.
- A set of limited and inaccurate labels L, associated with I, for partially describing the underlying true target t^*.
- A knowledge base $KB = \{k_1, k_2, \ldots, k_m\}$ that describes the full semantic facts of the underlying true target t^*.

Step 1. Extraction of grounding from $\mathbf{L}$

- A list of groundings (g), partially describing the logical facts contained in the given L for the underlying true target t^*, is extracted. This grounding extraction (GE) step can be expressed as:

$$g = GE(L) = \{g_1, \cdots, g_s\}. \tag{1}$$

Step 2. Estimation of inconsistencies between g and $\mathbf{KB}$

- The inconsistencies (ic) between the extracted groundings g and the prior knowledge accumulated in KB are estimated with logical conclusions. Formally, this logical reasoning (LR) step can be expressed as

$$ic = LR(g, KB) = \{ic_1, \cdots, ic_u\}. \tag{2}$$

Step 3. Logical abduction

- The groundings in g are revised by logical abduction, which aims to reduce the estimated inconsistencies in ic. Formally, this logical abduction (LA) step can be expressed as

$$rg = LA(ic, g) = \{rg_1, \cdots, rg_w\}. \tag{3}$$

Step 4. Generation of MIATTs set

- The MIATTs set is generated by leveraging $\{I, L\}$ and rg to abduce multiple types of inaccurate true targets for each instance in I. Formally, this generation (G) step can be expressed as

$$MIATTs = G\left(\{I, L\}, rg\right) = \{t_n{}^{*} | n \in \{1, \cdots, N\}\}, \ N \geq 2. \tag{4}$$

Output:

- Each instance in I is assigned an MIATTs set $\{t_n{}^{*}\}_{n=1}^{N\geq 2}$, each being a partial but informative approximation of the corresponding underlying true target t^{*}.

This abductive reasoning–driven algorithm aims to ensure that, for each instance in the input raw data I, a corresponding $MIATTs$ set is generated that represents its underlying true target while exhibiting fewer inconsistencies with the knowledge base KB than its originally assigned limited and inaccurate label in L.

3.3. Analysis of algorithm

In this subsection, we analyze the implementation and difficulties of the abductive reasoning–driven algorithm for MIATT generation in practice.

3.3.1. Implementation

Previous studies [6][7][8][9] have detailed the implementation of this algorithm for real-world MIATTs generation in medical image segmentation tasks. Referring to Formulas (1)–(4), the implementation of this algorithm can be generally analyzed as follows:

Grounding extraction: Based on the provided limited and inaccurate labels L associated with the raw data instances I, meaningful semantics contained in L are extracted. These extracted semantics form the groundings g, which describe partial logical facts of the underlying true target t^{*}.

Inconsistency estimation: The groundings g are compared with the knowledge base KB, and inconsistencies (ic) are estimated via logical reasoning. Since the KB describes the complete semantic facts of t^{*}, the inconsistencies ic between g and KB reflect the degree to which the limited and inaccurate labels L deviate from the underlying true target.

Grounding revision via abduction: The groundings g are revised through logical abduction to produce revised groundings (rg), which aim to reduce inconsistencies with KB. The possible true target, from which rg can be extracted, serves as a plausible explanation for the reduced inconsistencies.

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MIATTs construction: The revised groundings rg are then used to abduce MIATTs, which contain groundings more consistent with KB for representing the underlying true target t^*. In practice, $\{I, L\}$ can be partitioned into several subsets, each partially containing rg and used to train a predictive model. The trained models are subsequently applied to infer the true target of each instance in I. For each instance, the multiple predicted true targets, together with the original label (if available), are aggregated into an MIATTs set that distributionally represents its underlying true target [8].

3.3.2. Difficulties

As the abductive reasoning–driven algorithm is designed from first principles, its implementation in real-world scenarios requires a necessary foundation for the subsequent abductive reasoning steps to generate MIATTs. This foundation includes raw data instances I with their corresponding limited and inaccurate labels L, which partially describe the underlying true target t^*, as well as a knowledge base $KB = \{k_1, k_2, \ldots, k_m\}$ that specifies the full semantic facts of t^*. Implementing the algorithm from scratch is challenging, as it requires expertise across multiple domains. Specifically, preparing L and KB depends on domain specialists, while carrying out the abductive reasoning steps for MIATT generation relies on expertise in AI or computer science. This challenge poses difficulties for individual domain experts to implement the algorithm in practice. To alleviate these difficulties, in Section 5, we will present a simplified solution for MIATT generation using retrievable real-world resources.

4. Boolean Algebra-Driven MIATTs Assessment

Boolean algebra [5], which is grounded in formal logic [2] to serve as a mathematical framework that enables systematic reasoning, simplification of expressions, and verification, provides the foundation for deriving an algorithm for MIATT assessment. Section 4.1 offers a brief review of Boolean algebra. Section 4.2 introduces the Boolean algebra–driven algorithm for MIATT assessment with respect to the definition of MIATTs. Section 4.3 analyzes its implementation and the associated challenges in practice.

4.1. Boolean algebra

Boolean algebra, introduced by George Boole in the mid-19th century [5], is a mathematical framework for representing and manipulating logical statements using binary values, typically 0 (false) and 1 (true). Unlike classical algebra over real numbers, Boolean algebra operates on a set $B = \{0, 1\}$ with operations such as conjunction ($\wedge$), disjunction ($\vee$), and negation ($\neg$), governed by a set of axioms and laws including commutativity, associativity, distributivity, identity, and complementation.

Formally, a Boolean algebra is a structure $(B, \vee, \wedge, \neg, 0, 1)$ satisfying:

- Commutativity: $a \vee b = b \vee a$, $a \wedge b = b \wedge a$;
- Associativity: $(a \vee b) \vee c = a \vee (b \vee c) \; = \; (a \vee b) \vee c = a \vee (b \vee c)$;
- Distributivity: $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$;
- Identity: $a \vee 0 = a$, $a \wedge 1 = a$;
- Complementation: $a \vee \neg a = 1$, $a \wedge \neg a = 0$.

Boolean algebra underpins digital logic design, computer architecture, and set theory, providing a formal language for reasoning about propositional logic, circuits, and decision-making processes. Its algebraic properties also enable systematic simplification of logical expressions and optimization of digital circuits [22][23].

Beyond engineering, Boolean algebra has influenced knowledge representation, search algorithms, and fuzzy logic extensions, making it a foundational tool for both theoretical and applied disciplines in mathematics, computer science, and artificial intelligence [24][25][26][27].

4.2. Algorithm: Boolean algebra-driven MIATTs assessment

Regarding its definition, the MIATTs set encodes partial aspects of a true target t^*. Using Boolean algebra, we propose to model the semantic facts $SF\,(t^*)$ as a set of Boolean variables and each inaccurate true target (IATT) ${t_n}^*$ as a subset of these variables. This allows us to estimate partial representativeness, collective coverage, and redundancy, which are then combined into an overall quality score for assessing an MIATTs set using standard Boolean operations $(\vee, \wedge)$. The stepwise procedure of this Boolean algebra-driven algorithm are described as follows.

Input:

- An MIATTs set $\{{t_n}^*\}_{n=1}^{N\geq 2}$.
- The full set of semantic facts $SF(t^*) = \{f_1, f_2, \ldots, f_m\}$ for the underlying true target t^*.

Step 1: Represent IATT as Boolean vectors

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- Represent each IATT $t_n{}^*$ as a Boolean vector $v_n \in \{0,1\}^m$, where:

$$v_n[i] = \begin{cases} 1, \; if \; IATT \; t_n{}^* \; encodes \; semantic \; fact \; f_i \\ \hfill 0, \; otherwise \end{cases}. \tag{5}$$

Step 2: Assess partial representation (Per-IATT quality)

- For each IATT $t_n{}^*$:

$$PartialRepresentation \left(t_n{}^*\right) = \frac{|SF(t_n{}^*)|}{|SF(t^*)|} = \frac{\sum_i v_n[i]}{m}. \tag{6}$$

Step 3: Assess collective coverage (set-level quality)

- Compute the Boolean union (logical OR) across the MIATTs set:

  $v_{union} = v_1 \vee v_2 \vee \ldots \vee v_N$

- Compute collective coverage score:

$$CollectiveCoverage = \frac{\sum_i v_{union}[i]}{m}. \tag{7}$$

Step 4: Assess redundancy / diversity

- Compute pairwise intersections (logical AND) between MIATTs:

  $Intersection \left(t_j{}^*, t_k{}^*\right) = v_j \wedge v_k.$

- Measure redundancy ratio:

$$Redundancy = \frac{\sum_{j<k} |v_j \wedge v_k|}{\sum_j |v_j|}. \tag{8}$$

Step 5: Overall quality score

- Combine the metrics into an aggregate score:

$$\begin{aligned} Q_{MIATTs} &= \alpha \cdot mean(PartialRepresentation) \\ &+ \beta \cdot CollectiveCoverage - \gamma \cdot Redundancy_{norm}. \end{aligned} \tag{9}$$

Output:

- The computed Q_{MIATTs} for assessing the overall quality score of the MIATTs set.

This Boolean algebra–driven algorithm aims to quantitatively assess the quality of an MIATT set using only Boolean algebra operations, with respect to the full set of semantic facts of the underlying true target. Boolean algebra here ensures efficient computation of union and intersection for large semantic spaces.

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4.3. Analysis of algorithm

In this subsection, we analyze the implementation and difficulties of the Boolean algebra–driven algorithm for MIATT assessment in practice.

4.3.1. Implementation

Referring to Formulas (5)–(9), the implementation for real-world scenarios can be generally analyzed as follows:

IATT representation as Boolean vector: Given the full set of semantic facts of the underlying true target t^*, $SF(t^*) = \{f_1, f_2, \ldots, f_m\}$, each IATT ${t_n}^*$ in an MIATT set is represented as an mmm-dimensional Boolean vector v_n. The i-th entry of v_n takes the value 1 if ${t_n}^*$ contains the semantic fact $f_i \in SF(t^*)$, and 0 otherwise. Since each Boolean vector v_n is directly associated with semantic facts in $SF(t^*)$, it can be equivalently expressed as $SF({t_n}^*)$. Correspondingly, the set $SF(t^*)$ itself can be represented as an mmm-dimensional Boolean vector with all entries equal to 1. In cases where a proxy or partial gold standard for t^* exists, the true target can also be represented by a Boolean vector v^*.

Partial representation: The partial representation of each IATT ${t_n}^*$ with respect to the underlying true target is assessed by $|SF\left({t_n}^*\right)| / |SF\left(t^*\right)|$. This can be conveniently computed as the sum of the Boolean vector v_n divides m, formalized as $\sum_i v_n[i]/m$. This value measures how much of the full semantic space each IATT individually covers. Ideally, the score lies between 0 and 1, reflecting that each IATT provides only a partial but informative approximation.

Collective coverage: The collective coverage of an MIATTs set is assessed by modeling the semantic facts it contains as the Boolean union of its constituent IATTs. Formally, this can be conveniently computed by Boolean operation $\vee$, and is given by $v_{union} = v_1 \vee v_2 \vee \ldots \vee v_N$. The collective coverage is then computed as the sum of the Boolean vector v_{union} divides m, formalized as $\sum_i v_{union}[i]/m$. Particularly, if $v_{union} = v^*$, the MIATTs collectively cover all semantic facts of the underlying true target.

Redundancy: The redundancy of the MIATT set is quantified by measuring pairwise intersections (logical AND) between IATTs, formalized as $Intersection\left({t_j}^*, {t_k}^*\right) = v_j \wedge v_k$. Based on these intersections, the redundancy of the MIATT set is defined as $Redundancy = \sum_{j<k} |v_j \wedge v_k| / \sum_j |v_j|$. Lower redundancy indicates that MIATTs are capturing complementary aspects rather than overlapping the same semantic facts.

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Overall quality score: The overall quality score of an MIATT set is modeled as a weighted aggregation of partial representation, collective coverage, and redundancy, formalized as, formalized as $Q_{MIATTs} = \alpha \cdot mean(PartialRepresentation) + \beta \cdot CollectiveCoverage - \gamma \cdot Redundancy.$ $\alpha, \beta, \gamma \geq 0$ are weights reflecting importance of partial representation, coverage, and non-redundancy. Higher Q_{MIATTs} indicates better-quality MIATTs under Boolean-algebra-based evaluation.

4.3.2. Difficulties

As the Boolean algebra–driven algorithm is designed from first principles, its implementation in real-world scenarios requires a necessary foundation for the subsequent Boolean operation steps to assess MIATTs. This foundation includes the MIATTs set and the full set of semantic facts $SF(t^*) = \{f_1, f_2, \ldots, f_m\}$ for the underlying true target t^*. Implementing the algorithm from scratch is challenging, as it requires expertise across multiple domains. Specifically, constructing the full set of semantic facts $SF(t^*)$ typically depends on domain specialists as the underlying true target t^* is undefinable, while carrying out the Boolean operations for MIATT assessment relies on expertise in mathematics, AI or computer science. This challenge poses difficulties to implement the algorithm in practice. To alleviate these difficulties, in Section 5, we will present a simplified solution for MIATT assessment using retrievable real-world resources.

5. Simplified Logic-Driven Solutions Using Retrievable Real-World Resources

The multidisciplinary requirements make the direct implementations of the abductive reasoning-driven algorithm for MIATTs generation and the Boolean algebra-driven algorithm for MIATTs assessment challenging in real-world scenarios. Fortunately, for specific tasks, real-world resources can be conveniently retrieved—particularly with the previously accumulated task-specific data and model resources [6][7][8][9][10][11] and the emergence of large AI models [12], which are fundamentally transforming knowledge acquisition, extending the boundaries of productivity, and reshaping paradigms of human–machine collaboration. Building on such resources, in this section, we propose two simplified solutions corresponding to the logic-driven algorithms for MIATTs generation and assessment, and further summarizes them through comparative analysis of the original versus simplified approaches as well as through geometric visualization of commonly generated MIATTs and their assessment indicators.

5.1. Simplified solution for MIATTs generation

For specific tasks, real-world resources—such as previously accumulated task-specific datasets and models or task-related large AI models—can be conveniently retrieved. Leveraging these resources, we can directly employ AI predictive models or tools built on accumulated task-specific knowledge [6][7][8][9][10][11], together with existing task-related large AI models [12], to constitute a set of task-specific AI models (AIM). This AIM set enables mapping an instance to multiple predicted true targets for its underlying true target, which collectively form the generated MIATTs for that instance.

5.1.1. Formalization

Denoting by $AIM = \{p_1, p_2, \ldots, p_N\}$ the set of AI predictive models and tools, including those previously constructed from task-specific resources as well as existing task-related large AI models, the stepwise procedure of this solution is outlined as follows:

Input:

- Raw data instances I.
- A set of AI models $AIM = \{p_1, p_2, \ldots, p_t\}$ for mapping an instance into predicted multiple true targets for the underlying true target t^*.

Step 1: Prediction of multiple potential true targets

- Predict multiple potential true targets for I with AIM:

$$\begin{aligned} MIATTs = AIM(I) &= \{p_1(I), p_2(I), \ldots, p_N(I)\} \\ &= \{t_n{}^* | n \in \{1, \cdots, N\}\},\ N \geq 2. \end{aligned} \tag{10}$$

Output:

- Each instance in I is assigned an MIATTs set $\{t_n{}^*\}_{n=1}^{N\geq 2}$, each being a partial but informative approximation of the corresponding underlying true target t^*.

5.1.2. Analysis

Within the abductive reasoning–driven framework for MIATTs generation, the input set of AI models (AIM)—comprising previously constructed predictive models or tools as well as existing task-related large models—acts as a collection of plausible explanations for the predicted multiple true targets that exhibit consistency with the underlying true target. This interpretation is logically grounded in the nature of abductive reasoning itself, which seeks the most plausible explanation for observed data given incomplete knowledge. Since each model in AIM is trained on domain-specific data or equipped with generalizable knowledge from large-scale pretraining, its predictions capture different but overlapping aspects of the underlying true target. Consequently, the outputs of AIM can be reasonably viewed as plausible candidate explanations rather than arbitrary guesses, thereby aligning with the abductive principle of generating hypotheses consistent with available evidence.

In this way, the use of retrievable real-world resources provides a practical and simplified alternative to the full abductive reasoning–driven approach. The proposed simplification substantially lowers implementation barriers while preserving the essential abductive rationale for MIATTs generation, since the collective predictions of AIM approximate the abductive search space for plausible explanations in a computationally efficient manner.

5.2. Simplified solution for MIATTs assessment

For specific tasks, once an MIATTs set is generated by task-specific AIM retrievable from real-world resources, a probable true target can be summarized to approximate the underlying true target. This summarization is feasible because each IATT within the MIATTs set can be viewed as covering a partial but reliable proportion of the underlying true target. By integrating these partial coverages, the MIATTs collectively yield a probable true target that captures the essential semantic structure of the ground truth. Consequently, this summarized probable true target can serve as a reference for assessing the quality of the MIATTs set from which it is derived, ensuring a self-consistent and task-adapted evaluation process.

5.2.1. Formalization

The stepwise procedure of this solution is outlined as follows:

Input:

- An MIATTs set $\{t_n{}^*\}_{n=1}^{N\geq 2}$ generated by task-specific AIM.

Step 1: Approximate probable true target

$$\check{t^*} = mean\left(\{t_n{}^*\}_{n=1}^{N\geq 2}\right). \tag{11}$$

Step 2: Represent IATT as Boolean vectors

- Represent each IATT $t_n{}^*$ as a Boolean vector $v_n \in \{0,1\}^m$, where:

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$$v_n[i] = \begin{cases} 1, \ if \ abs\left(t_n{}^*(i) - \check{t^*}(i)\right) < \delta \\ 0, \ otherwise \end{cases}. \tag{12}$$

Step 3: Assess partial representation (Per-IATT quality)

- The partial representation for each IATT $t_n{}^*$ is the same as Formula (6).

Step 4: Assess redundancy / diversity

- The redundancy is the same as Formula (8).

Step 5: Overall quality score

- Combine the metrics into an aggregate score:

$$Q_{MIATTs} = \alpha \cdot mean(PartialRepresentation) - \gamma \cdot Redundancy. \tag{13}$$

Output:

- The computed Q_{MIATTs} for assessing the overall quality score of the MIATTs set.

5.2.2. Analysis

Within this solution, summarizing a probable true target from the MIATTs generated by task-specific AIM and using it as a reference to assess the quality of the originating MIATTs set is both logically grounded and methodologically feasible. The rationale is threefold. First, each IATT in the MIATTs set represents a partial semantic grounding of the underlying true target. When multiple IATTs are generated by AIM trained on task-specific data or derived from task-related large models, their collective coverage is expected to approximate the semantic space of the true target. Second, by aggregating these partial groundings, a probable true target can be inferred, which functions analogously to the consensus in ensemble learning [28], where multiple weak learners jointly approximate an unknown label. Third, this summarized probable true target provides a stable reference for evaluating the quality of the MIATTs set itself, since higher-quality MIATTs should yield a more coherent and consistent approximation of the underlying true target. Although the accuracy of this approximation depends on the reliability of the underlying AIM, weighting mechanisms or credibility assessments can be incorporated to mitigate biases from low-quality models.

Building on the approximated probable true target obtained from the MIATTs, as expressed in Formula (11), we reformulate the representation of each IATT as Boolean vectors, as shown in Formula (12). In the

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simplified assessment, the collective coverage indicator is omitted because its contribution is effectively redundant: the approximated probable true target already subsumes this aspect, rendering the coverage of an MIATTs set with respect to its approximated probable true target nearly constant (≈1). Thus, excluding this indicator streamlines the assessment without compromising its evaluative rigor. Specifically, since the probable true target is derived from the union of all IATTs in the MIATTs set, its semantic space is by definition fully covered, which makes the collective coverage indicator redundant. In this case, the indicator is constant at a value of 1, contributing no additional discriminative power to the assessment. Consequently, the overall quality score is redefined with respect to only two indicators: partial representation and redundancy. This adjustment highlights the contribution of individual IATTs while simultaneously controlling for overlaps among them. In this way, leveraging MIATTs derived from retrievable real-world resources offers a practical and simplified alternative to the full Boolean algebra–driven approach. The proposed simplification substantially lowers the implementation barrier while preserving the essential advantages of Boolean algebra operations in assessing MIATTs quality.

5.3. Summary

5.3.1. Comparative analysis of the original versus simplified approaches

The comparisons between the original logic-driven algorithms and their simplified solutions, with respect to MIATTs generation and assessment, are illustrated in Fig. 1 and Fig. 2, respectively. These visual summaries highlight how retrievable real-world resources serve as a practical foundation for lowering implementation barriers while maintaining the essential logical underpinnings of the full approaches.

The use of retrievable real-world resources provides practical and simplified alternatives to both the abductive reasoning–driven and Boolean algebra–driven algorithms, lowering implementation barriers while preserving their essential logical foundations for MIATTs generation and assessment.

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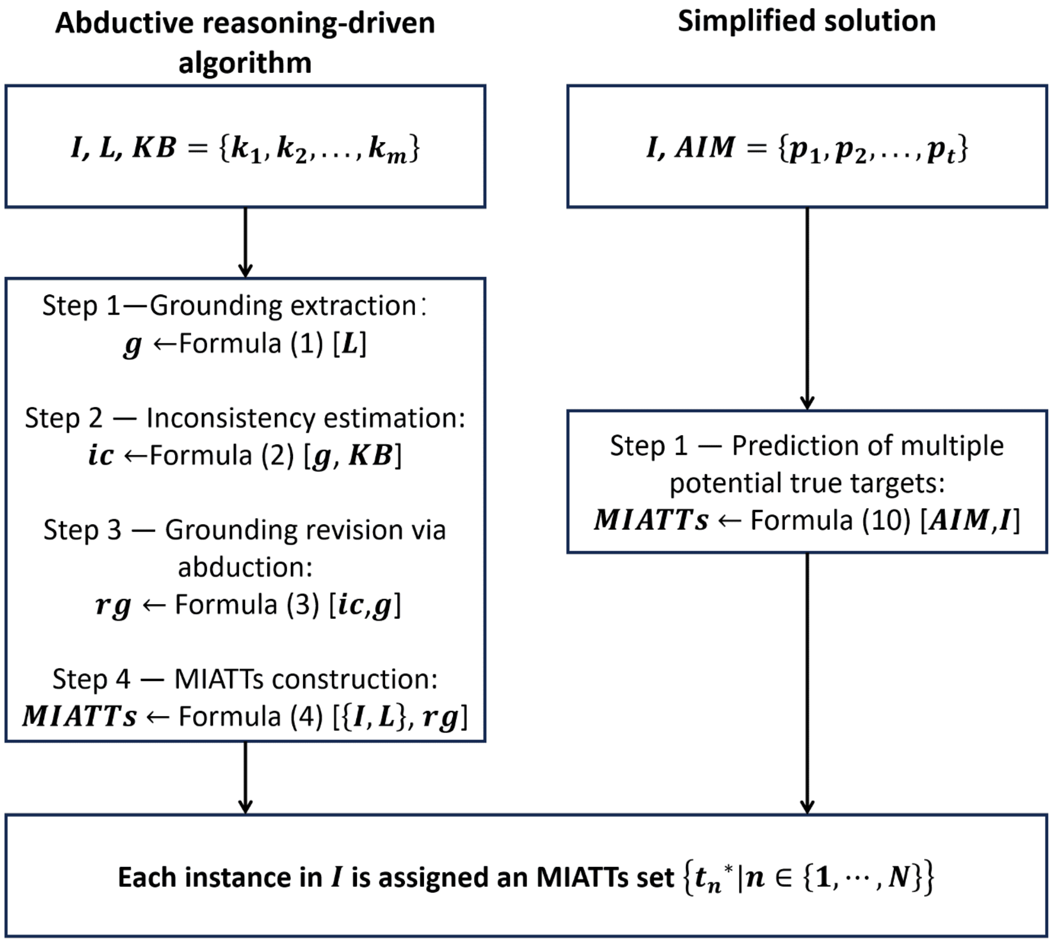

Figure 1. Comparison between the full abductive reasoning–driven algorithm for MIATTs generation and its simplified solution. The simplified approach leverages retrievable real-world resources, such as task-specific predictive models and large AI models (AIM), to reduce multidisciplinary requirements while retaining the essential abductive rationale for constructing MIATTs sets.

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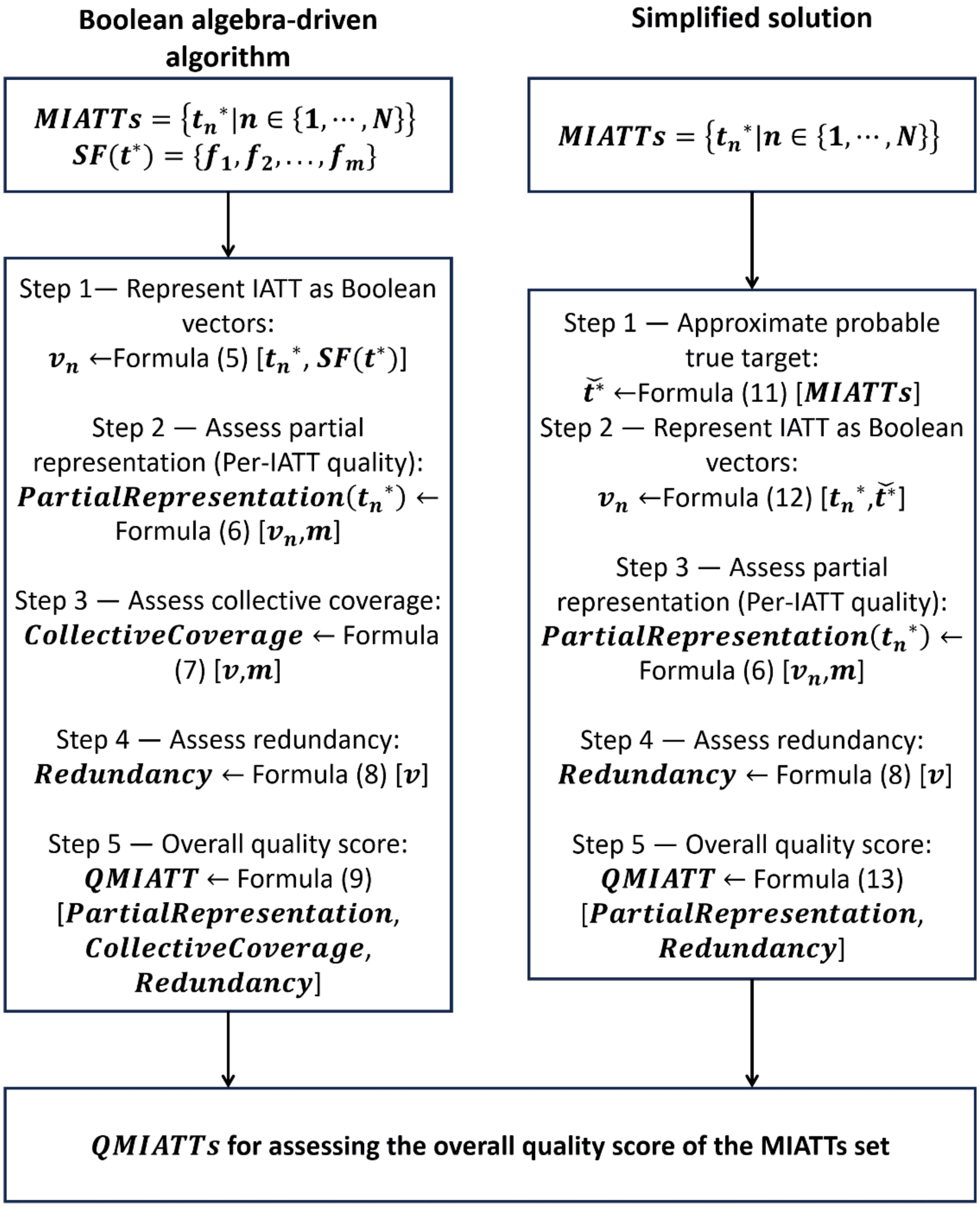

Figure 2. Comparison between the full Boolean algebra–driven algorithm for MIATTs assessment and its simplified solution. The simplified approach employs approximated probable true targets derived from the MIATTs generated by task-specific AIM to streamline assessment, lowering implementation barriers while preserving the core Boolean algebra principles of partial representation and redundancy.

5.3.2. Geometric visualization of commonly generated MIATTs and their assessment indicators

Commonly, the union of the generated MIATTs set $\bigcup_{n=1}^{N} t_n{}^*$ should approximate full coverage of the underlying true target t^*, while inevitably introducing some additional noise. Regarding the underlying true target t^*, the geometric relationships are illustrated through two complementary visualizations: Fig. 3 depicts the internal geometry of commonly generated MIATTs, showing how multiple approximate true targets collectively form a distributional approximation around t^*; while Fig. 4 illustrates the geometry of their assessment indicators, highlighting how measures such as partial representation and redundancy map onto interpretable dimensions of informativeness and overlap. Together, these visualizations provide an intuitive understanding of how MIATTs relate to the underlying true target and how their quality can be systematically evaluated.

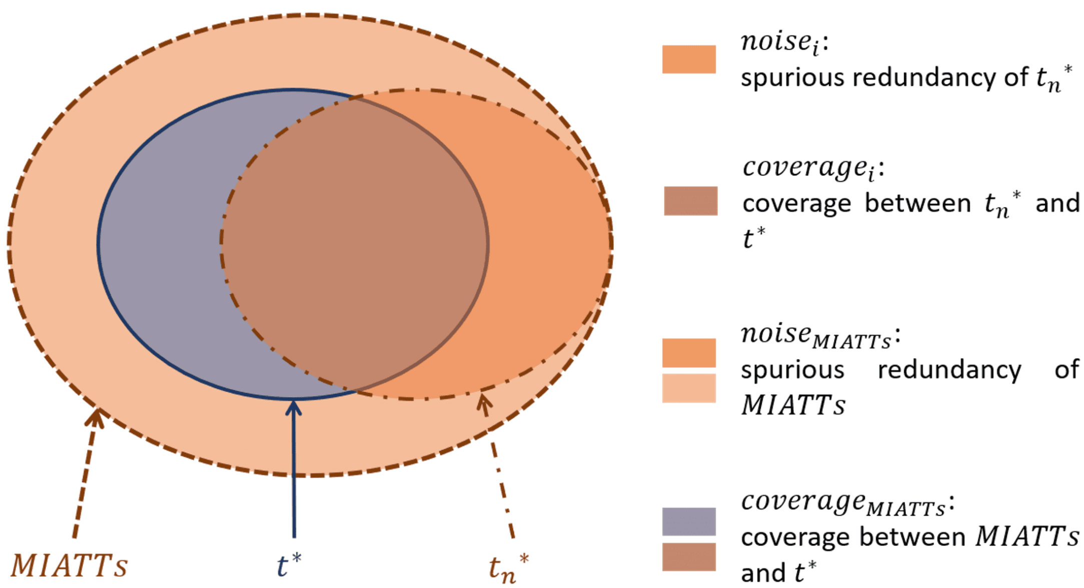

Figure 3. Depiction for the internal geometry of commonly generated MIATTs.

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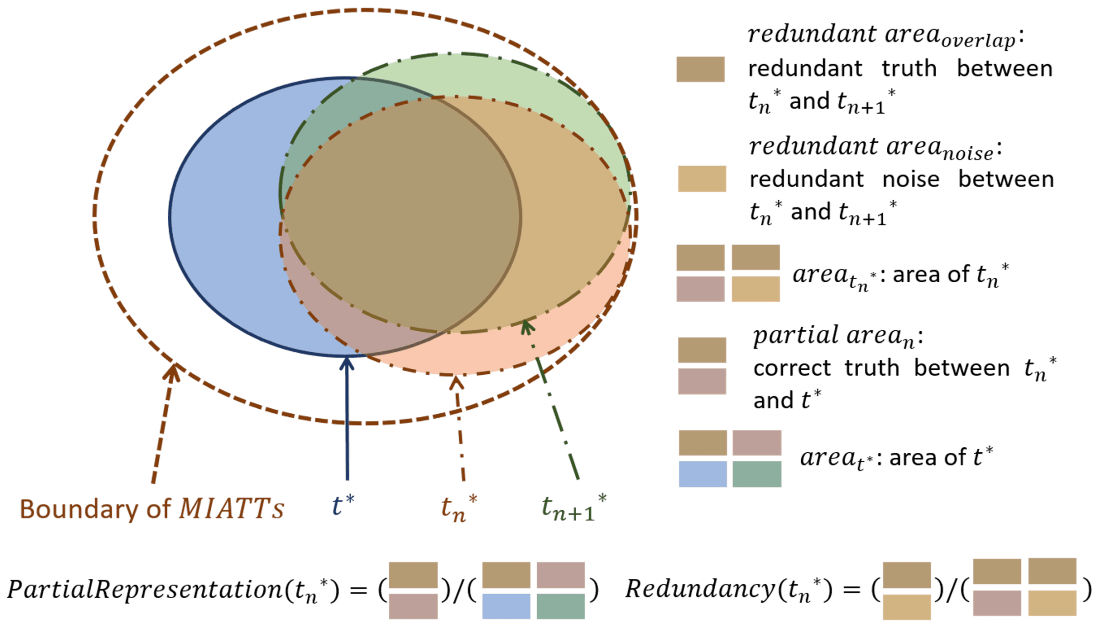

Figure 4. Illustration for the geometry of assessment indicators for commonly generated MIATTs.

Building on these visual insights, the subsequent discussion on the physical interpretations of MIATTs and their assessment indicators (Section 6) synthesizes these geometric intuitions into a broader conceptual framework, emphasizing their dual role as both approximations to the underlying true target and as operational tools for quality control in weak supervision.

6. Physical Interpretations for MIATTs and Their Assessment Indicators

MIATTs carry inherent physical meaning, as each IATT represents a partial, plausible approximation of the underlying true target, enabling insight into which aspects of the target are captured. Likewise, the assessment indicators—partial representation, collective coverage, and redundancy—have tangible interpretations, reflecting the informativeness, completeness, and overlap of the MIATTs set. Together, interpreting both MIATTs and their assessment indicators in physical terms bridges the gap between abstract algorithmic outputs and actionable insights, enhancing the utility of MIATTs-based approaches in real-world applications.

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6.1. Physical interpretations of MIATTs

By definition, a MIATTs set consists of multiple inaccurate but plausible approximations (IATTs) of the underlying true target t^*, each covering a subset of its semantic facts $SF(t^*)$. Through the abductive reasoning–driven algorithm, these IATTs are logically generated from limited evidence, such that their collective union increases the probability of covering the entirety of $SF(t^*)$. Similarly, in the simplified solution based on retrievable real-world resources (e.g., task-specific AIM or large AI models), aggregating diverse model predictions enhances semantic coverage by capturing complementary aspects of the underlying true target. Nonetheless, both approaches inevitably introduce erroneous information due to the incompleteness of evidence or model imperfections. Consequently, the generated MIATTs set is highly likely to contain full semantic facts of t^*, while also incorporating a degree of noise with diversity.

The MIATTs set can be endowed with deeper theoretical grounding by situating it within established frameworks in information theory, logic, and machine learning, thereby underscoring its practical significance in real-world applications. When the MIATTs set achieves high coverage of the underlying target's semantic facts $SF(t^*)$ but also admits erroneous information, its physical meaning can be interpreted through four complementary perspectives:

- **Information-theoretic perspective: Noisy cover approximation**

  From an information-theoretic viewpoint, MIATTs resemble a noisy cover of the underlying true target t^*. That is, while the union of inaccurate true targets collectively covers most or all semantic facts of $SF(t^*)$, it inevitably introduces spurious or redundant facts. This parallels Shannon's classic distinction between signal and noise in communication channels, where the integrity of the transmitted message depends on separating true information from distortion [14]. Hence, MIATTs can be conceptualized as a mixture of signal (semantic fact) and noise (errors).

- **Logical and Boolean algebra perspective: Noisy upper approximation**

  From a logical standpoint, MIATTs align closely with concepts in rough set theory. Each inaccurate true target provides only a partial representation, but collectively, their union can probably form an upper approximation of the true semantic set. Formally, $SF(t^*) \subseteq \bigcup_{n=1}^{N} SF(t_n{}^*) \subseteq SF(t^*) \cup \epsilon$, where ϵ denotes spurious elements. This framing follows Pawlak's [15] treatment of rough sets, where upper approximations guarantee coverage but risk introducing irrelevant or noisy elements. Within

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Boolean algebra, MIATTs thus constitute a superset approximation that is logically sound but imperfect.

- **Machine learning perspective: Noisy supervision signal**

  In the domain of machine learning, MIATTs can be interpreted as a form of weak supervision. Rather than supplying clean and exact labels, MIATTs provide a probabilistic training signal characterized by a balance between coverage of true facts and injection of erroneous information. As Zhou [16] emphasizes, such noisy labels can still be highly valuable for model training: the true signal enables convergence toward the target distribution, while the noise compels models to develop robustness or exploit statistical regularities. In this sense, MIATTs serve as a signal-to-noise supervision mechanism.

- **Intuitive Analogy: A Blurry but useful map**

  Finally, an intuitive metaphor helps illustrate the above perspectives. MIATTs can be likened to a blurry yet information-rich map. Much like a traveler navigating with a map that marks nearly all the correct landmarks but also includes fictitious ones, a model trained on MIATTs must learn to distinguish reliable semantic cues from misleading artifacts. This analogy echoes the broader treatment of uncertainty and approximate reasoning in artificial intelligence systems [13].

Taken together, these interpretations suggest that the physical meaning of MIATTs is that of a noisy upper approximation to the true semantic facts. In practice, this means that while MIATTs guarantee broad semantic coverage of the underlying true target, they also inevitably introduce redundancy and noise with diversity. Rather than being a flaw, this property provides an imperfect yet valuable supervision signal: it enables evaluation and learning in scenarios where the true target is uncertain, inaccessible, or inherently unobservable. Thus, MIATTs serve as a bridge between idealized theoretical targets and the imperfect information available in real-world domains, offering a pragmatic pathway for advancing machine learning in complex, knowledge-sparse environments.

6.2. Physical interpretations of MIATTs assessment indicators

The probability that "the generated MIATTs set is highly likely to contain all semantic facts of t^*, while also incorporating a degree of noise" directly supports the establishment of the simplified overall quality score. Since full semantic coverage is already ensured, the evaluation no longer needs a collective coverage indicator. Instead, quality depends primarily on two aspects: the extent to which each IATT provides informative partial representation of t^*, and the extent to which redundancy reflects noise or overlap among MIATTs. From a physical perspective, the simplified Formula (13) $Q_{MIATTs} = \alpha \cdot mean(PartialRepresentation) - \gamma \cdot Redundancy$, logically captures the trade-off between informativeness and noise, offering a concise yet principled way to assess MIATTs quality under the simplified Boolean algebra–driven framework.

The term $mean(PartialRepresentation)$ measures the average proportion of the underlying semantic facts $SF(t^*)$ that each IATT contributes. Physically, this can be viewed as the useful signal strength contained in the MIATTs set. Just as in information theory where effective communication depends on maximizing the transmitted signal that carries relevant information [14][29], a higher $PartialRepresentation$ indicates that individual IATTs provide meaningful, non-trivial coverage of the underlying true target's semantic space. This aligns with the principle of efficient representation: each IATT acts as a "signal carrier" that reveals part of the underlying true target.

The $Redundancy$ term reflects the overlap or repetition among IATTs, which corresponds to inefficiency or wasted capacity in physical systems. In communication theory, redundancy beyond a certain level is equivalent to introducing noise that does not increase effective information content but consumes representational capacity [14][30]. Within MIATTs, redundancy implies that multiple IATTs cover the same semantic facts, diminishing the diversity and informativeness of the set. Physically, this can be understood as "information interference" or "echo signals" that blur the clarity of representation.

Formula (13) therefore expresses a trade-off: maximizing informative coverage through $PartialRepresentation$ while penalizing inefficiency through $Redundancy$. A higher score corresponds to MIATTs that behave like a signal-dominant approximation of $SF(t^*)$, whereas a lower score reflects a noise-dominant representation. In practice, this quality score provides a concise and principled tool for evaluating MIATTs in domains where the true target is uncertain or inaccessible, allowing practitioners to distinguish between useful and noisy supervision signals. This practical value extends to real-world machine learning applications where imperfect labels or approximations must be systematically assessed [31].

6.3. Summary

Taken together, these physical interpretations highlight the dual role of MIATTs and their associated assessment indicators. On the one hand, MIATTs themselves serve as physically meaningful approximations to the underlying true target, capturing its semantic coverage while inevitably admitting a degree of redundancy and noise. This interpretation grounds the idea of weak supervision in a concrete representation that is not purely abstract, but instead reflects the practical reality of learning from multiple, imperfect signals. On the other hand, the assessment indicators—such as partial representation and redundancy—provide physically interpretable tools for quantifying the informativeness and quality of these approximations. They operationalize the process of distinguishing useful diversity from harmful noise, thereby functioning as a form of quality control.

This duality is of particular practical importance. MIATTs offer a robust way of encoding uncertain or inaccessible truth into a structured supervision signal, while their assessment indicators ensure that such signals remain interpretable, measurable, and optimizable. Together, they form a coherent framework that bridges the gap between theoretical logic-driven formalisms and practical machine learning applications. In real-world domains—such as natural language understanding, medical diagnosis, and open-ended decision-making—where the exact true target is unattainable, this provides not only a principled substitute for ground truth but also a practical mechanism to ensure that learning systems can reliably benefit from it.

7. Discussion

Two complementary logic-driven algorithms are proposed for MIATTs generation and assessment. For generation, the abductive reasoning–driven algorithm treats revised groundings—obtained through deduction to minimize inconsistencies between provided labels and the knowledge base for the underlying true target—as plausible abductive explanations. This enables the construction of MIATTs sets that remain logically consistent with the underlying true target, thereby providing a principled pathway from uncertain or noisy inputs toward semantically meaningful approximations. For assessment, the Boolean algebra–driven algorithm represents IATTs and MIATTs as Boolean vectors over semantic facts, allowing their quality to be assessed through Boolean operations. The resulting overall quality score balances the informativeness of partial representation against the redundancy arising from overlap or noise, yielding a concise yet rigorous assessment metric. Together, these two algorithms offer a logically grounded and mutually reinforcing framework for advancing the paradigm of EL-MIATTs.

Two simplified solutions that rely on retrievable real-world resources are proposed to alleviate the multidisciplinary challenges of implementing fully the two logic-driven algorithms. By leveraging previously accumulated task-specific data and model resources and task-related large AI models, the simplified approaches substantially reduce implementation barriers while preserving the essential rationale of abductive reasoning and Boolean algebra. In practice, this means that MIATTs can be generated using readily available predictive models or large pretrained AI models, and their quality can be assessed using simplified Boolean formulations. These solutions not only improve feasibility but also align EL-MIATTs with current trends in AI, where weak supervision and large AI model-based inference are becoming dominant.

The physical interpretations of MIATTs and their assessment indicators underscore the fact that in many real-world domains—such as natural language understanding, medical diagnosis, or decision-making under uncertainty—supervision signals are plentiful but inherently imperfect. MIATTs capture this practical reality by framing supervision not as a single accurate true target but as a noisy yet informative distributional approximation. Correspondingly, assessment indicators such as partial representation and redundancy serve as interpretable measures that balance informativeness against diversity (overlap or noise), thereby translating abstract logical constructs into physically meaningful quantities. This duality between approximation and assessment demonstrates the practicality of MIATTs: they not only provide a conceptual lens for understanding weak supervision but also offer an operational framework for systematically managing its quality in practice.

These advances for scientifically generating and assessing MIATTs eventually guarantee the practical value of MIATTs, which lies in their ability to transform the challenges of imperfect supervision into opportunities for robust learning: By aggregating multiple approximate targets, MIATTs reduce the brittleness of relying on a single label, offering safeguards against noise, biases, or errors; MIATTs provide distributional supervision, enabling learning systems to approximate the underlying truth by reasoning over partial signals rather than enforcing rigid correctness; The diversity of MIATTs naturally complements the multi-perspective outputs of large AI models, making them particularly suitable for weakly supervised or open-ended tasks; Domains such as healthcare, knowledge graph construction, and natural language understanding can directly benefit from MIATTs, especially where exact ground truth is elusive. Thus, MIATTs serve as a bridge between idealized ground-truth–based learning and realistic weak supervision, providing a principled yet practical foundation for future EL-MIATTs development.

8. Conclusion, Limitations, and Future Work

In this paper, bridging theory and practice in implementing EL-MIATTs, we proposed two complementary logic-driven algorithms as the foundation of EL-MIATTs: an abductive reasoning–driven algorithm for MIATTs generation and a Boolean algebra–driven algorithm for MIATTs assessment.

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Together, these methods provide a principled framework for transforming uncertain, noisy, or inaccurate supervision signals into logically consistent approximations of the underlying true target and for systematically evaluating their quality through interpretable indicators. To enhance feasibility in real-world applications, we further proposed simplified solutions that leverage retrievable resources, including task-specific predictive models and large AI models, thereby lowering implementation barriers while retaining the essential abductive and Boolean rationale. The physical interpretations of MIATTs and their assessment indicators further highlight their practical value as both conceptual and operational tools for managing imperfect supervision.

Nevertheless, several limitations remain. First, the current abductive reasoning–driven algorithm relies on the availability and quality of external knowledge bases, which may introduce biases or incompleteness. Second, the Boolean algebra–driven assessment, while principled, assumes a tractable representation of semantic facts; scalability to high-dimensional or unstructured domains remains a challenge. Third, the simplified solutions, though more practical, inevitably trade off some of the theoretical guarantees of the full logic-driven framework. These limitations suggest that further refinement is needed to balance logical rigor with scalability and efficiency.

Future research can proceed in several promising directions. One is to integrate probabilistic reasoning and differentiable logic to enhance the robustness of MIATTs generation beyond purely symbolic abduction. Another is to extend the Boolean algebra–driven assessment toward hybrid measures that combine logical consistency with statistical learning metrics. Moreover, leveraging large AI models [12] not only as retrievable resources but also as active reasoning agents offers exciting potential for scaling EL-MIATTs to open-domain tasks. Finally, systematic empirical studies across diverse application domains, such as healthcare, knowledge graph construction, and natural language understanding, are essential to validate the effectiveness and generalizability of the EL-MIATTs framework.

Taken together, MIATTs embody a practical response to the challenge of imperfect supervision. By offering a structured yet flexible framework for generating and assessing MIATTs, they transform the limitations of an undefinable true target into opportunities for robust, distributional evaluation and learning. With further refinement and empirical validation, the MIATTs-based framework of EL-MIATTs [1] have the potential to become a foundational paradigm for machine learning in domains where the true target remains uncertain or inaccessible.

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