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Peer Review

Review of: "Scrambling in Charged Hairy Black Holes and the Kasner Interior"

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The authors of this paper set an interesting goal of probing the black hole interior from the boundary theory by the AdS/CFT correspondence. They use the numerical solution of AdS hairy black holes in the “Einstein-Maxwell-Scalars” model to calculate the Lyapunov exponent that characterizes the chaotic behavior of the boundary theory. They study various numerical dependences of the Lyapunov exponent on the bulk parameters, such as the scalar coupling, the axion coupling, and in particular, the Kasner exponents that characterize the black hole interior. The following are my questions and comments that may help to make the paper clearer:

(a) The calculation in the paper is not really about OTOCs but rather the mutual information, which I think are two different probes of chaotic behavior. The authors should be clear about whether these two ways of defining the Lyapunov exponent are the same or not. Moreover, in the calculation of the mutual information, the authors should specify what the subregions A and B are.

(b) Since there is no shock wave in the ansatz of the black hole metric (equation (9)), it is not clear how the shock wave parameters enter the numerical study. In particular, in the calculation of mutual information, it would be nicer to have some picture of how the minimal surface looks in the shock wave background.

(c) Related to the profile of the minimal surface, it is not clear to me how the turning point (or the critical radius mentioned in the paper) is obtained. In the paper, it mentions they use the function FindRoot in Mathematica. However, the background metric is only obtained numerically using the shooting method; I don’t know how the analytical expression for the critical radius, like the one in the background without scalar deformation (equation (60)), is possible. Also, it is not convincing to say there is only one turning point by looking at the numerical plots (Fig. 3), since there are many other

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parameter regions that may give multiple turning points, and they can dominate the contribution to the area of the minimal surface.

(d) In the numerical studies of the Lyapunov and the Kasner coefficients, it is not clear to me why the results depend only on the ratio of scalar deformation and temperature. As far as I know, the black hole could still have scalar hair when the temperature is smaller than some critical value (that is what happens in holographic superconductors). So, it should have some qualitative changes in this phase transition. Also, in the superconducting phase, even though the scalar deformation is zero, it is not simply the aRN black hole as studied in the paper.

(e) There are some errors in the captions of the figures in section III. For example, Fig. 7, Fig. 8, and Fig. 9 have the same captions, but they don't plot the same thing. Also, in these numerical studies, it would be nicer to mention which equations are used to obtain them.

(f) It may make the main result in the paper more interesting (in sections III and IV) by including more discussion on physics. For this purpose, it may be more interesting to represent the dependences of the Lyapunov and the Kasner coefficients on the physically measurable quantities, such as boundary temperature, charge density, scalar condensate, etc., rather than the axion coupling, scalar coupling, or scalar deformation as done in the paper.

(g) Another suggestion is that it may be more interesting to study the relation between chaotic behavior and geometry inside a black hole in a simpler model of a holographic superconductor (consider the action with the terms in equations (1) and (2) only). The phase transition in a holographic superconductor may have interesting effects on chaotic behavior, and the corresponding change in the geometry inside the black hole can also be studied.

Declarations

Potential competing interests: No potential competing interests to declare.