Qeios

Peer Review

Review of: "A Prime-Based Continued Fraction Constant ρ = [2; 3, 5, 7, 11, ...]: Convergence, Prime-Indexed Engel/Egyptian Expansions, and Conditional Estimates Involving the Riemann Hypothesis"

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This manuscript studies the continued fraction constant

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whose partial quotients are the prime numbers. The author provides a systematic theoretical analysis that extends previous computational work on prime-based continued fractions. The main contributions include:

1. Establishing explicit bounds for denominator growth via primorial products and the Chebyshev function.
2. Deriving the asymptotic relation $\log Q_n \sim p_{n+1}$ with a precise error budget.
3. Proving that the irrationality exponent of ρ is exactly 2.
4. Exploring Engel and Egyptian expansions indexed by primes, offering a novel comparative framework.

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The treatment of the Riemann Hypothesis is careful and appropriate. Rather than claiming new equivalences, the manuscript relies only on well-known conditional bounds for the Chebyshev function, presented transparently as conditional refinements. The revised title and the section heading "Conditional Estimates Involving the Riemann Hypothesis" ensure clarity and avoid any misleading interpretation.

The paper is clearly written and well-structured, beginning with motivation, moving through background results, theoretical analysis, numerical validation, and discussion. Figures and tables are plentiful and well-prepared, effectively supporting the claims. Importantly, the author commits to providing an anonymized reproducibility bundle with scripts and data, which aligns with Qeios' emphasis on openness and transparency.

Strengths

- A rigorous mathematical analysis combined with empirical validation.
- Careful framing of results under RH, avoiding overstatement.
- Clear structure and accessible exposition.
- Availability of reproducibility resources.

Recommendation

This work is original, well-executed, and fits the scope of Qeios as an exploratory but rigorous contribution in number theory. It improves upon earlier literature by providing both theoretical results and reproducible computations. The presentation is clear, the mathematics is correct, and the narrative is well-balanced.

Decision: Accept for publication without revisions.

Declarations

Potential competing interests: No potential competing interests to declare.