Review of: "Archetypal Resonances Between Realms: The Fractal Interplay of Chaos and Order"

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Archetypal Resonances Between Realms:

The Fractal Interplay of Chaos and Order

Comments to the editors:

The topic of the paper is surely of great interest to a wider audience, and it is absolutely recommendable for publication in your journal after some improvements.
BUT: The outcome does not really keep the promises formulated in the abstract. The style of the paper is sort of a lengthy popularization of science, where the interrelations of psychologist C.G. Jung and physicist W. Pauli deliver the basic motivation. As a summary, one can say that the paper tries a revival of the early enlightenment's "more geometrico" point of view, and its holistic view of the universe. Nevertheless, the description of mathematical facts in the first 6 pages contains half-truths and partly makes confusing statements. There is a lack of precision and proper definitions. The content seems to focus on the "realms of order and chaos" as undefined concepts of common speech, but the essence of the paper is the contrast between deterministic processes and randomization. Science popularization addresses non-specialists. This means that authors are responsible for proper and correct translations of specialists' scientific language to "readable texts." I try to clarify these critics in the comments to the authors, which could be of interest also for you.

A pity that the styles do not cope with mathematical formulas and Greek letters and symbols! This makes reading the following text not convenient.

Comments to the authors:

Overall estimation:

The topic treated in the paper is surely of great interest to many readers, and it could even be part of a general education course for non-specialists.

I interpret the paper as a revival of the "more geometrico point of view" of the early enlightenment and its holistic approach to the world (see e.g., B. Spinoza).

But, in my opinion, it contains misunderstandings, half-truths, and a lack of precision in its mathematical part, at least in the first pages. By connecting human psychology with particle physics and thus forming “the real universe,” the authors should consider the fact of the relatively short existence of humanity in relation to the universe. To avoid the “smell” of clustering illusion and apophenia[1], one could emphasize that archetypes and “images of the world” are only metaphors, and that, according to psychology and philosophy (c.f. E. Kant’s “What can I know/understand?”), we can only construct images/abstractions/models of what actually is supposed to be “reality.”

Proposals for improvements

Already at the very beginning, when starting with “The fractal interplay of chaos and order,” one could/should at least give an explanation of what you mean by “order” and “chaos.” Are these concepts meant as colloquialisms or as mathematical objects? “Order” can be seen as a matter of scale, occurs differently at different levels and scopes, and mathematically seen, can be based on deterministic processes, Euclidean and non-Euclidean symmetries and functions, while “chaos” has to do with randomized processes and can be linked to “order” via statistics and stochastics. Some clarification occurs in your paper not before page 9.

Among the many ideas and references formulated at the beginning and aiming for the pages > 7, one could explicitly call them “motivation for …”, e.g., for a holistic view of the universe, and/or for developing a sort of “meta-viewpoint” for C.G. Jung’s “collective unconsciousness” and particle entanglement in actual physics. By the way, more concise language would be clearer for your reader. Not everything forms a “realm”; often “domain” or even “field” would be big enough and could diminish the agglomeration of repetitions.

The key properties of fractals are not perceptible formulated on the first pages. Here a hint like “see pages 9” could help. Note that the fractal objects shown in table 1 stem from a pre-computerized time and are, in their simplicity, already classical objects. One should also keep these objects apart from Julia and Mandelbrot sets. The first is due to a sort of bi-, tri-, n-furcation process leading to a specific real dimension in the limit, i.e., one has to perform infinitely (!) many steps to call a fractal a fractal (see Hausdorff-Besicovich)! In addition, they are *deterministic* processes, whereby also the best computers only deliver a finite set of steps, i.e., a “pre-fractal” which needs *infinitely* many steps to be called a “fractal”. Thus, they are, from the mathematical point of view, not at all directly connected with “chaos”. Furthermore, I miss the key concept “model” at the very beginning, as it is basic for all intellectual treatment of nature/reality versus mathematics, and within mathematics, too. The concept “model” occurs in your paper too late. Deterministic processes, even if they are handled by computers calculating with reals of 15 and more decimals, cannot perform infinitely many steps, and attractors found via convergence criteria using a finite epsilon have to be mapped to a discrete pixel on the screen. One sees a metaphor of what is meant in the ideal Platonic world. And even though Mandelbrot and Julia sets look at some sections like chaos, they are still deterministic!

When speaking of archetypes, one would expect some more examples besides the only Ouroboros (or “Uroboros”; note that “Oroboros” is a misprint). Would not “symmetry,” graphically represented by the equilateral triangle, the square, and perhaps the “yin and yang” symbol (the latter, in some graphic versions, even related to the Ouroboros), not be worthy to mention, too? Especially as “symmetry” occurs also in physics?

Page 3, “From ancient symbols…”: Replace “quantum mechanics” by “quantum physics” throughout the paper.
Last 2 lines: I do not agree with your statements! Fractals, like the von-Koch snowflake, the Menger sponge, the Hilbert snakes, and many more, were discovered and described during the first third of the 20th century, without computers! These objects can, as an idea, be seen as rough models of parts of natural objects. This idea, formulated by Mandelbrot in “The Fractal Geometry of Nature” with many examples, deals *with finite steps of fractals which model abstractions of some parts of natural objects*. And without randomization, they cannot model clouds!

Page 4, para.1: The Fibonacci sequence is described in 1202 by Fibonacci; this is in *medieval* times, not in antiquity! The sequence itself has nothing to do with fractals!!
It is not true that “it preserves part/whole relationships,” as e.g., the ratios 3:5 and 5:8 are not equal! Yes, the limit ratio for phi, delivering the golden ratio, is a celebrated fact of the Fibonacci sequence. It connects the ancient Greek “exhaustion method” (best represented by a *continuous fraction*) applied to proportions within the regular pentagon. It delivers the ratio Major:Whole = minor:Major, which leads to the well-known quadratic equation with phi, -phi^-1 as solutions. For phyllotaxis, the first ratios, e.g., 2:3, 3:5,.., are good models. Note that only parts of the “golden spiral,” which models arrangements of seeds in sunflowers, etc., are good models. Note that a mathematical golden spiral, or e.g., a logarithmic spiral, always winds infinitely often around its pole and extends to infinity. No natural object has this behaviour!

Page 5, para. 3: “Fractal geometry *is* a holistic approach…”. A statement without proof! “Traditional geometries…” are neither only linear, nor only Euclidean! Note that fractals, without randomizing them, cannot model coastlines, etc. They are still deterministic! The term “irregular pattern” assumes already the existence of a “pattern.” This connects the topic treated to actual AI research based on “very big data,” where one tries to find patterns and relations/coincidences. Once again, the infinitely many steps of, say, the von Koch polygon applied to a piece of a coastline would end up in an area where waves and wavelets act and deliver a “dynamic coast.” For this area of water and sand, one can use the “neither 1- nor 2-dimensional limit object" as a metaphor.

Page 5, para. 4: “At the heart of … notation of self-similarity.” There is no necessity to restrict fractals to Euclidean planes and spaces. One can look at sutures on the cranium and find spherical von Koch polygons as metaphors for them; the pictures of Julia sets model feedback loops in the Gauss plane, which has the structure of a Möbius plane and models the 1-dimensional (!) line of complex numbers. If fractals absolutely need a heart, then one should put it at the feedback loop property.

Pare 5, last but one para.: See comments to “self-similarity” above. When stating “…each mirroring the grandeur of a whole set…” in bloomy language, you neglect the fact that the main body of the Mandelbrot set is connected with the *cardioid* (i.e., the trace of a boundary point of a circle, which rolls on a congruent circle), while the bases of dendrites are circular regions.

Page 5, last para.: “This set bridging real and imaginary numbers…” This sentence can lead to misunderstandings! The real numbers come via the interpretation of complex numbers as 2-vectors in the real Gauss plane. This has nothing to do with the Mandelbrot set!

Page 9, para. 1: “…the Mandelbrot set emerges from repeating mathematical functions.” This can be misunderstood. The functions are always fixed, as long as their coefficients are not randomly distorted. What is repeated is the calculation of the function values. And here you can already “see” what happens if to one abscissa value there are more than one, e.g., 2, ordinate values: you get a bifurcation process.

Page 9, last para: Here, by mixing two totally different procedures, you receive a figure 4 that cannot be understood. Please delete Figure 4! Throwing a coin is never a bifurcation process! And it is never a fractal, as claimed at page 12! It has to do with probability theory, and, with “big data” of casts, one can structure this set by formulating that the chance to get a number or image is 50:50. From one coin, there is no “doubling sequence” of coins. When you throw the T-coin twice, you will not get one T-coin and one H-coin. For visualizing a bifurcation process, you must use a correct example!

Page 10, para. 1: An isosceles triangle with rows of obstacles in an order like the numbers in the Pascal triangle, where a sphere rolls down from top to bottom, delivers, after very many attempts, the Gaussian distribution as a result, thus extracting the order of a stochastic process. The discrete values at each row are indeed the binomial coefficients of the Pascal triangle.

Page 10, Fig. 5: Of course, there are many number sequences and number series one can find in a Pascal triangle, but without adding ratios, the value phi does not appear. Therefore, putting the letter phi without further explanation over the right Pascal triangle does not make sense.

Page 11, para. 2: Yes, the universe operates on a mix of laws and randomness. But this trivial fact is independent of coin flips and fractals, isn’t it? The last sentence hits the point when using the word “symbolizing”. Here is the first time I feel sure that you really deal with metaphors, especially when letting order and chaos perform a harmonious dance.

Page 12, “Fractals: Bridging the gap…”: This can be misunderstood! Fractals ARE deterministic processes. One can introduce a random modification of the for a fractal fixed feedback loops and therewith get a new class/”realm” of objects, named “randomized fractals”. Therefore, the headline could be “Randomized Fractals Filling the Gap Between…”. When the authors reproduce a table of another paper, they should explain the content more detailedly. Table 1 shows classical examples of fractals together with their Hausdorff-Besicovich dimensions. It would be helpful to the reader if he finds how this dimension is calculated. In addition, the size of randomization influences the random Hausdorff dimension! Therefore, the golden mean will occur only for very special randomizations! In fact, one can modify this dimension to a larger extent.

For connecting randomization with fractals and other objects, two well-known examples could be of interest:

a) Sierpinski curtains: Take three fixed points A, B, C forming a triangle, give A the value red, B the value green, and C the value blue. Choose a point X in the plane of A,BC, and a randomly chosen color of RGB. Connect X with the triangle vertex indicated by the chosen color, then the midpoint of this segment delivers the new point X'.. Repeat over and over again, and you will get the Sierpinski gasket as a “limit figure” (if one could perform infinitely many repetitions).

Of course, one could modify this construction further by replacing the midpoint of the segment with points with randomly distorted ratios R(X',X,A) or R(X',X,B) or R(X',X,C).

The point X need not even lie in the plane of ABC, and this procedure can be applied to more than three colored fixed points.

b) Random construction of Pi: Let it rain onto the unit square. Count all raindrops and those, the distance of which to a fixed vertex of the square is less than or equal to 1. Then the ratio of these drop numbers approximates Pi.

These two examples show that fractals and the golden mean are not the only interesting objects in our universe.

Final remark: I am in fact grateful to the authors for their paper. It made me reflect over many things that I would not have noticed without the stimulus of the paper.

[1] Peter Brugger: *From Haunted Brain To Haunted Science. A Cognitive Neuroscience View of Paranormal and Pseudoscientific Thought.* In: J. Houran, R. Lange (Hrsg.): *Hauntings and Poltergeists. Multidisciplinary Perspectives.* McFarland & Co., Jefferson 2001, ISBN 0-7864-0984-3