Review of: "Can the electromagnetic fields form tensors if D = ϵE and H = B/μ?"

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The author consider a material where, in its frame of reference, $\mathbf{E} = \mathbf{D}/\varepsilon$ and $\mathbf{B} = \mu\mathbf{H}$.

The author makes the mistake of imagining that there is a material for which
this relationship is valid in any frame of reference.

If such a material existed, it would not be possible to have
both $-\mathbf{E}, \mathbf{B}$ components of a tensor and $\mathbf{H}, \mathbf{D}$ components of a tensor,
and then his conclusion would be valid.

Now, since $-\mathbf{E}, \mathbf{B}$ and $\mathbf{H}, \mathbf{D}$ are tensors - and this is,
in my opinion, the most fundamental assumption of electromagnetism - we must,
by contraposition deduce that there is no material for which
$\mathbf{E} = \mathbf{D}/\varepsilon$ and $\mathbf{B} = \mu\mathbf{H}$ in any reference frame.

If, for simplicity's sake, we take only reference frames
whose metric tensor is $(+1, -1, -1, -1)$,
we have, in the linear regime of matter $F_{ij} = (1/2)a_{ij}{}^{kl}\mathrm{G}_{kl}$

with

$$\begin{aligned}
F_{01} &= -F_{10} = -E_x \\
F_{02} &= -F_{20} = -E_y \\
F_{03} &= -F_{20} = -E_z \\
F_{12} &= -F_{21} = B_x \\
F_{31} &= -F_{13} = B_y \\
F_{12} &= -F_{21} = B_z
\end{aligned}$$

$$\begin{aligned}
\mathrm{G}_{01} &= -\mathrm{G}_{10} = H_x \\
\mathrm{G}_{02} &= -\mathrm{G}_{20} = H_y \\
\mathrm{G}_{03} &= -\mathrm{G}_{20} = H_z \\
\mathrm{G}_{12} &= -\mathrm{G}_{21} = D_x \\
\mathrm{G}_{31} &= -\mathrm{G}_{13} = D_y \\
\mathrm{G}_{12} &= -\mathrm{G}_{21} = D_z
\end{aligned}$$

In a vacuum, non-zero terms are

$$a^{0123} = 1$$
$$a^{0132} = -1$$
$$a^{0231} = 1$$
$$a^{0213} = -1$$
$$a^{0312} = 1$$
$$a^{0321} = -1$$
$$a^{1023} = 1$$
$$a^{1032} = -1$$
$$a^{2031} = 1$$
$$a^{2013} = -1$$
$$a^{3012} = 1$$
$$a^{3021} = -1$$
$$a^{2301} = 1$$
$$a^{3201} = -1$$
$$a^{3102} = 1$$
$$a^{1302} = -1$$
$$a^{1203} = 1$$
$$a^{2103} = -1$$
$$a^{2310} = 1$$
$$a^{3210} = -1$$
$$a^{3120} = 1$$
$$a^{1320} = -1$$
$$a^{1230} = 1$$
$$a^{2130} = -1$$

$$a_{01}{}^{23} = -1$$
$$a_{01}{}^{32} = 1$$
$$a_{02}{}^{31} = -1$$
$$a_{02}{}^{13} = 1$$
$$a_{03}{}^{12} = -1$$
$$a_{03}{}^{21} = 1$$
$$a_{10}{}^{23} = -1$$
$$a_{10}{}^{32} = 1$$
$$a_{20}{}^{31} = -1$$
$$a_{20}{}^{13} = 1$$
$$a_{30}{}^{12} = -1$$
$$a_{30}{}^{21} = 1$$
$$a_{23}{}^{01} = 1$$
$$a_{32}{}^{01} = -1$$
$$a_{31}{}^{02} = 1$$
$$a_{13}{}^{02} = -1$$
$$a_{12}{}^{03} = 1$$
$$a_{21}{}^{03} = -1$$
$$a_{23}{}^{10} = 1$$
$$a_{32}{}^{10} = -1$$
$$a_{31}{}^{20} = 1$$
$$a_{13}{}^{20} = -1$$
$$a_{12}{}^{30} = 1$$
$$a_{21}{}^{30} = -1$$

In the frame of reference of the material considered by the author, the constitutive tensor components are

$$a_{01}{}^{23} = -\varepsilon$$
$$a_{01}{}^{32} = \varepsilon$$
$$a_{02}{}^{31} = -\varepsilon$$
$$a_{02}{}^{13} = \varepsilon$$
$$a_{03}{}^{12} = -\varepsilon$$
$$a_{03}{}^{21} = \varepsilon$$
$$a_{10}{}^{23} = -\varepsilon$$
$$a_{10}{}^{32} = \varepsilon$$
$$a_{20}{}^{31} = -\varepsilon$$
$$a_{20}{}^{13} = \varepsilon$$
$$a_{30}{}^{12} = -\varepsilon$$
$$a_{30}{}^{21} = \varepsilon$$
$$a_{23}{}^{01} = \mu$$
$$a_{32}{}^{01} = -\mu$$
$$a_{31}{}^{02} = \mu$$
$$a_{13}{}^{02} = -\mu$$
$$a_{12}{}^{03} = \mu$$
$$a_{21}{}^{03} = -\mu$$
$$a_{23}{}^{10} = \mu$$
$$a_{32}{}^{10} = -\mu$$
$$a_{31}{}^{20} = \mu$$
$$a_{13}{}^{20} = -\mu$$
$$a_{12}{}^{30} = \mu$$
$$a_{21}{}^{30} = -\mu$$

Lorentz transform with a speed along z axis are

$$a'^0 = \alpha a^0 - \beta a^3$$
$$a'^3 = -\beta a^0 + \alpha a^3$$
$$a_0' = \alpha a_0 + \beta a_3$$
$$a_3' = \beta a_0 + \alpha a_3$$

with

$$\alpha = 1/\sqrt{1 - v^2}$$
$$\beta = v/\sqrt{1 - v^2}$$
$$\alpha^2 - \beta^2 = 1$$

It is easy to obtain the constitutive equations of the material at a speed v along z azis:

$$E'_x = (1/\varepsilon')D'_x + \xi' H'_y$$
$$E'_y = (1/\varepsilon')D'_y - \xi' H'_x$$
$$E'_z = (1/\varepsilon)D'_z$$
$$B'_x = \mu' H'_x + \xi' D'_y$$
$$B'_y = \mu' H'_y - \xi' D'_x$$
$$B'_z = \mu H'_z$$

with

$$1/\varepsilon' = \alpha^2/\varepsilon - \beta^2\mu$$
$$\mu' = \alpha^2\mu - \beta^2/\varepsilon$$
$$\xi' = \alpha\beta(1/\varepsilon - \mu)$$

These expressions can be obtained either from field component transformations or from tensor components transformation.