Description of ΩΩ, Ω(ccc)Ω(ccc), and Ω(bbb)Ω(bbb) Dibaryon States in Terms of a First-Order Hexaquark Mass Formula

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Funding: The author(s) received no specific funding for this work.

Potential competing interests: The author(s) declared that no potential competing interests exist.

Abstract

Recently discovered tetraquark and pentaquark structures have renewed interest in multiquark states. In the spirit of these results, the masses of postulated hexaquark structures using a first-order mass formula are determined. The hexaquarks model is applied to ΩΩ, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryon states. The first-order model suggests that the ΩΩ, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ hexaquarks have masses of 3.53, 9.78, and 28.8 GeV/c^2, respectively.

Introduction

1.1 Near Term History

The possibility that hadrons could exist with structures beyond conventional qq or qqq quark configurations was noted by Gell-Mann[1]. Previous searches of multiquark systems have appeared, but most were later invalidated following additional analysis[2]. However, a recent sample of experimental work suggests the existence of both tetraquark[3-5] and pentaquark[6] structures. These studies open the possibility of additional exotic quark structures[7].

Given the existence of tetraquark and pentaquark structures, it is possible that more complex quark configurations exist. These configurations would include hexaquarks and these structures are considered in this paper. Although somewhat speculative, these states are of theoretical interest and merit attention.

Although no hexaquark states have been confirmed, experimental effort is suggestive of possible hexaquark configurations[8-10]. These possible hexaquark states arise from interpretation of two baryon-baryon resonances.

The first possible hexaquark candidate was observed in exclusive and kinematically complete measurements[8,9] of the fusion reactions p n → d π^0 π^0 and p n → d π^+ π^- that revealed a narrow resonance-like structure in the total cross section at a mass ≈ 2380 MeV/c^2 with a width ≈ 70 MeV/c^2. Additional evidence for this structure has been found in the p n → p p π^0 π^- reaction. This possible state has been interpreted as a dibaryon hexaquark state[8,9].

Although not confirmed as a second candidate hexaquark state, the BESIII Collaboration[10] suggests the existence of a possible p p-bar hexaquark resonance structure by studying the reaction $J/\Psi \rightarrow \gamma\ \eta'\ \pi^+\ \pi^-$. Two models were utilized to characterize the $\eta'\ \pi^+\ \pi^-$ line shape around 1.85 GeV/c^2. One model explicitly incorporates the opening of a decay threshold in the mass spectrum, and another uses a coherent sum of two resonant amplitudes. Both fits reproduce the data with good agreement, and suggest the existence of either a broad state around 1.85 GeV/c^2 with strong couplings to the p p-bar final states or a narrow state just below the p p-bar mass threshold. The models suggest either a p p-bar molecular state or bound state with greater than 7σ significance.

Previous theoretical studies investigated a limited set of hexaquark candidates based primarily on combinations of baryon structures. Theoretical studies include a (1) chiral constituent quark model[11], (2) baryon-antibaryon resonance model using a chromomagnetic interaction[12], (3) dispersion relation technique[13], (4) string model using the strong coupling regime of quantum chromodynamics[14], and (5) coupled channels resonating group method[15]. The theoretical calculations were limited in scope to a small set of quark configurations in these studies (i.e., uuddsQ systems (Q = c or b)[11], q q q q-bar q-bar q-bar systems[12], dibaryon resonances based on u, d, and s quarks[13], systems based on a light quark (q) with unit mass (m) and a heavy quark (Q) with a mass of m, 2m, 3m, 4m, and 5m with q^3Q^3 and $\bar{q}^3\ Q^3$ configurations[14], and only u and d quark systems[15]).

1.2 Current Study

In this paper, selected ΩΩ and analogue hexaquark structures (i.e., ΩΩ, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$) are investigated using a first-order mass formula that was previously used to examine tetraquark[16-26], pentaquark[27], and hexaquark and other exotic quark configurations[28]. The model's extension to the ΩΩ hexaquark and analogue structures is a logical extension of previous efforts. In particular, the ΩΩ hexaquark is assumed to be comprised of two weakly coupled Ω baryon structures.

Liu and Geng[29] recently proposed that one charmonium exchange is responsible for the formation of the $\Omega_{ccc}\Omega_{ccc}$ dibaryon predicted by one boson exchange calculations. This same approach can be extended to the strange and bottom analogues that suggest the existence of the ΩΩ and $\Omega_{bbb}\Omega_{bbb}$ dibaryons. Ref. 29 suggests that the Coulomb interaction may break up the $\Omega_{ccc}\Omega_{ccc}$ pair but not the $\Omega_{bbb}\Omega_{bbb}$ and ΩΩ dibaryons. In addition, Liu and Geng[29] calculated the binding energies for the dibaryon clusters and determined the energy to be less than about 10 MeV. The binding energy is much smaller that the inherent dibaryon mass.

2.0 Model Formulation

Zel'dovich and Sakharov[30,31] proposed a semiempirical mass formula that provides a prediction of mesons and baryons in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest 1S state. These mass formulas are used as the basis for deriving a first-order hexaquark mass formula. In particular, the model utilized in this paper assumes the hexaquark is partitioned into two baryon clusters with the interaction between the clusters providing a minimal contribution to the mass. In addition, zero angular momentum is assumed to exist between the clusters.

The baryon (b) mass (M) formula of Refs. 30 and 31 is:

$$M_b = \delta_b + m_1 + m_2 + m_3 + Z(1a)$$

$$Z = \frac{b_b}{3}\left[\frac{m_0^2}{m_1 m_2}\sigma_1 \cdot \sigma_2 + \frac{m_0^2}{m_1 m_3}\sigma_1 \cdot \sigma_3 + \frac{m_0^2}{m_2 m_3}\sigma_2 \cdot \sigma_3\right](1b)$$

where the m_i labels the three baryon quarks (i = 1, 2, and 3) and δ_b and b_b are 230 MeV and 615 MeV, respectively[31]. In Eq. (1), m_1 , m_2, and m_3 are the mass of the first, second, and third quark comprising the baryon, m_o is the average mass of first generation quarks[32,33], and the σ_i (i = 1, 2, and 3) are the spin vectors for the quarks incorporated into the baryon.

The last term in Eq. 1 represents the spin-spin interaction of the quarks and $q_i{\cdot}\sigma_j$ is the scalar product of the quark spin vectors. $\sigma_i{\cdot}\sigma_j$ has the value -3/4 and +1/4 for pseudoscalar and vector configurations, respectively.

In formulating the hexaquark mass formula, effective quark masses provided by Griffiths[32] are utilized. The effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively. These masses are utilized in Eq. 1.

These six quarks are arranged in three generations: [d(-1/3 e), u(+2/3 e)], [s(-1/3 e), c(+2/3 e)], and [b(-1/3 e), t(+2/3 e)][33]. The three generations are specified by the square brackets and the quark charges (in terms of the unit charge e) are given within parentheses.

The first-order mass formula used in this paper partitions the hexaquark into two baryon clusters. In this investigation, each baryon cluster in the hexaquark (ΩΩ, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$) is limited to a single flavor (i.e., sss, ccc, and bbb) that results in sss+sss, ccc+ccc, and bbb+bbb hexaquark configurations. Considering the spin-parity value of the $3/2^+$ of the Ω^- baryon[33], vector plus vector (Eq. 2) clusters are used to describe the hexaquark states.

The weak coupling structure is incorporated to minimize model complexity, which is consistent with an initial first-order formulation. In addition, the hexaquark mass formula is assumed to have the following form for the vector ($\mathbf{M_{bv}}$) coupling:

$$\mathbf{M = M_{bv}(1) + M_{bv}(2) + \Phi\ (2)}$$

where Φ defines the interaction between the baryon clusters, and $\mathbf{M_{bv}(i)}$ represents the use of vector coupling in Eq. 1 for the ith cluster (i=1,2). Within the scope of this mass formula, the baryon-baryon cluster interaction is assumed to be sufficiently small, relative to the cluster masses, to be ignored. Accordingly, Eq. 2 represents a quasimolecular six quark system whose basic character is a weakly bound baryon-baryon system.

The mass relationships of Eqs. 1 and 2 do not predict the total angular momentum of the final hexaquark state, but do permit primitive spin coupling to be specified for the individual baryon clusters. In addition, the angular momentum between the clusters is assumed to be zero. Specific angular momentum assignments based on the first-order mass formula for the hexaquark states are provided in subsequent discussion.

3.0 Weak Coupling Approximation

The weak coupling approximation is based on fundamental quantum chromodynamics (QCD) arguments[34,35]. Recent one boson exchange[29] as well as lattice gauge calculations provide numerical simulation results that support these QCD arguments[34,35].

3.1 Fundamental Quantum Chromodynamics Arguments

The quark charges are related to the number of colors (N_c) incorporated into the QCD formulation[34,35]. For example, the first generation quark charges within SU(N_c) are:

$$Q_d = \frac{1}{2}\left[\frac{1}{N_c} - 1\right] \quad (3)$$

and

$$Q_u = \frac{1}{2}\left[\frac{1}{N_c} + 1\right] \quad (4)$$

For conventional QCD using 3 colors, the expected d and u electric charges are obtained. The importance of QCD expansions involving $1/N_c$ is outlined in this section to illustrate the weak coupling assumption.

A key assumption of the first-order mass formula of Eq. 2 is weak coupling between the two clusters. In particular, the model utilized in this paper assumes the hexaquark is partitioned into two clusters with the interaction between the clusters providing a minimal contribution to the hexaquark mass. Within the scope of this mass formula, the cluster-cluster interaction is assumed in Eqs. 1 and 2 to be weak, and sufficiently small to be ignored.

This assumption is justified because QCD can be investigated as an expansion in $1/N_c$[36,37]. The large N_c limit reduces to a field theory of weakly interacting meson-like objects. The physical situation with $N_c = 3$ retains many of the characteristics of the $N_c \rightarrow \infty$ limit, and further justifies the weak coupling approximation.

The $1/N_c$ expansion[36,37] is well accepted in elementary particle physics and leads to the Okubo-Zweig-Iizuka (OZI) rule[38-40] and the Skyrme model[41,42]. In fact, in the $1/N_c$ expansion, QCD is reduced to a weakly interacting meson theory, and the meson-meson and associated cluster-cluster interactions are regarded to be small[36,37]. This situation is also a characteristic of Eqs. 1 and 2.

3.2 One Boson Exchange Model

Liu and Geng[29] recently utilized the extended one boson exchange (OBE) model to study baryon-baryon interactions containing heavy quarks (i.e., bottom, charm, and strange). The results of the OBE calculations support the weak coupling approach utilized in the first-order mass formula approach of this paper. In particular, the ΩΩ, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ binding energies were 1.6, 5.1 – 5.8, and 5.7 MeV, respectively. These binding energies are small relative to the masses of the baryon-baryon systems (See Table 1). The OBE calculations of Liu and Geng[29] assign a J^{π} value of 0^+ to these dibaryons.

3.3 Lattice Gauge Calculations

Green et al.[43] add support to the comments summarized in Sections 3.1 and 3.2. Ref. 43 presented the first study of baryon-baryon interactions in the continuum limit of lattice QCD. Green et al.[43] determined the binding energy of the H dibaryon. The H dibaryon is a scalar six-quark state with flavor content of uuddss. The calculation is performed at six values of the lattice spacing, using improved Wilson fermions at the SU(3)-symmetric point with $m_{\pi} = m_K \approx 420$ MeV/c^2. Ref. 43 estimated the binding energy in the continuum limit to be 4.56 MeV.

An additional QCD calculation predicted that the $\Omega_{ccc}\Omega_{ccc}$ dibaryon had a binding energy of 5.7 MeV[44]. Ref 44 noted that this indicates there is a strong attractive interaction between the $\Omega_{ccc}\Omega_{ccc}$ pair. The term strong is an important consideration in binding energy calculations. However, within the context of the first-order mass formula, the 5.7 MeV is insignificant in comparison to the $\Omega_{ccc}\Omega_{ccc}$ mass noted in Table 1. This further supports neglecting the cluster-cluster interaction Φ in comparison to the baryon mass (See Eq. 2). For completeness, it should be noted that a quark model study[45] did not support the existence of a $\Omega_{ccc}\Omega_{ccc}$ dibaryon. Ref. 45 suggests a negligible value for the cluster-cluster interaction Φ.

4.0 Results and Discussion

The model results address the description of ΩΩ, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryon states in terms of a first-order hexaquark mass formula. Ω, Ω_{ccc}, and Ω_{bbb} baryon masses are determined from the basic mass formula of Eq. 1. The dibaryon mass, modeled in terms of a first-order hexaquark mass formula, is provided by Eq. 2. The ΩΩ, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ masses and J^{π} values noted in Sections 4.1 – 4.3 are summarized in Table 1.

Table 1 ΩΩ, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ Dibaryon State Mass and J^{π} Values Derived from a First-Order Hexaquark Mass Formula[a]

System	Mass (MeV/c^2)	J^{π}
ΩΩ	3,525	$0^+, 1^+, 2^+, 3^+$
$\Omega_{ccc}\Omega_{ccc}$	9,775	$0^+, 1^+, 2^+, 3^+$
$\Omega_{bbb}\Omega_{bbb}$	28,842	$0^+, 1^+, 2^+, 3^+$

[a] The number of mass value significant figures does not represent the model accuracy.

4.1 ΩΩ Hexaquark State

Eq. 2 is used as the basis to calculate the mass of the candidate **ΩΩ** hexaquark[29] state

$$M(\mathbf{\Omega\Omega}) = \mathbf{M_{\Omega}(1) + M_{\Omega}(2) + \Phi} \quad (5)$$

where $\mathbf{M_{\Omega}(i)}$ (i = 1,2 labels the individual baryon cluster). $\mathbf{M_{\Omega}(i)}$ is determined from Eq. 1 and m_1, m_2, and m_3 are strange quarks representing the Ω^- quark structure[33]. Using Eqs. 1, 2, and 5, provides a ΩΩ hexaquark mass of 3.53 GeV/c^2.

The Ω^- has a $J^\pi = 3/2^+$ angular momentum structure[33]. Since the first-order model assumes zero angular momentum between the two clusters, the ΩΩ hexaquark has a $3/2^+ \times 0 \times 3/2^+$ configuration that allows 0^+, 1^+, 2^+, and 3^+ J^π values. The model only has the aforementioned primitive angular momentum coupling that includes the 0^+ assignment suggested in Ref. 29.

4.2 $\Omega_{ccc}\Omega_{ccc}$ Hexaquark State

Eq. 2 is used as the basis to calculate the mass of the candidate $\mathbf{\Omega_{ccc}\Omega_{ccc}}$ hexaquark[29] state

$$M(\mathbf{\Omega_{ccc}\Omega_{ccc}}) = \mathbf{M_{\Omega(ccc)}(1) + M_{\Omega(ccc)}(2) + \Phi} \quad (6)$$

where $\mathbf{M_{\Omega(ccc)}(i)}$ (i = 1,2 labels the individual baryon cluster). $\mathbf{M_{\Omega(ccc)}(i)}$ is determined from Eq. 1 and m_1, m_2, and m_3 are charm quarks representing the Ω_{ccc} quark structure. Using Eqs. 1, 2, and 6, provides a $\mathbf{\Omega_{ccc}\Omega_{ccc}}$ hexaquark mass of 9.78 GeV/c^2.

Based on the Ω^- structure, the Ω_{ccc} has a presumed $J^\pi = 3/2^+$ angular momentum assignment. Since the first-order model assumes zero angular momentum between the two clusters, the $\mathbf{\Omega_{ccc}\Omega_{ccc}}$ hexaquark has a $3/2^+ \times 0 \times 3/2^+$ configuration that allows 0^+, 1^+, 2^+, and 3^+ J^π values. The model only has the aforementioned primitive angular momentum coupling that includes the 0^+ assignment suggested in Ref. 29.

4.3 $\Omega_{bbb}\Omega_{bbb}$ Hexaquark State

Eq. 2 is used as the basis to calculate the mass of the candidate $\mathbf{\Omega_{bbb}\Omega_{bbb}}$ hexaquark[29] state

$$M(\mathbf{\Omega_{bbb}\Omega_{bbb}}) = \mathbf{M_{\Omega(bbb)}(1) + M_{\Omega(bbb)}(2) + \Phi} \quad (7)$$

where $\mathbf{M_{\Omega(bbb)}(i)}$ (i = 1,2 labels the individual baryon cluster). $\mathbf{M_{\Omega(bbb)}(i)}$ is determined from Eq. 1 and m_1, m_2, and m_3 are bottom quarks representing the Ω_{bbb} quark structure. Using Eqs. 1, 2, and 7, provides a $\mathbf{\Omega_{bbb}\Omega_{bbb}}$ hexaquark mass of 28.8 GeV/c^2.

Based on the Ω^- structure, the Ω_{bbb} has a presumed $J^\pi = 3/2^+$ angular momentum assignment. Since the first-order model assumes zero angular momentum between the two clusters, the $\mathbf{\Omega_{bbb}\Omega_{bbb}}$ hexaquark has a $3/2^+ \times 0 \times 3/2^+$ configuration that allows 0^+, 1^+, 2^+, and 3^+ J^π values. The model only has the aforementioned primitive angular momentum coupling that includes the 0^+ assignment suggested in Ref. 29.

5.0 Model Uncertainties and Weaknesses

Although the first-order mass approach provides reasonable results in the description of tetraquark[16-26], pentaquark[27], and hexaquark and other exotic quark configurations[28], it has a number of uncertainties and weaknesses. These include:

1. The quark masses are model dependent and can assume a range of values[32,33]

2. The cluster-cluster interaction is not definitively known. Although it is small relative to the combined cluster masses forming a tetraquark, pentaquark, or hexaquark, its value has not been well established. However, fundamental QCD arguments, one boson exchange calculations[29], QCD Calcuations[43,44], and quark model calculations[45] suggest the cluster-cluster interaction is negligible relative to the individual cluster masses.

3. The angular momentum coupling is primitive. Hexaquark angular momentum is defined by the individual cluster J^{π} values, but zero angular momentum is assumed between the clusters. This approach allows a range of J^{π} values, but does not define a definitive angular momentum configuration.

4. Only quark-quark effects are allowed in Eq. 1. There is no consideration of higher order or gluon influenced terms.

5.0 Conclusions

Model results address the description of $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryon states in terms of a first-order hexaquark mass formula. Ω, Ω_{ccc}, and Ω_{bbb} baryon masses are determined from the basic mass formula of Eq. 1. The dibaryon mass, modeled in terms of a first-order hexaquark mass formula, is provided by Eq. 2.

$\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryon masses in terms of a first-order hexaquark mass formula are determined to be 3.53, 9.78, and 28.8 GeV/c^2. The states are predicted to have the following range of J^{π} values: 0^+, 1^+, 2^+, and 3^+. A 0^+assignment is predicted in Ref. 29.

The predicted masses of these states provide a crucial test of the validity of the proposed first-order mass formula and its weak coupling structure. Moreover, this mass formula provides a general framework to calculate hexaquark masses.

References

1) M. Gell-Mann, Phys. Lett. **8**, 214 (1964).

2) Particle Data Group, J. Phys. **G37**, 075021 (2010).

3) LHCb Collaboration, Phys. Rev. Lett. **112,** 222002 (2014).

4) BESIII Collaboration, Phys. Rev. Lett. **115,** 112003 (2015).

5) The D0 Collaboration, Phys. Rev. Lett. **117,** 022003 (2016).

6) LHCb collaboration, Phys. Rev. Lett. **115**, 072001 (2015).

7) The ATLAS collaboration, Phys. Lett. **B764**, 11 (2017).

8) WASA-at-COSY Collaboration, Phys. Rev. Lett. **112**, 202301 (2014).

9) M. Bashkanov, H. Clement, and T. Skorodko, arXiv:1502.07500v5 [nucl-ex] 5 Dec 2016.

10) BESIII Collaboration, Phys. Rev. Lett. **117,** 042002 (2016).

11) S. Pepin and Fl. Stancu, Phys. Rev. **D57**, 4475 (1998).

12) M. Abud, F. Buccella, and F. Tramontano, Phys. Rev. **D81**, 074018 (2010).

13) S. M. Gerasyuta and E. E. Matskevich, Phys. Rev. **D82**, 056002 (2010).

14) J. Vijande, A. Valcarce, and J. –M. Richard, Phys. Rev. **D85**, 014019 (2012).

15) F. Huang, Z-Y. Zhang, P-N, Shen, and W-L. Wang, Chinese Physics **C39**, 071001 (2015).

16) J. J. Bevelacqua, First-Order Tetraquark Mass Formula, Physics Essays **29**, 198 (2016).

17) J. J. Bevelacqua, Description of the X(5568) and Proposed 750 GeV/c^2 State in Terms of a First-Order Tetraquark Mass Formula, Physics Essays **29**, 367 (2016).

18) J. J. Bevelacqua, Fusion of doubly heavy mesons into a tetraquark, Physics Essays **31**, 167 (2018).

19) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed X(3872), Physics Essays **32**, 469 (2019).

20) J. J. Bevelacqua, Description of the X(6900) as a Four Charmed Quark State in Terms of a First-Order Tetraquark Mass Formula, QEIOS **KLXLKJ**, 1 (2020). https://doi.org/10.32388/KLXLKJ.

21) J. J. Bevelacqua, Description of the X(2900) as an Open Flavor Tetraquark in Terms of a First-Order Mass Formula, QEIOS **OVLMEB**, 1 (2020). https://doi.org/10.32388/OVLMEB.

22) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed $Z_{cs}(3985)^-$, QEIOS **GLTEU2**, 1 (2021). https://doi.org/10.32388/GLTEU2.

23) J. J. Bevelacqua, Possible Tetraquark Explanation for the X(6200), QEIOS **J6AFYW**, 1 (2021). https://doi.org/10.32388/J6AFYW.

24) J. J. Bevelacqua, Possible Tetraquark Explanation for the T_{cc}^+, Qeios **OMDGAQ**, 1 (2021). https://doi.org/10.32388/OMDGAQ.

25) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed $Z_{cs}(4000)+$ and $Z_{cs}(4220)+$, Qeios **PPLMWV**, 1 (2021). https://doi.org/10.32388/PPLMWV.

26) J. J. Bevelacqua, Possible Tetraquark Explanations for the Proposed X(4630) and X(4685), Qeios. **7EGF85**, 1 (2021). https://doi.org/10.32388/7EGF85.

27) J. J. Bevelacqua, First-Order Pentaquark Mass Formula, Physics Essays **29**, 107 (2016).

28) J. J. Bevelacqua, Description of Selected Hexaquark States in Terms of a First-Order Mass Formula, Physics Essays **31** (1), 104 (2018).

29) M. Z. Liu and L. S. Geng, *Chin. Phys. Lett.* **38,** 101201 (2021).

30) Ya. B. Zel'dovich and A. D. Sakharov, *Kvarkovaia struktura i massy sil'novzaimodeistvuyushchikh chastits*, Yad. Fiz. **4**, 395 (1966).

31) A. D. Sakharov, Mass formula for mesons and baryons, Sov. Phys. JETP **51**, 1059 (1980).

32) D. Griffiths, *Introduction to Elementary Particles*, 2nd ed., (Wiley-VCH, Weinheim, 2008).

33) Particle Data Group, Review of Particle Physics, Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).

34) A. Abbas, Phys. Lett. **B 238**, 344 (1990).

35) A. Abbas, J. Phys. **G 16**, L163 (1990).

36) G. 't Hooft, Nucl. Phys. **B72**, 461 (1974).

37) E. Witten, Nucl. Phys. **B160**, 57 (1979).

38) S. Okubo, Phys. Lett. **5**, 1975 (1963).

39) G. Zweig, CERN Report No.8419/TH412 (1964).

40) J. Iizuka, Prog. Theor. Phys. Suppl. **37**, 38 (1966).

41) T.H.R. Skyrme, Proc. R. Soc. **A260**, 127 (1961).

42) T.H.R. Skyrme, Nucl. Phys. **31**, 556 (1962).

43) J. R. Green et al. Phys. Rev. Lett. **127**, 242003 (2021).

44) Y. Lyu, H. Tong, T. Sugiura, S. Aoki, T. Doi, T. Hatsuda, J. Meng J, and T. Miyamoto, *Phys. Rev. Lett.* **127,** 072003 (2021).

45) J. M. Richard, A. Valcarce, and J. Vijande, *Phys. Rev. Lett.* **124,** 212001 (2020).