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The origin of dark energy and dark matter: the galactic antigravitation

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Abstract

In standard cosmology ordinary matter consists of only 5% of the universe. Alternative cosmology is developed including exclusively observable baryonic materials. In this model gravitation is given by a mediation process represented by spherically symmetric spatial circulations denoted as keplerons, which are emitted by massive objects. The velocity of circulation follows the Kepler's law reproducing the Newtonian gravitation including also the Schwartzshield correction, but at long distances the accelerating expansion of space converts gravitation into antigravitation. Inside galaxies attractive gravitation predominates due to the elliptic geometry, but in the intergalactic space hyperbolic geometry brings to a repulsive interaction between galaxies. The attractive force of hypothetic dark matter is replaced by external compression caused by repulsive antigravitation between galaxies. The repulsive galactic interaction does not decrease with distance and even increases when the recession velocity is close to the speed of light. This relativistic increase gives the predominant contribution to the dark energy. The external compression can explain why the orbiting velocities of stars are the same in the spiral arms of galaxies, why mass density of nebulae clusters is enhanced and why the gravitational lensing is so intensive. The model derives the overall mass of universe as 1.74x10⁵³ kg, which is in accordance to the estimated number of galaxies.

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1. Introduction



Dark matter and dark energy are thought to account for approximately 95% of the matter in the universe (NASA, 2021). The dark matter is supposed to enhance the power of gravitation in spiral galaxies (Rubin, 1980) and galaxy clusters (Zwicky, 1937), since the amount of ordinary matter is far too less explaining the orbiting velocities of stars in the spiral arms and mass distribution in the clusters. Further evidence is given by the enhanced intensity of gravitational lensing (Taylor, 1998). The origin of dark energy stems from the proposal of Einstein introducing the Λ cosmological constant present everywhere in the universe, the role of this energy is to balance the attractive power of gravitation.

Though the Λ -CMD model (Cold Dark Matter) can offer proper explanation for many cosmological observations, the theory is still struggling with a few unanswered questions:

- Why is dark matter concentrated in the external zone of astronomical objects?
- Why one cannot observe any non-baryonic particles (Weakly Interacting Massive Particles, WIMP)? (Bertone, 2010, Joss, 2016)
- What is the physical origin of dark energy?

A few alternative cosmologies have already been developed in which modified gravitational rules were suggested to replace the role of dark matter. In these theories deviations from the classical gravitation law are assumed at large distances from the galactic center. This assumption can help to reproduce the flat shape of the radial acceleration relation (RAR) in the case of spiral galaxies (Milgrom, 1983; Chae, 2020). These proposals can partially answer for the above points, but the origin of dark energy still remains obscure, furthermore, the applied mathematics has arbitrary elements by including adjustable parameters.

In the following we make an attempt to develop a new cosmology by modifying the gravitational laws including both attractive and repulsive force between massive objects. It is important that this modification is based on strictly *a priory* physical principle (Rockenbauer, 2022). In this effort the methods of classical and quantum field theories are combined:

- In the theory of Einstein gravitation is explained by the spatial curvatures created by the mass of physical objects;
- In the theory of quantum electrodynamics (QED) virtual bosons emitted and absorbed by elementary particles are considered as mediators for the electromagnetic interaction.

The former is a classical theory while the latter is based on the quantum nature of elementary particles. We suggest a semi-classical field theory of gravitation, which is a mediation procedure without quantum effect. The non-quantum nature of gravitational mechanism follows from the information obtained for gravitation, which has a continuous character. We are on the opinion that the applied mathematical procedure should reflect the nature of available information. This standpoint is supported by the long failure for developing a consequent quantum theory for gravitation. The suggested mediators are emitted and absorbed by the mass of elementary objects and represented by spherically symmetric circulations in the spacetime, which creates spatial curvatures due to the Lorentz contraction effective in a circularly moving non-inertial frame. (Though the transformation laws of special relativity are valid for inertial frames, the rule of Lorentz contraction can be applied in non-inertial systems as well, since any motional frame can be decomposed into infinitesimal parts. While in an inertial frame Lorentz contraction causes an apparent shortening, the curvature in a rotating system is already a real



phenomenon.) We call these mediators as keplerons for the honor of Kepler, since the velocity of circulations follows Kepler's law. The mediators in question have no mass and spin they propagate with the speed of light similarly to the photons. The keplerons have their impact through the spatial curvatures created by the circulation of space, since the circumference becomes shorter due to the Lorentz contraction while the radius remains the same. This spatial asymmetry leads to the trajectories of free fall according to the theory of general relativity. Since the largest curvature is related to the largest contraction, the kepleron model also fulfills the principle of shortest path.

The sign of spatial curvature determines whether the gravitation is attractive or repulsive. In the following we will show that this sign is inverted by the accelerating expansion of space (AES) and this inversion can turn the attractive gravitation into a repulsive force between galaxies. Furthermore, we will demonstrate that the inversion of gravitation can interpret the origin of dark matter and dark energy.

2. Theory

Circulation of space

How can we develop a model, where the gravitation is attractive at short distances and becomes repulsive at large distances? The starting point is the spatial geometry produced by the individual baryons, which builds up all macroscopic objects. The addition rules of spatial curvatures determine their metric tensors. According to Einstein's concept of gravitation attraction and repulsion can be related to the sign of curvature in spacetime. In this paper we follow the Schwartzshield metric (Schwartzshield, 1916) and focus our attention to the predominant spherically symmetric term in the 4x4 dimensional metric tensor. This limitation is justified for spherical astronomical objects, like planets or stars. Though the shape of galaxies has generally low symmetry, their structure can yield to little anisotropic effect at large distances, e.g. at 10 million light-year (Mly) away from the galactic center. For interpretation of the cosmic gravitational waves in the LIGO experiments (Barish, 1999) the space anisotropy plays a crucial role, this question is outside of the scope of present paper.

Above the boundary 10 Mly the receding velocities of galaxies follow the Hubble law, that is u = HD, where D is the distance between galaxies. In the following analysis we discuss how this motion can be connected to a circular motion of space around the massive objects. Assumption of this spatial motion is based on the Kepler-Newton law (KNL) in which the v velocity of orbiting is independent of the m mass of the circulating planet and depends only on the M mass of the central object provided by that m is small compared to M. According to KNL: $v^2R = GM$, where R is the radius and $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ is the general gravitational constant. Since the circulating velocity v is the same even for infinitely small mass it is justified to assume circulation for the massless light or empty space around the massive objects. This is the point where we can borrow the basic concept of QED, in which the electromagnetic interactions are mediated by virtual photons emitted and absorbed by the charge of the elementary particles. In the field theory of gravitation the circulating space can be considered as a mediator emitted and absorbed by the massive objects. These mediators can propagate at the speed of light similarly to the photons. Obviously, the circulation of space has no spin, thus we cannot



speak about fermions or real bosons and for this reason we propose a new name for this mediator: the kepleron. (In the former paper we denoted this mediator as screw: Rockenbauer, 2015). It should be also noted that the kepleron mediated mechanism cannot be classified as a theory of quantum field, it is actually a semi-classic field theory.

Intensity distribution of keplerons follows the KNL, thus it is proportional to the M mass of the emitting objects and decreases with the relation of $1/R^2$. The product of the square of v velocity and R rotation radius is constant according to the KNL: $v^2R = GM$. The mechanism of mediation for keplerons is based on the spatial curvature created by the Lorentz contraction $\sqrt{1-\left(\frac{v}{c}\right)^2}$, where v is the circumferential velocity. The curvature is related to the asymmetric character of contraction, namely, contraction can take place in the circumference while the circulation radius is not altered. The reduced ratio of circumference versus radius defines an elliptic (Riemannian) geometry. This ratio determines the radial curvature RC defined in the polar r, θ , φ metric by $RC = g_{\varphi\varphi}^2 - g_{\eta r}^2$:

$$RC = \left(\frac{\text{Circumference}}{2R\pi}\right)^2 - 1 \qquad \textit{text}(1)$$

This definition postulates an opposite sign what we introduced in the earlier paper (Rockenbauer, 2015) norder to make *RC* negative for attraction. If no circulation takes place, *RC* is zero. The curvature increases with the square of *v* velocity due to the Lorentz contraction:

$$RC = -\frac{v^2}{c^2}$$
 (2)

The negative sign of curvature corresponds to the attractive direction of gravitation. By applying the KNL and defining the potential energy of gravitation as a product of curvature with mc^2 :

$$V_{qr} = RC \cdot mc^2 = -\frac{GMm}{R} \tag{3}$$

Deriving by *R* we obtain the Newtonian force law:

$$F_{gr} = -\frac{GMm}{R^2}$$
 (4)

This force law can be interpreted as a product of kepleron intensity GM/R^2 with the m mass.

The purpose of above elaboration was to show that the kepleron circulations can well reproduce the classical gravitation law. Even the relativistic correction of Schwartzshield can be included by making use the law of energy conservation. When two massive objects have an interaction the original energy of objects will change for supplying the gravitational energy. The modified energy of object will affect also the kepleron intensity and causes a relativistic term proportionally to $1/R^3$ (Rockenbauer, 2020). Any further relativistic effect like singularities could have minor importance since only large? distances are considered in this paper.



The outlined model of spatial circulations is equivalent with the concept of general relativity based on spatial curvatures. Einstein also suggested an equivalence rule between gravitation and inertial force in his elevator example. The same equivalence can be extended between the inertial force and gravitation in a merry go round. Furthermore, the kepleron concept is also consistent with the principle of shortest path in optics, since the orbit of massive objects bends toward the domain of space, where the curvature has a maximum due to the largest Lorentz contraction.

Expansion of space

The basic law of physical cosmology was discovered by Hubble, who found that the recessional velocity of galaxies is proportional to their proper distance from the Milky Way, that is u = HD. The presently accepted value of Hubble's constant is $H = 70 \text{ (km/s)/Mpc} = 2.3 \times 10^{-18} \text{ l/s}$ (Freedman, 2021). Assuming that the radius of kepleron expands with space, that is R = D, this expansion modifies the curvature in space but in this case the radius of keplerons becomes shorter due to the Lorentz contraction, while the circumference remains the same. In this case the ratio of circumference versus radius is increased with respect to the ratio in the Euclidean geometry – consequently a hyperbolic (Bolyai-Lobachevsky) geometry is manifested.

The *RC* value can be derived from Eq. (1):

$$RC = \frac{u^2/c^2}{1 - u^2/c^2} \tag{5}$$

In this case the radial curvature is positive and when the recession velocityu is small compared to c an analogous formula can be obtained with Eq. (2)

$$RC = \frac{u^2}{c^2} = \frac{H^2 R^2}{c^2} = \left(\frac{R}{R_{HS}}\right)^2$$
 (6)

Here $R_{HS} = c/H = 13.78$ Gly denotes the Hubble sphere, this distance can be covered by light during the total age of universe. This relation is valid if R is small compared to R_{HS} , but, anomalously, for galaxies of extremely far away – close to R_{HS} – the curvature is larger compared to the approximation in Eq. (6).

The positive sign of RC represents repulsion mediated by the inverted keplerons. We speak about inverted keplerons when the u recession velocity exceeds the v circulation velocity. No change can be assumed for the GM/R^2 intensity of the originally emitted keplerons, only RC is changed due to the spatial expansion, thus the force of antigravitation exerted on the m mass can be given as:

$$F_{agr} = \frac{GMmH^2}{c^2} = GMm/R_{HS}^2 \qquad (7)$$

In the transitional domain where the velocities u and v are comparable gravitation and antigravitation can cancel each other, while in the domain where u is close to c an enhanced RC value is given by Eq. (5) and the kepleron gravitational



force is obtained:

$$F_{\text{kepleron}} = \frac{GMm(u^2 - v^2)}{\left(u^2R_{HS}^2 + v^2R^2\right)\left(1 - u^2/c^2\right)}$$
(8)

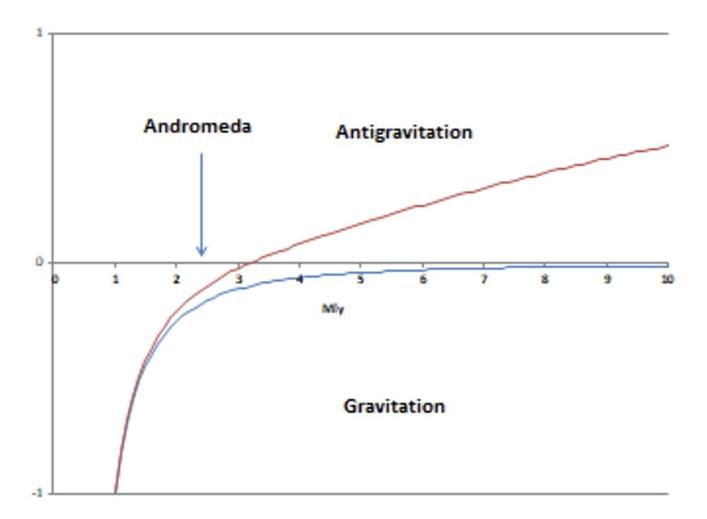


Figure 1. The Newtonian and kepleron gravitational forces are compared, where ID = 3.26 Mly, the former curve is blue, the latter one is red. The distance is given in Mly unites.

The above considerations allow to interpreting the dark energy of the Λ -CMD cosmology as a sum of galactic antigravitation (GAG). The GAG force of Eq. (8) has a very unusual feature: its power does not decrease with distance, and even can increase for galaxies at extremely high distances where the recession velocity is close to c (See Figure 2).



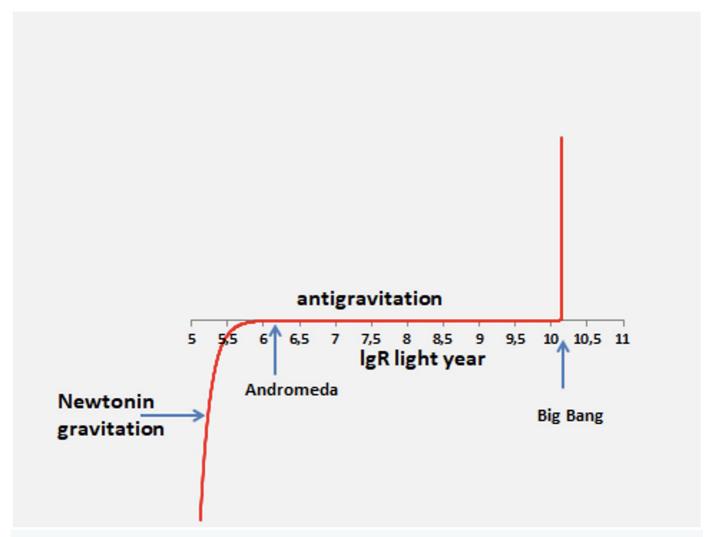


Figure 2. The kepleron gravitational force is given as a function of logarithmic distance in ly units. Left part: Newtonian law, middle: weak antigravitation, right part: relativistic antigravitation

At intermediate distances antigravitation between galaxies does not decrease, but it is rather weak and equal to the usual gravitational force between two galaxies at the largest observable distance, namely at R_{HS} . All galaxies in the observable universe can contribute to the uniform repulsive energy in space, which integrates the impact of masses in the whole universe. Since all physical interactions propagate with the speed of light, the antigravitation energy accumulates the impact of the entire history of universe and it seems that material motions in the early universe can have a decisive imprint on the present structure of astronomical objects. The important role of early galaxies is expressed in the dominance of dark energy, which is defined as a driving force for the accelerating expansion of universe. Originally Einstein introduced this energy by the Λ cosmological constant to counterbalance the gravitation in the field equation of general relativity. In our cosmology this repulsive energy is not an *ad hoc* constant in space, but expresses the repulsive gravitational energy of ordinary matter. In other words, there is no need to speak about vacuum energy or energy of the empty space (Sean, 2001). Our model also supports the concept of flat universe (Shirley, 2008) since the antigravitation is very weak between individual galaxies except where the distance is close to R_{HS} .

The GAG force also explains the stability of rotating spiral galaxies and the enhanced mass density of complex



astronomical objects. Any selected object is surrounded by hundred billions of galaxies and their repulsive force creates an external compression. The effect of this universal compression is equivalent to the attractive force in the hypothetical dark matter, for this reason the kepleron concept can replace the dark matter in all its aspects. It means that all results gained by the Λ -CDM theory (NASA, 2021) can be utilized on the one hand, but there is no need to assume arbitrary distribution of dark matter, or to search for any non-baryonic particles, on the other hand.

3. Discussion and Results

Gravitation is inverted into antigravitation if the v circumferential velocity of keplerons at R distance from the center of massive object becomes equal to the u velocity of space expansion:

$$R_{ID}^{3} = \frac{GM}{H^2}$$
 (9)

The third power of inversion distance (ID) is proportional to the mass of astronomical objects. In the case of Milky Way (Carlesi, 2022) the mass is estimated around 2.3x10⁴² kg and consequently the gravitation is compensated at the distance 3.26 Mly. In this estimation the overall mass includes the hypothetical dark matter, as well. Smaller inversion distance is obtained if only the ordinary matter is taken into account. If interstellar gas and dust as well as contribution of the halo are included (Gupta, 2012), the observable mass of the Milky Way can be around 0.5x10⁴² kg, which yields an ID of 2 Mly. This ID is less than the average distance between neighboring galaxies (c.a. 4 Mly), that is, practically all galaxies repel each other. Systematic recession from the Milky Way can be observed for galaxies farther away than 10 Mly, in this domain the overall mass of astronomical objects can be large enough to produce adequately strong tossing power. However, no efficient tossing power is expected between the Milky Way and Andromeda Galaxy separated by 2.5 Mly in the Local Group, where the distance is close to ID. In this case the random character of galactic motion determines whether the nebulae are approaching or moving away from each other.

We can define ID for any well separated massive objects. ID increases with the cubic root of mass. For electrons ID is 1 cm, for hydrogen atoms 20 cm, for heavier atoms around 1 m, for the Earth 1 ly, for the Sun 300 ly, for galaxies around a few Mly, respectively. Since in condensed matters the separation of atoms is around 10^{-10} m, the spatial curvatures created by individual atoms are added together if ID is close to the width of curvature at atomic level. In this case the amplitude of spatial curvatures is indeed proportional to the overall mass of condensed objects and the respective ID increases with the cubic root of mass. The summation rule of gravitation, however, is not valid in the intergalactic space if the number of hydrogen atoms is less than 30 in a cubic meter.

A special ID can be defined for the entire observable universe, where the size is given by $R_{HS} = c/H$. The standard cosmological model allows expansion rates greater than c. This assumption allows a space domain outside the $R_{HS} = c/H$ Hubble sphere but no information can be obtained from this external zone. It means that the overall gravitational energy should be calculated inside the Hubble sphere, and the force becomes zero at the limit R_{HS} . This condition is equivalent in the kepleron model when the force of gravitation can be cancelled by antigravitation. Applying Eq. (9) for the whole



universe its total M_U mass can be given:

$$M_{IJ} = \frac{c^3}{GH} = 1.74 \times 10^{53} \,\mathrm{kg}$$
 (10)

This value is compatible with the astronomic estimation for the number of galaxies, since M_U is 350 billion times larger than the mass of Milky Way, and the number of galaxies is estimated between 200 billions and 2 trillions (Camarillo, 2018, Castelvecchi, 2016, Lauer, 2021), It is also important to emphasize that the product of the M_U mass of universe and the Hubble constant can be given by two basic constants of physics: the speed of light and the general gravitational constant. This expresses the fact that the AES is governed by the overall mass of universe.

The power of GAG is rather weak when only two galaxies are considered. This fact can be visualized by comparing the power of antigravitation with that of the gravitation existing in the outer arm of Milky Way at 43 500 ly from the center (Goodwin, 2012). Here the attraction force is 100 billion times stronger than the repulsion between two galaxies of average size. The overall compression of 350 billion galaxies can enhance the stabilizing force by 3.5 times, which is close to the six times enhancement suggested in the CDM model. This estimation is valid if the Milky Way is indeed an average galaxy in size.

The kepleron model can be also tested by making use the proportion of dark energy in the universe (NASA, 2021). The sum of antigravitation energy is proportional to the square of M_{U_1} if GAG is independent of the distance. (Note: All galactic interactions should be considered twice, since the present of an object A interacts with the past of B and the present of B interacts with the past of A, respectively.) Then the ratio of GAG to the energy of observable matter:

$$\frac{E_{GAG}}{M_U c^2} = \frac{GMan_{UH}}{c^3} = 1 \tag{11}$$

In the Λ -CMD model the proportion of dark energy in the universe is estimated around 65-70% (NASA, 2021). In Eq. (11) the overall GAG energy is equal to the energy of ordinary matter. This ratio is derived from the truncated Eq. (6) RC formula, which significantly underestimates the GAG energy for galaxies with large recessional velocities. The large proportion of dark energy can be assessed to the relativistic effect of galaxies receding from each other at the velocities close to c, that is the major contribution to the GAG energy are given by galaxy pairs of extremely far distance. It means that the early history of universe has a predominant imprint on the present universe.

There is a clear example, where the kepleron model offers more convincing explanation than the Λ -CDM cosmology: this is the flat dependence of orbital velocities in the arms of spiral galaxies. (Joss, 2016). In the Milky Way this velocity is between 210-220 km/s. The constant value is contrary to the KNL, where the velocity has to decrease with the radius of spiral arms according to the rule $v^2 = GM/R$. This contradiction is resolved in the CDM model by assuming a particular distribution of dark matter concentrated mainly in the external zone of the spiral galaxy. There is, however, a rather weak point in this argument: if dark matter can govern mass distribution of the ordinary matter by the gravitational law, why is the same law not effective for the distribution of dark matter itself? Maybe a further unknown force exists organizing the distribution of dark matter? Further disputable point is the lack of evidence for detecting any non-baryonic particles, the



WIMPs? Our model based on the assumption of universal compression can avoid all these logical traps. The pressure is defined by the ratio of power and compressed surface. The surface of spiral arms is proportional to the product of the *R* radius and thickness, where the thickness is the same in each spiral arms (Bland-Hawthorn, 2016) and, consequently, the external pressure will decrease linearly with *R*, similarly to the centrifugal force. This circumstance clearly explains why the orbital velocity of stars is independent of the radius of arms. The universal compression can also explain the anomalously large mass density in the globular clusters (Zwicky, 1937), and so is the case for the enhanced intensity of gravitational lensing (Taylor, (1998).

There are further phenomena that the CDM model cannot properly describe, while the kepleron concept can offer straightforward picture. Look at a few examples! Why do the majority of galaxies have a spiral structure indicating rotation of the whole nebula? The reason is that the symmetry of universal compression is not perfectly isotropic in space and can be different for individual galaxies. The anisotropic compression can yield to torques causing the galaxies to rotate. This concept is in line with the observation that the cosmic microwave background is also anisotropic (Riess, 1998 and Perlmutter, 1999). Recently oscillation of stars was observed in the Milky Way (Poggio, 2020). This oscillation can be created by an inhomogeneous compression in the plane of galactic disk.

Empty or rarely populated domains separate the spiral arms in the galaxies. This distribution can be partially related to the ID of individual stars. For the Sun this is 300 ly, for larger stars this can be as large as 1000 or 1500 ly. Consequently, when a star is separated from its closest neighbors farther than 2000 ly, it can be outside of the attractive zone and repels other stars in the nebula. This feature yields to almost empty zones between the spiral arms separated with ca. 10 kly gaps, which exceed the ID of constituting stars. The thickness of arms is about 1 to 2 kly, which is rather narrow compared to the length of spiral arms. This geometry also can be explained by external compression, which produces flat distribution of stars, since in the direction perpendicular to the plane of the galactic disk the centrifugal force is weak.

The age of Milky Way is close to the age of universe allowing more than a dozen revolutions, but the number of arms is much less. (Sparke, 2007) The reason is the stability limit of galaxies: at short distances the attractive gravitation predominates connecting together the stars, but at long distances, the weak but infinitely long-range repulsion can prevail. At the largest scale, the associations of nebulae are generally arranged into sheets and filaments surrounded by immense voids (Tully, 2008 and Bradley, 2013). These astronomic formations also show that the universal compressional power predominates at this large scale.

Another astronomical phenomenon that needs to be reinterpreted is the question of non-observable Great Attractor (Dressler,1994). Due to the repulsion between galaxies it is more correct to assume a "Great Repulsor" formed by the spatial inhomogeneity of galaxies, which tosses the Milky Way in the direction of the constellation of Centaurus at about 600 km/s.

4. Conclusions



Here we derived a cosmology in which the whole universe is built up exclusively from observable materials and the stability and structure of galaxies are interpreted by a universal compression. Gravitation is mediated by keplerons between massive objects and attraction can turn into repulsion at large distances. The attractive gravitation inside galaxies is interpreted by their elliptic geometry, while the empty intergalactic space has a hyperbolic geometry causing repulsion between galaxies. It means that the expansion of universe is coming from a geometric requirement. The GAG explains the origin of dark energy and plays the same role which is attributed to dark matter in the Λ-CDM cosmology. The proposed concept can avoid any arbitrary and non-controllable assumptions for distribution of the non-observable dark matter and there is no need to assume existence of any non-baryonic particles. The kepleron model can suggest a value for the total amount of mass in the universe, which is compatible with astronomic estimations.

Our concept relies on the isotropic component of the 4x4 metric of spacetime in the theory of general relativity. Elaboration of a more complete theory is under development in which the kepleron concept can be combined with the field equation of Einstein.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References and notes

- Barish, Barry C.; Weiss, Rainer, 1999; LIGO and the Detection of Gravitational Waves. Physics Today 52, 44.
- Bertone, G. 2010, Nature. 468, 389.
- Bradley W. Carroll; Dale A. Ostlie, 2013, An Introduction to Modern Astrophysics (International ed.). Pearson. pp. 1173–1174.
- Camarillo, Tia, Dredger, Pauline, Ratra, Bharat. 2018, Astrophysics and Space Science. 363, 268.
- Carlesi, Edoardo, Hoffman, Yehuda, Libeskind, Noam I. 2022 Monthly Notices of the Royal Astronomical Society.513, 2385.
- Castelvecchi, Davide. 2016, Nature. 537, 459.
- Chae, Kyu-Hyun, Lelli, Federico, Desmond, Harry, McGaugh, Stacy S., Li Pengfei, and Schombert, James M. 2020,
 The Astrophysical Journal, 904, 51.
- Dressler, Alan, 1994. "Voyage to the Great Attractor: Exploring Intergalactic Space". New York, NY: Alfred A. Knopf. p. 355. ISBN 978-0-394-58899-5.
- Freedman, Wendy L. 2021, The Astrophysical Journal. 919, 16.
- Goodwin, S. P., Gribbin, J., Hendry, M. A. 1998, The Observatory. 118, 201.
- Gupta, A., Mathur, S. Krongold, Y., Nicastro, F., Galeazzi, M. 2012, The Astrophysical Journal.756, L8.
- Joss, Bland-Hawthorn, Ortwin, Gerhard, 2016, Annual Review of Astronomy and Astrophysics.54, 529.
- Lauer, Tod R. at al. 2021, The Astrophysical Journal. 906, 77.



- NASA Science Universe Dark Energy, Dark Matter. NASA Science. Retrieved 23 May 2021.
- Perlmutter, S. et al. 1999. The Astrophysical Journal. 517, 565
- Poggio, E. at al. 2020, Nature Astronomy. 4, 590.
- Riess, Adam G. et al, 1998, The Astronomical Journal. 116, 1009.
- Rockenbauer, A. 2015, Indian J. Physics 89, 389.
- Rockenbauer, A. 2020, "Útikalauz a fizikához Newtontól Higgsig", Budapest, Scolar, pp. 127.
- Rockenbauer, A. 2022, Femme Harmone 2,108.
- Rubin, V., Thonnard, W.K. Jr., Ford, N. 1980, The Astrophysical Journal. 238, 471.
- Sean. Carroll 2001, Living Reviews in Relativity.4, 1.
- Schwarzschild, K. (1916). "Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie".
 Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften. 7: 189–196.
- Shirley Ho; Hirata; Nikhil Padmanabhan; Uros Seljak; Neta Bahcall.2008, Physical Review D.78, 043519.
- Sparke, Linda S., Gallagher, John S. 2007, Galaxies in the Universe: An Introduction. p. 90.
- Zwicky, F. 1937. The Astrophysical Journal. 86, 217.
- Taylor, A.N. et al. 1998, The Astrophysical Journal. 501, 539.
- Tully, R. Brent et al. 2008, The Astrophysical Journal 676, 184.