

Peer Review

Review of: "Parameterized Deceleration in $f(Q, C)$ Gravity: A Logarithmic Approach"

Zahra Davari¹

1. Korea Institute for Advanced Study, Seoul, South Korea

The paper presents an interesting and relevant approach by applying $f(Q, C)$ gravity to cosmic acceleration, contributing meaningfully to ongoing discussions on alternatives to the Λ CDM model.

The inclusion of observational data analysis (OHD and Pantheon+SH0ES) and cosmographic diagnostics adds value to the research. However, there are several inconsistencies that the authors need to address by revisiting their calculations and clarifying the methodology. Despite these issues, the foundation of the research is promising. If the authors resolve these discrepancies, clarify the derivations, and provide stronger comparisons with

Λ CDM predictions, the paper could offer valuable insights. Only after addressing these concerns could the paper be considered scientifically sound and suitable for publication.

Specific Comments and Suggestions:

1-Clarification of the Functional Form (Equation 27):\\

While the introduction discusses the mathematical advantages of the logarithmic parameterization, it does not provide a clear physical explanation for why this form, in particular, should be preferred over other non-divergent models. Why a logarithmic form captures the dynamics of cosmic acceleration more effectively than polynomial or other commonly used parameterizations should be explained.\\

2-Referencing Prior Explanations:\\

Since the motivation for the functional form of Equation (27) is addressed in the introduction, it is suggested that the authors briefly reference that explanation when introducing this equation in Section III. This would enhance the clarity and logical flow of the manuscript by directly connecting the theoretical motivation to the mathematical formulation.\\

3-Physical Interpretation of Parameters α and β :

The physical interpretation of the parameters α and β in equation 27 is not clearly explained. What roles do they play in the cosmic evolution scenario? Clarifying this would enhance the readers' understanding of the model.

4-Inconsistencies in $q(z)$ Form at $z=0$:

In the paper, it's mentioned that at $z=0$ (present time or current epoch), the equation simplifies to:

$$q(z)=q_0+q_1 \log[\alpha-\beta]$$

However, Equation (27) actually provides a different limiting form of the deceleration parameter, expressed as $q(z)=q_0+q_1 \log[\alpha]-\beta$

5-Choice of Nonlinearity (Equation 31):

The model uses $f(Q,C)=\gamma_1 Q^n+\gamma_2 C$

While there is some justification citing earlier works, the paper does not explain why this specific nonlinear form is most appropriate for their study. A more thorough justification would strengthen the argument.

6-Visual Representation Suggestion:

Including a plot that visually compares $q(z)$ for various parameter choices would provide better intuition about the model's behavior before analyzing this model by observational data.

7-Marginalized Constraints in Table 1:

The paper presents marginalized constraints in Table 1 but does not clarify the prior distributions used for the parameters. Could the authors explain what priors were assumed?

8-MCMC Convergence Justification:

Could the authors justify why using 100 walkers and 10,000 steps was sufficient for convergence? Were any diagnostics, such as the Gelman–Rubin statistic or autocorrelation checks, applied?

9-Discrepancy in $q(0)$ Value:

The calculated present-day deceleration parameter $q(0)$ based on your best-fit parameters is approximately 1.21. This value suggests a decelerating universe, which is inconsistent with both current cosmological observations and the standard Λ CDM model that predicts $q(0)\approx-0.55$. Could the authors clarify this apparent discrepancy? Is there a specific reason for this deviation from the expected accelerating behavior of the universe?

10-Mismatch Between Table 1 and Table 2 Results:\\

Using the best-fit parameters provided in Table 1, the computed value of $q(0)$ is approximately 1.23, which differs significantly from the reported value of -0.2815 in Table 2. Could the authors clarify if there are additional corrections or assumptions applied in the calculation of $q(0)$ that were not explicitly mentioned in the manuscript?

11- Comparison with Λ CDM Model:\\

The current value of q_0 reported (-0.2815 for OHD and -0.2584 for Pantheon+SHoES(in pervious comments I have doubt about these values)) is not compared with the expected Λ CDM value (-0.55). This omission makes it difficult to assess the model's physical relevance.

It would be beneficial if the authors could provide a more detailed explanation of the values reported in Table 2 for the various cosmological quantities, particularly q_0 , z_t , w_0 . Additionally, I suggest comparing these results with the corresponding values predicted by the standard Λ CDM model. Such a comparison would help assess the consistency of the proposed model with current observational constraints and highlight any potential deviations or improvements over the standard model.

12-Unnecessary Re-Derivation:\\

In the manuscript, the authors define a relation for the deceleration parameter $q(z)$ in Equation (27) and subsequently derive the Hubble parameter $H(z)$ in Equation (30). Later, the authors appear to derive Equation (37) by applying a reverse process using $H(z)$ again. Could the authors clarify the necessity of this step, as it seems to retrace the same derivation in reverse?

13-Transition Redshift Discussion:\\

How do these transition redshift values compare to those predicted by the Λ CDM model, which generally suggests a transition around 0.7 ? Could the authors provide a discussion of this comparison?

14-Equation of State Behavior:\\

Why does the model specifically remain in the quintessence phase without crossing into the phantom regime? Is this behavior inherent to the $f(Q,C)$ model or due to specific parameter choices?

15-Inconsistencies Between EoS Parameter and r-s Diagnostic:\\

This seems to highlight a potential inconsistency between two diagnostic tools used in the paper:

EoS parameter suggests the universe remains in the quintessence regime throughout cosmic evolution.

r-s trajectory in Figure 9 initially falls into the Chaplygin gas regime before eventually approaching the Λ CDM point.

This could indicate:

A misinterpretation of the r-s diagnostic or a potential issue with how the statefinder parameters were calculated.

A feature of the model where, despite having a quintessence-like equation of state, the geometric evolution could temporarily exhibit similarities to the Chaplygin gas model in certain cosmic epochs.

A clear explanation from the authors would be essential to understand if this is a natural feature of the model or an artifact of the analysis.

16-Suggested Revision for Figure 9:\\

It is recommended to revise Figure 9 by including horizontal and vertical reference lines corresponding to the standard Λ CDM model values ($r=1$, $s=0$). Adding these reference lines would enhance the clarity of the statefinder diagnostic and provide a clearer visual comparison between the behavior of the proposed model and the standard cosmological model.

Declarations

Potential competing interests: No potential competing interests to declare.