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An Empirical Examination of Collateralization in Financial Markets

ABSTRACT

This paper presents a new model for pricing financial instruments under different collateral arrangements. Moreover, we show that the model-implied price of a collateralized contract is very close to its market price, which suggests that the model is fairly accurate. Further, we find empirical evidence that asset prices in cleared markets are determined in a similar way to those in OTC markets. This practice is questionable, as the clearing process has changed the risk structure that affects outcomes. In fact, cleared derivatives are not economically equivalent to their OTC counterparts.

Key words: unilateral/bilateral collateralization, partial/full/over-collateralization, asset pricing, plumbing of the financial system, swap premium spread.

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1. Introduction

Collateralization is an essential element in the plumbing of the financial system that allows financial institutions to reduce economic capital and credit risk, free up lines of credit, and expand the range of counterparties. All of these factors contribute to the growth of financial markets.

The reason collateralization of financial derivatives and repos has become one of the most important and widespread credit risk mitigation techniques is that the Bankruptcy Code contains a series of "safe harbor" provisions to exempt these contracts from the "automatic stay". The automatic stay prohibits the creditors from undertaking any act that threatens the debtor's asset, while the safe harbor, a luxury, permits the creditors to terminate derivative and repo contracts with the debtor in bankruptcy and to seize the underlying collateral. This paper focuses on safe harbor contracts (e.g., derivatives and repos), but many of the points made are equally applicable to automatic stay contracts.

Financial derivatives can be categorized into three types. The first category is over-the-counter (OTC) derivatives, which are customized bilateral agreements. The second group is cleared derivatives, which are negotiated bilaterally but booked with a clearinghouse. Finally, the third type is exchange-traded/listed derivatives, which are executed over an exchange. The differences between the three types are described in detail on the International Swap Dealers Association (ISDA) website (see ISDA (2013)).

Under the new regulations (e.g., Dodd-Frank Wall Street Reform Act), certain 'eligible' OTC derivatives must be cleared with central counterparties (CCPs) (see Heller and Vause (2012) and Pirrong (2011)). Hull (2011) further recommends mandatory CCP clearing of all OTC derivatives. Meanwhile, Duffie and Zhu (2011) suggest a move toward the joint clearing of interest rate swaps and credit default swaps (CDS) in the same clearinghouse.

Ottonello, etc. (2022) study the design of macroprudential policies based on quantitative collateral-constraint models and find the desirability macroprudential policies critically depends on the specific form of collateral used in debt contracts. Bianchi, etc. (2020) develop a quantitative model that focuses on collateral inefficiencies arising from prices that affect borrowing limits and individual agents not internalizing such price effects.

Devereux, etc. (2019) analyze how predictions of collateral-constraint models very with different timing assumptions. Du, etc. (2023) investigate how market participants price and manage counterparty credit risk using confidential trade repository data on single-name CDS transactions.

The posting of collateral is regulated by the Credit Support Annex (CSA) that specifies a variety of terms including the threshold, the independent amount, and the minimum transfer amount (MTA), etc. The threshold is the unsecured credit exposure that a party is willing to bear. The minimum transfer amount is the smallest amount of collateral that can be transferred. The independent amount plays the same role as the initial margin (or haircut). The CSA was originally designed for OTC derivatives, but more recently has been updated for cleared/listed derivatives. For this reason people in the financial industry often refer to collateralized contracts as CSA contracts and non-collateralized contracts as non-CSA contracts.

We define effective collateral threshold as the threshold plus the MTA. The collateral is called as soon as the mark-to-market (MTM) value rises above the effective threshold. A positive effective threshold corresponds to partial/under-collateralization where the posting of collateral is less than the MTM value. A negative effective threshold represents over-collateralization where the posting of collateral is greater than the MTM value. A zero-value effective threshold equates with full-collateralization where the posting of collateral is equal to the MTM value.

From the perspective of collateral obligations, collateral arrangements can be unilateral or bilateral. In a unilateral arrangement, only one predefined counterparty has the right to call for collateral. Unilateral agreements are generally used when the other counterparty is much less creditworthy. In a bilateral arrangement, on the other hand, both counterparties have the right to call for collateral.

Upon default and early termination, the values due under the ISDA Master Agreement are determined. These amounts are then netted and a single net payment is made. All of the collateral on hand would be available to satisfy this total amount, up to the full value of that collateral. In other words, the collateral to be posted is calculated on the basis of the aggregated value of the portfolio, but not on the basis of any individual transaction.

The use of collateral in financial markets has increased sharply over the past decade, yet analytical and empirical research on collateralization is relatively sparse. Collateral management is often carried out in an ad-hoc manner, without reference to an analytical framework. Comparatively little research has been done to analytically and empirically assess the economic significance and implications of collateralization. Such a quantitative and empirical analysis is the primary contribution of this paper.

Due to the complexity of quantifying collateralization, previous studies seem to turn away from direct and detailed modeling of collateralization (see Fuijii and Takahahsi (2012)). For example, Johannes and Sundaresan (2007), and Fuijii and Takahahsi (2012) characterize collateralization via a cost-of-collateral instantaneous rate (or stochastic dividend or convenience yield). Piterbarg (2010) regards collateral as a regular asset in a portfolio and uses the replication approach to price collateralized contracts. All of the previous works focus on full-collateralization only.

We obtain the CSA data from two investment banks. The data show that only 8.21% of CSA counterparties are subject to unilateral collateralization, while the remaining 91.79% are bilaterally collateralized. The data also reveal that 61.63% of CSA counterparties have a zero threshold, and the remaining 38.37% use a positive threshold ranging from 25,000 to 750,000,000. Moreover, all CSA counterparties in the data maintain a positive MTA ranging from 500 to 60,000,000, which means that the effective thresholds are always greater than zero. In other words, contracts in OTC markets are partially collateralized due to positive effective thresholds, whereas contracts in cleared/listed markets are over-collateralized as all CCPs/Exchanges require initial margins. Therefore, full-collateralization does not exist in the real world. The reason for the popularity of full-collateralization is its mathematical simplicity and tractability.

This article makes a theoretical and empirical contribution to the study of collateralization by addressing several essential questions concerning the posting of collateral. First, how does collateralization affect expected asset prices? To answer this question, we develop a comprehensive analytical framework for pricing financial contracts under different (partial/full/over and unilateral /bilateral) collateral arrangements in different (OTC/cleared/listed) markets.

In contrast to other collateralization models in current literature, we characterize a collateral process directly based on the fundamental principal and legal structure of the CSA agreement. This framework shows that collateralization can always improve recovery and reduce credit risk. If a contract is over-collateralized (e.g., a repo or cleared contract), its value is equal to the risk-free value. If a contract is partially collateralized (e.g., an OTC derivatives), its CSA value is less than the risk-free value but greater than the non-CSA risky value.

Second, how can the model be empirically verified? To achieve the verification goal, this paper empirically measures the effect of collateralization on pricing and compares it with model-implied prices. This calls for data on financial contracts that have different collateral arrangements but are similar otherwise. We use the interest rate swap contract data from two investment banks for the empirical study, as interest rate swaps collectively account for around two-thirds of both the notional and market value of all outstanding derivatives.

The mid-market swap rates quoted in the market are based on hypothetical counterparties of AA-rated quality or better. Dealers use this market rate as a reference when quoting an actual swap rate to a client and make adjustments based on many factors, such as credit risk, liquidity risk, funding cost, operational costs and expected profit, etc. Unlike most other studies, this study mainly concentrates the analysis on swap adjustments/premia related to credit risk and collateralization, which are to be made to the mid-market swap rates for real counterparties.

Prior research has primarily focused on the generic mid-market swap rates and results appear puzzling. Sorensen and Bollier (1994) believe that swap spreads (i.e., the difference between swap rates and par yields on similar maturity Treasuries) are partially determined by counterparty default risk. Whereas Duffie and Huang (1996), Hentschel and Smith (1997), Minton (1997) and Grinblatt (2001) find weak or no evidence of the impact of counterparty credit risk on swap spreads. Collin-Dufresne and Solnik (2001) and He (2001) further argue that many credit enhancement devices, e.g., collateralization, have essentially rendered swap contracts risk-free. Meanwhile, Duffie and Singleton (1997), and Liu, Longstaff and Mandell (2002) conclude that both credit and liquidity risks have an impact on swap spreads. Moreover, Feldhütter and Lando (2008) find that the liquidity factor is the largest component of swap spreads. It seems that there is no clear-cut answer yet regarding the relative contribution of the liquidity and credit factors. Maybe, the recently revealed LIBOR scandal can partially explain these conflicting findings.

Unlike the generic mid-market swap rates, swap premia are determined in a competitive market according to the basic principles of supply and demand. A client who wants to enter a swap contract first contacts a number of swap dealers and asks for a swap rate. After comparing all quotations, the client chooses the most competitive rate. The determination of swap premia in an investment bank is complex. In fact, most of the time it is carried out in an ad-hoc manner and is largely based on the experiences of the traders. A swap premium is supposed to cover operational, liquidity, funding, and credit costs as well as a profit margin. If the premium is too low, the dealer may lose money. If the premium is too high, the dealer may lose the competitive advantage.

Unfortunately, we do not know the detailed allocation of a swap premium, i.e., what percentage of the adjustment is charged for each factor. Thus, a direct empirical verification is impossible.

To circumvent this difficulty, this article uses an indirect process to verify the model empirically. We define a *swap premium spread* as the difference between the swap premia of two collateralized swap contracts that have exactly the same terms and conditions but are traded with different counterparties under different collateral agreements. We reasonably believe that if two contracts are identical except counterparties, the premium spread should reflect the difference between two counterparties' unsecured credit risks only, as all other risks and costs are identical.

Empirically, we find quite a number of CSA swap pairs in the data, where the two contracts in each pair have different counterparties but are otherwise the same. The test results demonstrate that the model-implied swap premium spreads are very close to the market swap premium spreads, indicating that the model is quite accurate. To further check on the robustness of the conclusion, we estimate a regression model where the model-implied premium spreads are used as a dependent variable and the market premium spreads as the explanatory variable. The estimation results show that the constant term is insignificantly different from zero; the slope coefficient is close to 1 and the adjusted R^2 is very high. This suggests that the implied premium spreads explains nearly all of the market premium spreads.

Third, why does collateralization have different pricing impacts in different markets? The contract data from two investment banks reveal that cleared swaps have dramatically increased since 2011, reflecting the financial institutions' compliance to regulatory requirements. We find evidence that the economical determination of swap rates in cleared markets is the same as that in OTC markets, as all clearinghouses claim that cleared derivatives would replicate OTC derivatives, and promise that the transactions through the clearinghouses would be economically equivalent to similar transactions handled in OTC markets.

Although the practice recommended by CCPs is popular in the market, in which derivatives are continuously negotiated over-the-counter as usual but cleared and settled through clearinghouses, some market participants cast doubt on CCPs' economic equivalence claim. They find that cleared contracts have actually significant differences when compared with OTC trades. Some firms even file legal action against the clearinghouses, and accuse them of fraudulently inducing the firms to enter into cleared derivatives on the premise the contracts would be economically equivalent to OTC contracts (see Pengelly (2011)).

In fact, there are many differences between cleared markets and OTC markets. The first difference is that cleared derivatives are over-collateralized as all CCPs require initial margins, whereas CSA derivatives in OTC markets are partially collateralized as all CSA counterparties maintain a positive MTA. The second difference is that cleared derivatives are subject to unilateral collateralization because only CCPs have the right to call for collateral, while OTC derivatives are most probably under bilateral collateralization. The third difference is that OTC collateralization always sets a MTA to avoid the workload associated with a frequent transfer of insignificant amount of collateral between firms, whilst CCPs mark contracts to market daily and charge variation margins in response to changes in market values without a MTA.

The fourth difference is that variation margin in clearing markets is a linear function of daily market value changes. Whereas, collateral posted in OTC markets is a nonlinear function of daily market value changes. When the MTM value is greater than the threshold and the daily value change exceeds the MTA, collateral is called; otherwise, no transfer of collateral occurs. This non-linearity is the root cause of the complexity of pricing collateralized OTC derivatives.

The fifth difference is that a CCP mitigates credit risk via novation and multilateral netting. The novation process splits a contract into two – one setting out a sale and the other setting out the countervailing purchase – and substitutes the clearinghouse for the counterparty in each half of the transaction. Accordingly, the clearinghouse takes on credit risk on behalf of the original counterparties. The multilateral netting process enables offsets across market participants, which further ameliorates credit risk (see Cont and Kokholm (2011) and McPartland, et al. (2011)). In general, novation and multilateral netting processes change the risk structure that affects asset prices.

The last difference is that in cleared markets, a party receiving variation margin owns the funds and may withdraw them from the clearinghouse, while in OTC markets, a party posting collateral maintains ownership of the assets and receives interest (or coupons) on the assets. Many CCPs use the price alignment interest (PAI) adjustment or similar – a daily cash payment – to correct the difference in interest on variation margin between OTC contracts and CCP products (see Cont, Mondescu and Yu (2011)).

Even trivial variations between cleared and OTC markets can result in potentially large valuation discrepancy (see Pengelly (2011)). Given many differences between them, this paper demonstrates that cleared derivatives are not economically equivalent to their OTC counterparts.

Finally, what is the time-variation on the impact of collateralization? We find strong evidence that collateralization affects asset prices. The effects are time varying. The difference between the CSA value of a partially collateralized asset and the risk-free value reflects the cost of bearing unsecured credit risk. The cost increases as counterparty credit quality deteriorates. When counterparty risk is low, the risk-free value and the CSA value are almost coincident. However, when counterparty risk soars, the price difference surges, and then reaches the peak during the financial crisis. These results are in line with economic intuition and corroborate our theoretical analysis.

The remainder of this paper is organized as follows: Section 2 discusses unilateral collateralization. Section 3 elaborates bilateral collateralization. Section 4 presents empirical evidence. The conclusions and discussion are provided in Section 5. All proofs and some detailed derivations are contained in the appendices.

2. Collateralization

A unilateral collateral arrangement is sometimes used when a higher-rated counterparty deals with a lower-rated counterparty, in which only one party, normally the lower-rated one, is required to deliver collateral to guarantee performance under the agreement. Typical examples of unilaterally collateralized contracts include: repos, cleared/listed derivatives, sovereign derivatives, and some OTC derivatives.

A repurchase agreement, also known as a repo, is a short-term collateralized loan in which a cash-rich party lends money to a borrower and receives securities as collateral until the loan is repaid. Only the borrower is required to post collateral to the lender. The collateral in a repo is intended to protect the lender against default by the borrower. The lender therefore has to ensure that the loan is over collateralized. The degree of over-collateralization is measured by initial margin or haircut. While a cleared contract is a contract cleared and settled through a clearinghouse. Only clearing members are required to post a substantial amount of liquid collateral to the CCP as initial and variation margins.

Sovereign derivatives are OTC derivatives traded with sovereign entities. Sovereigns have been participants in OTC derivatives markets for decades. A majority of sovereigns have a one-way CSA in place. They have historically used their superior credit and bargaining power to obtain these favorable contracts from dealers. As a result, any exposures that sovereigns might have are always collateralized. Exposures that the dealers have, however, are unsecured (see AFME-ICMA-ISDA (2011)).

There are a small percentage of CSA counterparties in OTC markets subject to unilateral collateral arrangements (e.g., 8.21% in our sample data), in which only counterparties (normally much less creditworthy firms) are required to post collateral to dealers. Unlike cleared contracts that are over collateralized, CSA derivatives in OTC markets are under partial-collateralization.

We consider a filtered probability space (Ω , $\mathcal{F}$, $\{\mathcal{F}_t\}_{t\geq 0}$, $\mathcal{P}$) satisfying the usual conditions, where Ω denotes a sample space, $\mathcal{F}$ denotes a σ -algebra, $\mathcal{P}$ denotes a probability measure, and $\{\mathcal{F}_t\}_{t\geq 0}$ denotes a filtration.

Since the only reason for taking collateral is to reduce/eliminate credit risk, collateralization analysis is closely related to credit risk modeling. There are two primary types of models that attempt to describe default processes in the literature: structural models and reduced-form models. Many people in the market have tended to gravitate toward the reduced-from models given their mathematical tractability and market consistency. In the reduced-form framework, the stopping (or default) time τ of a firm is modeled as a Cox arrival process (also known as a doubly stochastic Poisson process) whose first jump occurs at default and is defined by,

$$\tau = \inf \left\{ t : \int_0^t h(s, \Gamma_s) ds \geq \Delta \right\} \tag{1}$$

where $h(t)$ or $h(t, \Gamma_t)$ denotes the stochastic hazard rate or arrival intensity dependent on an exogenous common state Γ_t, and Δ is a unit exponential random variable independent of Γ_t.

It is well-known that the survival probability from time *t* to *s* in this framework is defined by

$$p(t,s) := P(\tau > s \mid \tau > t) = \exp\left(-\int_t^s h(u) du \right) \tag{2a}$$

The default probability for the period (*t*, *s*) in this framework is given by

$$q(t,s) := P(\tau \leq s \mid \tau > t) = 1 - p(t,s) = 1 - \exp\left(-\int_t^s h(u) du \right) \tag{2b}$$

In order to assess the impact of collateralization on pricing, we study valuation with and without collateralization respectively.

2.1 Valuation without collateralization

Let valuation date be *t*. Consider a financial contract that promises to pay a $X_T > 0$ from party *B* to party *A* at maturity date *T*, and nothing before date *T*. We suppose that party *A* and party *B* do not have a CSA agreement. All calculations are from the perspective of party *A*. The risk free value of the financial contract is given by

$$V^F(t) = E\left[D(t,T)X_T \middle| \mathcal{F}_t\right] \tag{3a}$$

where

$$D(t,T) = \exp\left[-\int_t^T r(u)du\right] \tag{3b}$$

where $E\{\bullet|\mathcal{F}_t\}$ is the expectation conditional on the $\mathcal{F}_t$, $D(t,T)$ denotes the risk-free discount factor at time *t* for the maturity *T* and $r(u)$ denotes the risk-free short rate at time *u* ($t \le u \le T$).

Next, we discuss risky valuation. In a unilateral credit risk case, we assume that party *A* is default-free and party *B* is defaultable. We divide the time period (*t*, *T*) into *n* very small time intervals (Δt). In our derivation, we use the approximation $\exp(y) \approx 1 + y$ provided that *y* is very small. The survival and default probabilities of party *B* for the period (*t*, $t + \Delta t$) are given by

$$\hat{p}(t) := p(t, t+\Delta t) = \exp(-h(t)\Delta t) \approx 1 - h(t)\Delta t \tag{4a}$$

$$\hat{q}(t) := q(t, t+\Delta t) = 1 - \exp(-h(t)\Delta t) \approx h(t)\Delta t \tag{4b}$$

The binomial default rule considers only two possible states: default or survival. For the one-period (*t*, $t + \Delta t$) economy, at time $t + \Delta t$ the contract either defaults with the default probability $q(t, t+\Delta t)$ or survives with the survival probability $p(t, t+\Delta t)$. The survival payoff is equal to the market value $V^N(t+\Delta t)$ and the default payoff is a fraction of the market value: $\varphi(t+\Delta t)V^N(t+\Delta t)$, where φ is the recovery rate. The non-CSA value of the contract at time *t* is the discounted expectation of all possible payoffs and is given by

$$V^N(t) = E\left\{\exp(-r(t)\Delta t)\left[\hat{p}(t) + \varphi(t)\hat{q}(t)\right]V^N(t+\Delta t) \middle| \mathcal{F}_t\right\} \approx E\left\{\exp(-b(t)\Delta t)V^N(t+\Delta t) \middle| \mathcal{F}_t\right\} \tag{5}$$

where $b(t) = r(t) + h(t)(1 - \varphi(t)) = r(t) + s(t)$ denotes the (short) risky rate and $s(t) = h(t)(1 - \varphi(t))$ denotes the (short) credit spread.

Similarly, we have

$$V^N(t + \Delta t) = E\left\{ \exp(-b(t + \Delta t)\Delta t) V^N(t + 2\Delta t) \middle| \mathcal{F}_{t+\Delta t} \right\} \tag{6}$$

Note that $\exp(-b(t)\Delta t)$ is $\mathcal{F}_{t+\Delta t}$ -measurable. By definition, an $\mathcal{F}_{t+\Delta t}$ -measurable random variable is a random variable whose value is known at time $t + \Delta t$. Based on the *taking out what is known* and *towe*r properties of conditional expectation, we have

$$\begin{aligned} V^N(t) &= E\left\{ \exp(-b(t)\Delta t) V^N(t + \Delta t) \middle| \mathcal{F}_t \right\} = E\left\{ \exp(-b(t)\Delta t) E\left[\exp(-b(t + \Delta t)\Delta t) V^N(t + 2\Delta t) \middle| \mathcal{F}_{t+\Delta t} \right] \middle| \mathcal{F}_t \right\} \\ &= E\left\{ \exp\left(-\sum_{i=0}^{1} b(t + i\Delta t)\Delta t\right) V^N(t + 2\Delta t) \middle| \mathcal{F}_t \right\} \end{aligned} \tag{7}$$

By recursively deriving from t forward over T where $V^N(T) = X_T$ and taking the limit as Δt approaches zero, the non-CSA value of the contract can be obtained as

$$V^N(t) = E\left\{ \exp\left[-\int_t^T b(u)du \right] X_T \middle| \mathcal{F}_t \right\} \tag{8}$$

We may think of $b(u)$ as the risk-adjusted short rate. Equation (8) is the same as Equation (10) in Duffie and Singleton (1999), which is the market model for pricing risky bonds.

In theory, a default may happen at any time, i.e., a risky contract is continuously defaultable. This Continuous Time Risky Valuation Model is accurate but sometimes complex and expensive. For simplicity, people sometimes prefer the Discrete Time Risky Valuation Model that assumes that a default may only happen at some discrete times. A natural selection is to assume that a default may occur only on the payment dates. Fortunately, the level of accuracy for this discrete approximation is well inside the typical bid-ask spread for most applications (see O'Kane and Turnbull (2003)). From now on, we will focus on the discrete setting only, but many of the points we make are equally applicable to the continuous setting.

If we assume that a default may occur only on the payment date, the non-CSA value of the instrument in the discrete-time setting is given by

$$V^N(t) = E\left\{D(t,T)\left[p(t,T) + \varphi(T)q(t,T)\right]X_T \middle| \mathcal{F}_t\right\} = E\left[I(t,T)X_T \middle| \mathcal{F}_t\right] \tag{9}$$

where $I(t,T) = D(t,T)\left[p(t,T) + \varphi(T)q(t,T)\right]$ can be regarded as a risk-adjusted discount factor.

The difference between the risk-free value and the risky value is known as the *credit value adjustment* (CVA). The CVA reflects the market value of counterparty risk or the cost of protection required to hedge counterparty risk and is given by

$$CVA(t) = V^F(t) - V^N(t) = E\left\{D(t,T)\left[q(t,T)\left(1-\varphi(T)\right)\right]X_T \middle| \mathcal{F}_t\right\} \tag{10}$$

Since the recovery rate is always less than 1, we have $CVA(t) > 0$ or $V^N(t) < V^F$. In other words, the risky value is always less than the risk-free value. An intuitive explanation is that credit risk makes a financial contract less valuable.

2.2 Valuation with collateralization

Suppose that there is a CSA agreement between parties *A* and *B* in which only party *B* is required to deliver collateral when the mark-to-market (MTM) value arises over the collateral threshold *H*.

The choice of modeling assumptions for collateralization should be based on the legal structure of collateral agreements. According to the Bankruptcy Law, if the collateral value is greater than the default claim, creditors can only have a claim on the collateral up to the full amount of their default demand. Any excess collateral is returned to the estate of the failed institution for the payment of unsecured creditors. If the demand for default payment exceeds the collateral value, the balance of the demand will be treated as an unsecured claim and subject to its pro rate distribution under the Bankruptcy Code's priority scheme (see Garlson (1992), Routh and Douglas (2005), and Edwards and Morrison (2005)). The default payment under a collateral agreement, therefore, can be mathematically expressed as

$$P^D(T) = \begin{cases} X_T & \text{if } C(T) \geq X_T \\ C(T) + \varphi(T)\left(X_T - C(T)\right) = \varphi(T)X_T + C(T)(1-\varphi(T)) & \text{otherwise} \end{cases} \tag{11a}$$

or

$$P^D(T) = 1_{X_T \leq C(T)} X_T + 1_{X_T > C(T)}\left[C(T) + \varphi(T)\left(X_T - C(T)\right)\right] \tag{11b}$$

where 1_Y is an indicator function that is equal to one if *Y* is true and zero otherwise, and *C(T)* is the collateral amount at time *T*.

It is worth noting that the default payment in equation (11) is always greater than the original recovery, i.e., $P^D(T) > \varphi(T) X_T$, since $\varphi(T)$ is always less than 1. Said differently, the default payoff of a CSA contract is always greater than the default payoff of the same contract without a CSA agreement. That is why *the major benefit of collateralization should be viewed as an improved recovery in the event of a default.*

Let us consider repo/cleared/listed markets first. Contracts in these markets are always over collateralized, as the parties with collateral obligation are required to deposit initial margins and are also charged variation margins in response to changes in the market values. The total collateral (initial margin plus variation margin) posted at time *t* is given by

$$C(t) = V^C(t) - H(t) \tag{12}$$

where $V^C(t)$ is the CSA value of the contract at time *t* and $H(t)$ is the effective threshold. Note that for over-collateralization, the effective threshold is negative, i.e., $H(t) < 0$, which equals the negative initial margin. The collateral in equation (12) is a linear function of the asset value.

In general, initial margins are set very conservatively so that they are sufficient to cover losses under all scenarios considered. Also, the initial margins can be adjusted in response to elevated price volatility. Moreover, daily marking-to-market and variation margin settlement can further eliminate the risk that a loss exceeds the collateral amount. Thus, it is reasonable to believe that under over-collateralization the collateral amount is always greater than the default claim, i.e., $C(T) > X_T$.

At time *T*, if the contract survives with probability $p(t,T)$, the survival value is the promised payoff X_T and the collateral taker returns the collateral to the collateral provider. If the contract defaults with probability $q(t,T)$, the collateral taker has recourse to the collateral and obtains a default payment up to the full value of the promised payoff X_T. The remaining collateral $C(T) - X_T$ returns to the collateral provider. The CSA value of the over collateralized contract is the discounted expectation of the payoffs and is given by

$$V^C(t) = E\left[D(t,T)\left(q(t,T)X_T + p(t,T)X_T\right)\middle|\mathcal{F}_t\right] = V^F(t) \tag{13}$$

Equation (13) tells us that *the CSA value of an over-collateralized contract is equal to the risk-free value*. This result is consistent with the market practice in which market participants commonly assume that repos and cleared contracts are virtually free of default risk because of the implicit guarantee of the contracts provided by the clearinghouse and backup collateral.

It is worth keeping in mind that clearing does not eliminate any risk. It has no effect on the counterparty's default probability and does not improve the counterparty's credit rating. Instead, it uses some mitigation tools, e.g., collateralization, to perfectly hedge the credit risk, making a contract appear to be risk-free.

Next, we turn to OTC markets where usually $H(t) > 0$. If the value of the contract $V^C(t)$ is less than the effective threshold $H(t)$, no collateral is posted; Otherwise, the required collateral is equal to the difference between the contract value and the effective threshold. The collateral amount posted at time *t* can be expressed mathematically as

$$C(t) = \begin{cases} V^C(t) - H(t) & \text{if } V^C(t) > H(t) \\ 0 & \text{otherwise} \end{cases} \tag{14a}$$

or

$$C(t) = 1_{V^C(t) > H(t)}\left(V^C(t) - H(t)\right) \tag{14b}$$

In contrast to repo/cleared markets, collateral posted in OTC markets is a nonlinear function of daily market value changes. In fact, this discontinuous and state-dependent indicator function is the root cause of the complexity of collateralized valuation in OTC markets.

Since all CSA derivatives in OTC markets are partially collateralized, the default claim is almost certainly greater than the collateral amount. For a discrete one-period (*t*, *T*) economy, the collateral amount $C(t)$ posted at time *t* is defined in (14). At time *T*, if the contract survives, the survival value is the promised payoff X_T and the collateral taker returns the collateral to the collateral provider. If the contract defaults, the collateral taker possesses the collateral. The portion of the default claim that exceeds the collateral value is treated as an unsecured claim. Thus, the default payment is $C(T)+\varphi(T)\left(X_T - C(T)\right)$, where $C(T)=C(t)/D(t,T)$ is the future value of the collateral. Since the most predominant form of collateral is cash according to ISDA (2012), it is reasonable to consider the time value of money only for collateral assets. The large use of cash means that collateral is both liquid and not subject to large fluctuations in value. It can be seen from this, that collateral does not have any bearing on survival payoffs; instead, it takes effect on default payments only. The CSA value of the partially collateralized contract is the discounted expectation of all the payoffs and is given by

$$V^C(t) = E\left\{D(t,T)\left[q(t,T)\left(C(T)+\varphi(T)(X_T - C(T))\right) + p(t,T)X_T\right] \mid \mathcal{F}_t\right\} \tag{15}$$

Suppose that default probabilities are uncorrelated with interest rates and payoffs. We have the following proposition after some simple mathematics.

Proposition 1*: The unilateral CSA value of the partially collateralized single-payment contract is given by*

$$V^C(t) = E\left[F(t,T)X_T \mid \mathcal{F}_t\right] - G(t,T) \tag{16a}$$

where

$$F(t,T) = \left(1_{V^N(t)\le H(t)} + 1_{V^N(t)>H(t)} / \bar{I}(t,T)\right) I(t,T) D(t,T) \tag{16b}$$

$$G(t,T) = 1_{V^N(t)>H(t)} H(t)\bar{q}(t,T)\left(1-\varphi(T)\right) / \bar{I}(t,T) \tag{16c}$$

where $\bar{I}(t,T) = E\left(I(t,T) \mid \mathcal{F}_t\right)$. $I(t,T)$ and $V^N(t)$ are defined in (9).

Proof: See the Appendix.

We may think of $F(t,T)$ as the unilaterally CSA-adjusted discount factor and $G(t,T)$ as the cost of bearing unsecured credit risk. Proposition 1 tells us that the value of the unilaterally collateralized contract is equal to the present value of the payoff discounted by the unilaterally CSA-adjusted discount factors minus the cost of taking unsecured counterparty risk.

The valuation in equation (16) is relatively straightforward. We first compute $V^N(t)$ and then test whether its value is greater than $H(t)$. After that, the calculations of $F(t,T)$, $G(t,T)$ and $V^C(t)$ are easily obtained.

We discuss the following two cases. Case 1: $H(t)=0$ corresponds to full-collateralization. We have $V^C(t)=V^F(t)$ according to (16) where $H(t)=0$ and $V^N(t)>0$. That is to say: the CSA value under full-collateralization is equal to the risk-free value, which is in line with the results of Johannes and Sundaresan (2007), Fuijii and Takahahsi (2012), and Piterbarg (2010). Due to the mathematical tractability and simplicity, previous modeling works focus on full-collateralization only. As we point out above, however, full-collateralization does not appear to exist in any markets.

Case 2: $H(t)>0$ represents partial-collateralization. Equation (16) yields $V^N(t) \le V^C(t) < V^F(t)$ when $H>0$. In particular, $V^N(t)=V^C(t)$ when $H \to \infty$. Therefore, we conclude that the CSA value under partial-collateralization is less than the risk-free value but greater than the non-CSA value. Partial-collateralization that reflects the risk tolerance and commercial intent of firms is mostly seen in OTC markets.

Proposition 1 can be easily extended from one-period to multiple-periods. Suppose that a defaultable portfolio/contract has *m* netted cash flows. Let the *m* cash flows be represented as $X_i > 0$ with payment dates T_i, where *i* = 1,…,*m*. We derive the following proposition:

Proposition 2: *The unilateral CSA value of the partially collateralized multiple-payment contract is given by*

$$V^C(t) = \sum_{i=1}^{m} E\left[\prod_{j=0}^{i-1}\left(F(T_j,T_{j+1})\right)X_i \mid \mathcal{F}_t\right] - \sum_{i=0}^{m-1} E\left[\prod_{j=0}^{i-1}\left(F(T_j,T_{j+1})\right)G(T_i,T_{i+1}) \mid \mathcal{F}_t\right] \qquad (17a)$$

where

$$F(T_j,T_{j+1}) = \left(1_{J(T_j,T_{j+1}) \le H(T_j)} + 1_{J(T_j,T_{j+1})>H(T_j)} / \bar{I}(T_j,T_{j+1})\right) I(T_j,T_{j+1}) D(T_j,T_{j+1}) \qquad (17b)$$

$$G(T_j,T_{j+1}) = 1_{J(T_j,T_{j+1})>H(T_j)} H(T_j) \bar{q}(T_j,T_{j+1}) \left(1-\varphi(T_{j+1})\right) / \bar{I}(T_j,T_{j+1}) \qquad (17c)$$

$$J(T_j, T_{j+1}) = E\left[D(T_j, T_{j+1}) I(T_j, T_{j+1}) \left(V^C(T_{j+1}) + X_{j+1} \right) \middle| \mathcal{F}_{T_j} \right] \quad (17d)$$

Proof: See the Appendix.

The valuation in Proposition 2 has a backward nature. The intermediate values are vital to determine the final price. For a payment period, the current price has a dependence on the future price. Only on the final payment date T_m, the value of the contract and the maximum amount of information needed to determine the $J(T_{m-1}, T_m)$, $F(T_{m-1}, T_m)$ and $G(T_{m-1}, T_m)$ are revealed. This type of problem can be best solved by working backward in time, with the later value feeding into the earlier ones, so that the process builds on itself in a recursive fashion, which is referred to as *backward induction*. The most popular backward induction valuation algorithms are lattice/tree and regression-based Monte Carlo.

3. **Bilateral Collateralization**

A bilateral collateral arrangement enables the counterparties to pass collateral between each other to cover the net MTM exposure of the contracts. Under a two-way arrangement, the collateralization obligation is mutual and applicable to both the client and dealer. Bilateral collateralization most likely appears in OTC markets.

Two counterparties are denoted as *A* and *B*. The binomial default rule considers only two possible states: default or survival. Therefore, the default indicator Y_j for party *j* (*j*=*A*, *B*) follows a Bernoulli distribution, which takes value 1 with default probability q_j and value 0 with survival probability p_j, i.e., $P\{Y_j = 0\} = p_j$ and $P\{Y_j = 1\} = q_j$. The marginal default distributions can be determined by the reduced-form models. The joint distributions of a bivariate Bernoulli variable can be easily obtained via the marginal distributions by introducing extra correlations.

Consider a pair of random variables (Y_A, Y_B) that has a bivariate Bernoulli distribution. The joint probability representations are given by

$$p_{00} := P(Y_A = 0, Y_B = 0) = p_A p_B + \sigma_{AB} \quad (18a)$$

$$p_{01} := P(Y_A = 0, Y_B = 1) = p_A q_B - \sigma_{AB} \tag{18b}$$

$$p_{10} := P(Y_A = 1, Y_B = 0) = q_A p_B - \sigma_{AB} \tag{18c}$$

$$p_{11} := P(Y_A = 1, Y_B = 1) = q_A q_B + \sigma_{AB} \tag{18d}$$

where $E(Y_j) = q_j$, $\sigma_j^2 = p_j q_j$, $\sigma_{AB} := E[(Y_A - q_A)(Y_B - q_B)] = \rho_{AB}\sigma_A\sigma_B = \rho_{AB}\sqrt{q_A p_A q_B p_B}$ where ρ_{AB} denotes the default correlation coefficient and σ_{AB} denotes the default covariance.

For a two-way CSA, each party has an effective collateral threshold and is required to post collateral to the other as exposures arise. The collateral amount at *t* is given by

$$C(t) = \begin{cases} V^C(t) - H_B & \text{if } V^C(t) > H_B \\ 0 & \text{if } H_A \le V^C(t) \le H_B \\ V^C(t) - H_A & \text{if } V^C(t) < H_A \end{cases} \tag{19a}$$

or

$$C(t) = 1_{V(t)>H_B}\left(V^C(t) - H_B\right) + 1_{V(t)<H_A}\left(V^C(t) - H_A\right) \tag{19b}$$

where $H_B \ge 0$ and $H_A \le 0$ are the collateral thresholds for parties *B* and *A*, and $V^C(t)$ is the CSA value of the contract at time *t*.

Let valuation date be *t*. Consider a financial contract that promises to pay a X_T from party *B* to party *A* at maturity date *T*, and nothing before date *T* where $T > t$. The payoff X_T may be positive or negative, i.e. the contract may be either an asset or a liability to each party. All calculations are from the perspective of party *A*.

At time *T*, there are a total of four ($2^2 = 4$) possible states shown in Table 1. The CSA value of the contract is the discounted expectation of the payoffs and is given by the following proposition.

Proposition 3*: The bilateral CSA value of the partially collateralized single-payment contract is given by*

$$V^C(t) = E\left[L(t,T)X_T \middle| \mathcal{F}_t\right] - M(t,T) \tag{20a}$$

where

$$
\begin{aligned}
L(t,T) = D(t,T)\Big[&1_{0\le V_B^N(t)\le H_B(t)} I_B(t,T) + 1_{V_B^N(t)>H_B(t)} I_B(t,T)/\bar{I}_B(t,T) \\
&+1_{0\ge V_A^N(t)\ge H_A(t)} I_A(t,T) + 1_{V_A^N(t)<H_A(t)} I_A(t,T)/\bar{I}_A(t,T)\Big]
\end{aligned}
\tag{20b}
$$

$$
\begin{aligned}
M(t,T) = &1_{V_B^N(t)>H_B(t)} H_B(t)\bar{q}_B(t,T)\left(1-\varphi_B(T)\right)/\bar{I}_B(t,T) \\
&+1_{V_A^N(t)<H_A(t)} H_A(t)\bar{q}_A(t,T)\left(1-\varphi_A(T)\right)/\bar{I}_A(t,T)
\end{aligned}
\tag{20c}
$$

where $\bar{I}_j(t,T) = E\left(I_j(t,T)\middle|\mathcal{F}_t\right) = E\left(p_j(t,T)+\varphi_j(T)q_j(t,T)\middle|\mathcal{F}_t\right)$, $V_j^N(t) = E\left[I_j(t,T)X_T\middle|\mathcal{F}_t\right]$, and $j = A$ or B.

Proof: See the Appendix.

Table 1. Payoffs of a bilaterally collateralized contract

This table displays all possible payoffs at time *T*. In the case of $X_T > 0$, there are a total of four possible states at time *T:* i) Both *A* and *B* survive with probability p_{00}. The contract value is equal to the payoff X_T. ii) *A* defaults but *B* survives with probability p_{10}. The contract value is also the payoff X_T. Here we follow the two-way payment rule. iii) *A* survives but *B* defaults with probability p_{01}. The contract value is the collateralized default payment: $C(T)+\varphi_B(T)\left(X_T-C(T)\right)$. iv) Both *A* and *B* default with probability p_{11}. The contract value is also $C(T)+\varphi_B(T)\left(X_T-C(T)\right)$. A similar logic applies to the case of $X_T < 0$.

State		$Y_A=0, Y_B=0$	$Y_A=1, Y_B=0$	$Y_A=0, Y_B=1$	$Y_A=1, Y_B=1$
Comments		*A* & *B* survive	*A* defaults, *B* survives	*A* survives, *B* defaults	*A* & *B* default
Probability		p_{00}	p_{10}	p_{01}	p_{11}
Payoff	$X_T > 0$	X_T	X_T	$C(T)+\varphi_B(T)\left(X_T-C(T)\right)$	$C(T)+\varphi_B(T)\left(X_T-C(T)\right)$
	$X_T < 0$	X_T	$C(T)+\varphi_A(T)(X_T-C(T))$	X_T	$C(T)+\varphi_A(T)\left(X_T-C(T)\right)$

We may consider $L(t,T)$ as the bilaterally CSA-adjusted discount factor and $M(t,T)$ as the cost of bearing unsecured credit risk. Proposition 3 says that the value of the bilaterally collateralized contract is equal to the present value of the payoff discounted by the bilaterally CSA-adjusted discount factor minus the cost of taking unsecured counterparty risk.

In particular, if $H_A = H_B = 0$ (corresponding to bilateral full-collateralization), and $I_i(t,T)$ and X_T are uncorrelated, we have $L(t,T) = 1$, $M(t,T) = 0$, and thereby $V^C(t) = V^F(t)$. That is to say, under full-collateralization the bilateral CSA value of the contract is equal to the risk-free value.

Using a similar derivation as in Proposition 2, we can easily extend Proposition 3 from one-period to multiple-periods. Suppose that a defaultable portfolio/contract has *m* netted cash flows. Let the *m* cash flows be represented as X_i with payment dates T_i, where *i* = 1,…,*m*. Each cash flow may be positive or negative. The bilateral CSA value of the multiple payment contract is given by

$$V^C(t) = \sum_{i=1}^{m} E\left[\prod_{k=0}^{i-1}\left(L(T_k, T_{k+1})\right)X_i \mid \mathcal{F}_t\right] - \sum_{i=0}^{m-1} E\left[\prod_{k=0}^{i-1}\left(L(T_k, T_{k+1})\right)M(T_i, T_{i+1}) \mid \mathcal{F}_t\right] \tag{21a}$$

where

$$\begin{aligned} L(T_k, T_{k+1}) = D(T_k, T_{k+1})\Big[&1_{0 \le O_B(T_k, T_{k+1}) \le H_B(T_k)} I_B(T_k, T_{k+1}) + 1_{O_B(T_k, T_{k+1}) > H_B(T_k)} I_B(T_k, T_{k+1}) / \bar{I}_B(T_k, T_{k+1}) \\ &+ 1_{0 \ge O_A(T_k, T_{k+1}) \ge H_A(T_k)} I_A(T_k, T_{k+1}) + 1_{O_A(T_k, T_{k+1}) < H_A(T_k)} I_A(T_k, T_{k+1}) / \bar{I}_A(T_k, T_{k+1})\Big] \end{aligned} \tag{21b}$$

$$\begin{aligned} M(T_k, T_{k+1}) = &1_{O_B(T_k, T_{k+1}) > H_B(T_k)} H_B(T_k)\bar{q}_B(T_k, T_{k+1})\left(1 - \varphi_B(T_{k+1})\right) / \bar{I}_B(T_k, T_{k+1}) \\ &+ 1_{O_A(T_k, T_{k+1}) < H_A(T_k)} H_A(T_k)\bar{q}_A(T_k, T_{k+1})\left(1 - \varphi_A(T_{k+1})\right) / \bar{I}_B(T_k, T_{k+1}) \end{aligned} \tag{21c}$$

where $O_j(T_k, T_{k+1}) = E\left[D(T_k, T_{k+1}) I_B(T_k, T_{k+1})\left(V^C(T_{k+1}) + X_{k+1}\right) \middle| \mathcal{F}_{T_k}\right]$ and *j* = *A* or *B*.

Similar to Proposition 2, the individual payoffs in equation (21) are coupled and cannot be evaluated separately. The process requires a backward induction valuation.

4. **Empirical Results**

The objective of this paper is to assess the economic significance and implications of collateralization, which is essentially a matter of theoretical justification and empirical verification. We choose interest rate swaps for our empirical study, as they are the largest component of the global OTC derivative market, collectively accounting for around two-thirds of both the notional and market value of all outstanding derivatives.

Swap rate is the fixed rate that sets the market value of a given swap at initiation to zero. ISDA established ISDAFIX in 1998 in cooperation with Reuters (now Thomson Reuters) and Intercapital Brokers (now ICAP PLC). Each day ISDAFIX establishes average mid-market swap rates at key terms to maturity. These generic benchmark swap rates are based on a rigorously organized daily poll: An ICAP or Reuters representative canvasses a panel of dealers for their par swap rate quotes as of a specified local mid-day time. For any given swap term to maturity, the rate provided by the contributing dealer is the midpoint of where that dealer would itself offer and bid a swap for a certain notional. The mid-market benchmark rate for any given swap tenor is determined as a trimmed mean. Reuters and Bloomberg post the mid-market rates as soon as polling is completed.

In practice, the mid-market swap rates are generally not the actual swap rates transacted with counterparties but are instead the benchmarks against which the actual swap rates are set. A swap dealer that arranges a contract and provides liquidity to the market involves costs, e.g., hedging cost, credit cost, liquidity cost, operational cost, tax cost, and economic capital cost, etc. Therefore, it is necessary to adjust the mid-market swap rate to cover various costs of transacting and also to provide a profit margin to the dealer that makes the market. As a result, the actual price agreed for the transaction is not zero but a positive amount to the dealer.

Unlike the generic benchmark swap rates, swap premia are determined according to the basic principles of supply and demand. The swap market is highly competitive. In a competitive market, prices are determined by the impersonal forces of demand and supply, but not by manipulations of powerful buyers or sellers. If a premium is set too low, the dealer may lose money. If the premium is set too high, the dealer may lose the competitive advantage.

In contrast to most previous studies that focus on the generic swap rates, this article mainly studies swap adjustments/premia related to credit risk and collateralization. It empirically measures the effect of collateralization on pricing and compares it with model-implied prices.

A swap premium is supposed to cover the expected profit and all the expenses, including the cost of bearing unsecured credit risk. Unfortunately, however, we do not know what percentage of the market swap premium is allocated to the unsecured credit risk, which makes a direct verification impossible.

To circumvent this difficulty, we design an indirect verification process in which we select some CSA swap pairs, where the two contracts in each pair have exactly the same terms and conditions but are traded with different counterparties under different collateral agreements. It is reasonable to believe that the only difference between the two contracts in each pair is the difference in unsecured credit risk between two counterparties, as all other risks and costs are identical. Therefore, by taking credit risk and collateralization into account only, we can compare the model-implied swap premium spreads with the market swap premium spreads for these pairs.

We obtain the contract and counterparty information data from two investment banks. The trading dates are from May 6, 2005 to May 11, 2012. The data show that before 2011 the dealers traded interest rate swaps with a large number of counterparties. But sine 2011, the transactions have been concentrated in a few clearinghouses, which reflect financial institutions' compliance with the regulatory requirements. Consequently, we divide the data into two categories: the *OTC data set* containing all the transactions traded with regular counterparties and the *cleared data set* holding all the contracts cleared in CCPs.

The market data come from three independent sources: Bloomberg, Markit, and an investment bank. The data quality used in this study meets market makers' standard. The period of the market data is also from May 6, 2005 to May 11, 2012.

Let us examine the OTC data first. We find a total of 1032 swap pairs in the OTC data set, where the two contracts in each pair have the same terms and conditions but are traded with different counterparties under different collateral arrangements. We arbitrarily select one pair shown in Table 2.

Table 2: A pair of 20-year swap contracts

This table displays the terms and conditions of two swap contracts. They have different counterparties but are otherwise the same. We hide the counterparty names according to the security policy of the investment bank while everything else is authentic.

	Swap 1		**Swap 2**	
	Fixed leg	**Floating leg**	**Fixed leg**	**Floating leg**
Counterparty	*X*		*Y*	
Effective date	15/09/2005	15/09/2005	15/09/2005	15/09/2005
Maturity date	15/09/2025	15/09/2025	15/09/2025	15/09/2025
Day count	30/360	ACT/360	30/360	ACT/360
Payment frequency	Semi-annually	Quarterly	Semi-annually	Quarterly
Swap rate	4.9042%	-	4.9053%	-
Roll over	Mod_follow	Mod_follow	Mod_follow	Mod_follow
Principal	25,000,000.00	25,000,000.00	25,000,000.00	25,000,000.00
Currency	USD	USD	USD	USD
Pay/receive	Bank receives	Party *X* receives	Bank receives	Party *Y* receives
Floating index	-	3 month LIBOR	-	3 month LIBOR
Floating spread	-	0	-	0
Floating reset	-	Quarterly	-	Quarterly

An interest rate curve (see https://finpricing.com/lib/IrCurveIntroduction.html) is the term structure of interest rates, derived from observed market instruments that represent the most liquid and dominant interest rate products for certain time horizons. Normally the curve is divided into three parts. The short end of the term structure is determined using the LIBOR rates. The middle part of the curve is constructed using Eurodollar futures that require convexity adjustments. The far end is derived using mid swap rates. The LIBOR-future-swap curve is presented in Table 3. We use the investment bank's trading system to bootstrap the curve and get the continuously compounded zero rates.

Table 3: USD LIBOR-future-swap curve

This table displays the closing mid prices as of September 15, 2005

Instrument Name	Price
September 21 2005 LIBOR	3.6067%
September 2005 Eurodollar 3 month	96.1050
December 2005 Eurodollar 3 month	95.9100
March 2006 Eurodollar 3 month	95.8100
June 2006 Eurodollar 3 month	95.7500
September 2006 Eurodollar 3 month	95.7150
December 2006 Eurodollar 3 month	95.6800
2 year swap rate	4.2778%
3 year swap rate	4.3327%
4 year swap rate	4.3770%
5 year swap rate	4.4213%
6 year swap rate	4.4679%
7 year swap rate	4.5120%
8 year swap rate	4.5561%
9 year swap rate	4.5952%
10 year swap rate	4.6368%
12 year swap rate	4.7089%
15 year swap rate	4.7957%
20 year swap rate	4.8771%
25 year swap rate	4.9135%

As the payoff of an interest rate swap is determined by interest rates, we need to model the evolution of the floating rates. Interest rate models are based on evolving either short rates, instantaneous forward rates, or market forward rates (e.g., the LIBOR Market Model (LMM)). Since both short rates and instantaneous forward rates are not directly observable in the market, the models based on these rates have difficulties in expressing market views and quotes in term of model parameters, and lack agreement with market valuation formulas for basic derivatives. On the other hand, the object modeled under the LMM is market-observable. It is also consistent with the market standard approach for pricing caps/floors using Black's formula. They are generally considered to have more desirable theoretical calibration properties than short rate or instantaneous forward rate models. Therefore, we choose the LMM lattice proposed by Xiao (2011) for pricing collateralized swaps. We also implement the Hull-White trinomial tree to verify the results and ensure robustness of the valuation. This paper, however, only reports the results produced by the LMM lattice.

According to equation (21), we also need counterparty-related information, such as recovery rates, hazard rates and collateral thresholds. The CDS spreads and recovery rates are given in Table 4. We can compute the hazard rates via a standard calibration process (see J.P. Morgan [2001]).

Table 4: CDS premia and recovery rates

This table displays the closing CDS premia as of September 15, 2005 and recovery rates

Counterparty name	**Bank**	**Company *X***	**Company *Y***
6 month CDS spread	0.00031	0.000489	0.000808
1 year CDS spread	0.000333	0.00056	0.001017
2 year CDS spread	0.000516	0.000866	0.00154
3 year CDS spread	0.000664	0.001147	0.002114
4 year CDS spread	0.000848	0.00147	0.002768
5 year CDS spread	0.001012	0.001783	0.003439
7 year CDS spread	0.001334	0.002289	0.004283
10 year CDS spread	0.001727	0.002952	0.005281
15 year CDS spread	0.001907	0.003283	0.005814
20 year CDS spread	0.002023	0.003266	0.006064

30 year CDS spread	0.002021	0.00336	0.006461
Recovery rate	0.39213	0.35847	0.33872

Table 5: CSA agreement

This table provides the collateral thresholds and MTAs under the CSA agreements.

CSA agreement	**1**		**2**	
Counterparty name	Bank	Company *X*	Bank	Company *Y*
Threshold	0	0	0	0
MTA	500000	500000	500000	500000

The collateral thresholds and MTAs of the CSA agreements are displayed in Table 5. The effective collateral threshold is equal to the threshold plus the MTA.

Given the above information, we are able to compute the collateralized swap rates. We first use the LMM to evolve the interest rates and then determine the associated CSA-adjusted discount factors as well as the cost of bearing unsecured credit risk according to equation (21). Finally, we calculate the collateralized swap rates via backward induction method. The results are given in table 6.

Table 6: Swap rate results

This table presents the model-implied swap rates and premia as well as the dealer-quoted swap rates and premia, where Swap premium (in bps) = Swap rate – Generic swap rate, and Premium spread = Premium of swap 2 – Premium of swap 1.

	Swap 1		**Swap 2**		**Premium spread**	**Generic swap rate**
	Swap rate	**Premium**	**Swap rate**	**Premium**		
Model-implied	0.048780	0.09 bps	0.048790	0.19 bps	0.10 bps	0.048771
Dealer quoted	0.049042	2.71 bps	0.049053	2.82 bps	0.11 bps	

The 20-year generic mid-market swap rate is 0.048771 shown in Table 3. The swap rates of contracts 1 and 2 are given in Table 2 as 0.049042 and 0.049053. Accordingly, the market swap premia are 2.71 (0.049042 - 0.048771) basis points (bps) and 2.82 bps, respectively. These premia are charged for many expenses, e.g., operational, liquidity, funding, credit, etc., as well as profit margins. Although we do not know what percentage of the premia are allocated to cover the unsecured credit risks, we reasonably believe that the market premium spread, 0.11 bps in Table 5, should reflect the difference between the counterparties' unsecured credit risks only, as other factors are identical.

By taking credit risk and collateralization into account only, we calculate the model-implied swap rates as 0.048780 and 0.048790 shown in Table 6. Consequently, the implied swap premia are 0.09 bps and 0.19 bps. The results imply that only a small portion of a swap premium is attributed to unsecured credit risk. This is in line with the findings of Duffie and Huang (1996), Duffie and Singleton (1997), and Minton (1997). Table 6 shows that the model-implied swap premium spread is quite close to the dealer-quoted swap premium spread, suggesting that the model is fairly accurate in pricing collateralized financial instruments.

Repeating this exercise for the remaining pairs, we find that the model-implied swap premia fluctuate randomly around the market swap premia. The summary statistics of the market quoted premium spreads, the model-implied premium spreads, and the model-market premium spread differentials are presented in Table 7, where we refer to the differences between the model-implied premium spreads and the market quoted premium spreads as the *model-market premium spread differentials*. As can be seen from Table 7, the average of the model-market spread differentials is only -0.03 bps, which can be partly attributed to noises. The results indicate prima facie that the model performs quite well. The empirical tests corroborate the theoretical prediction on the impact of collateralization on swap rates.

Table 7: Summary statistics of implied swap premium spreads, market swap premium spreads and model-market swap premium spread differentials

All values are displayed in bps. Model-market premium spread differential = Model-implied premium spread – Market-quoted premium spread.

	Max	Min	Mean	Median	Std
Market quoted swap premium spreads	3.09	-5.25	-0.46	-0.16	1.80
Model-implied swap premium spreads	2.10	-5.33	-0.44	0.06	1.68
Model-market premium spread differentials	0.99	-1.18	-0.03	0.13	0.49

To determine the strength of the statistical relationship between the implied swap premium spreads and the market swap premium spreads, we present the estimate of the following regression model.

$$Y = a + bX + \varepsilon \tag{22}$$

where *Y* is the model-implied premium spread, *X* is the market premium spread, *a* is the intercept, *b* is the slope, and ε is the regression residual.

The results of this regression are shown in Table 8. The model is estimated directly using OLS. It can be seen from Table 8 that the implied premium spreads explicate nearly all of the market premium spreads with a slope coefficient close to 1. More important, the adjusted R^2 value is 0.9289, implying that approximately 93% of the market premium spreads can be explained by the implied spreads.

Table 8: Regression results for the OTC data

Slope	**Intercept**	**Adjusted R^2**	**Significance F**
0.8980	-0.0025	0.9289	0.0001

Next, we turn to cleared markets. We have found a total of 455 swap pairs in the cleared data set, where the two contracts in each pair have the same terms and conditions. The statistics of the market-quoted swap premium spreads for both the OTC data set and the cleared data set are presented in Table 9. It is shown from the table that the swap premium spreads in cleared markets behave in the same way as those in OTC markets, i.e., the market swap premium spreads in both cleared markets and OTC markets are statistical similar.

We find the evidence that under the new regulatory rules, derivative are continuously negotiated over-the-counter as usual, and then cleared and settled through a clearinghouse. Since clearinghouses claim that the cleared derivatives would replicate OTC derivatives and promise that the transactions through the clearinghouses would be economically equivalent to similar transactions handled in OTC markets, swap rates in cleared markets are determined in the same way as those in OTC markets.

Table 9: Statistics of market observed swap premium spreads

This table presents descriptive statistics for the market swap premium spreads in both OTC and cleared markets. All values are expressed in bps.

Market quoted swap premium spreads	**Max**	**Min**	**Mean**	**Median**	**Std**
In the OTC data set	3.09	-5.25	-0.46	-0.16	1.80
In the cleared data set	3.51	-4.41	-0.34	-0.21	1.27

Many market participants, however, have cast doubt on the claim of economic equivalence. They find that cleared contracts are actually significantly different from OTC trades (see Pengelly (2011)). There are many reasons to question this practice.

The first reason is that the results generated by this practice can not be interpreted by any known models or theories. As clearinghouses hide counterparties, we can only see the clearinghouses in the data. The two contracts in each cleared swap pair have exactly the same counterparty (i.e., the same CCP) and terms and conditions but different swap rates. In other words, they appear to be identical in everything except swap rates. This violates the single value rule: two assets with identical information must be traded at the same price.

Since the clearinghouse is the counterparty in all transactions, counterparty credit risk is always the same for a dealer. Consequently, the price of individual transactions should not be influenced by difference in counterparty credit risk, i.e., the swap premium spreads should be zero. Said differently, the presence of the swap premium spreads in cleared markets goes against the conventional wisdom and can not be explained theoretically and intuitively.

The second reason is that clearinghouses change the risk structure via novation and multilateral netting. The novation process splits a contract into two – one setting out a sale and the other setting out the countervailing purchase – and substitutes the clearinghouse for the counterparty in each half of the transaction. Accordingly, the clearinghouse takes on credit risk on behalf of the original counterparties. The multilateral netting enables offsets across market participants, which further ameliorates credit risk. In general, clearinghouses manage credit risk using proven methods and systems that are virtually always superior to what can be achieved by individual trading firms. Moreover, operations, books and fund transfers are all centralized and more efficient.

The final reason is that clearinghouses collect initial margins on all open positions and settle mark-to-market profits and losses in cash every day. Effectively, contracts in cleared markets are over-collateralized. Our theoretical study shows that over-collateralized derivatives in cleared markets are not economically equivalent to their partially collateralized counterparts in OTC markets.

Clearinghouses stand ready to act as counterparties to transactions with other market participants. The business of a clearinghouse closely resembles that of a specialized (monocline) insurer. Thus, the clearinghouse makes money in a similar way too – by charging fees to its members. For example, LCH Swap Clear does not levy a charge per transaction but instead charges its members an annual fee depending on their level of usage. Clearing members then pass on those charges to clients as part of their fee structure.

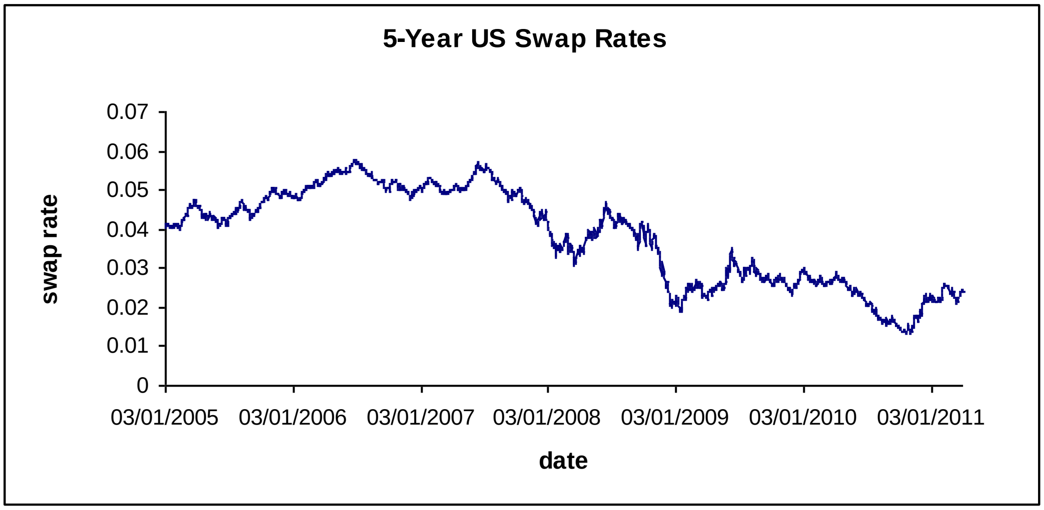
From a dealer's perspective, market making involves costs and executing a transaction in a clearinghouse also entails expenses. Dealers that make markets must be compensated for incurring these costs and expanses. As a consequence, dealers in cleared markets also collect a swap premium for a transaction. However, the swap premia charged in cleared markets should be different from those collected in OTC markets, as the cost and risk structures have changed. At least, if two contracts have exactly the same terms and conditions as well as the same counterparty (i.e., the clearinghouse), they should have the same premia, as the swap premia should not be influenced by difference in counterparty credit risk in cleared markets.

Finally, we examine how collateralization impacts swap rates over a period of time. We choose the USD mid-market swap rates from January 3, 2005 to April 5, 2011, and the CDS spreads for a generic AA from January 1, 2007 to December 30, 2011 for our study. The period of overlap is from January 1, 2007 to April 5, 2011. Unlike LIBOR bonds which carry AA default risk, the mid-market swap rates are very likely not impacted at all by default risk (see Collin-Dufresne, and Solnik (2001)). Therefore, we can regard the mid-market swap rates as the risk-free swap rates.

Figure 1 provides a plot of the time series of the 5-year swap rates. The classic explanation for the lower swap rates during the financial crisis is that the swap rates remain relatively unaffected by the financial crisis and have been notably declined since mid 2007 due to being swiftly adjusted to the monetary policy. The more recently revealed LIBOR scandal shows that some member banks manipulate LIBOR rates for at least two reasons. Routinely, traders sought particular rate submissions to benefit their financial positions. Later, during the 2007-2011 global financial crisis, they artificially lowered rate submissions to make their bank seem healthy.

Figure 1. Time series of 5-year swap rates

This diagram shows the time series of the daily US swap rates for maturity of 5 years from January 3, 2005 to April 5, 2011.

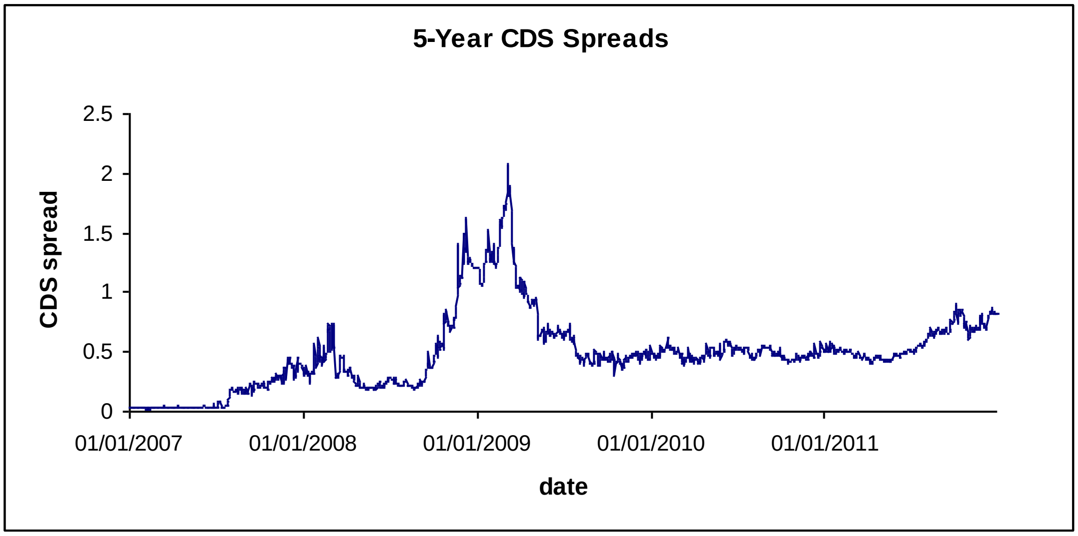


The time series plot of the 5-year CDS spreads is shown in Figure 2. The CDS spreads skyrocket during the global financial crisis period: spiking up in July 2007, remaining volatile for one and a half years, then spiking even higher in December 2008, and reaching the peak on March, 2009.

Assume that there is a 5-year interest rate swap. Party *A* pays a floating rate and party *B* pays a fixed rate. Further, Party *A* is risk-free and party *B* has a general AA credit quality. The collateral threshold for party *B* is 500,000. The results are displayed in Figure 3. From the time series plot, we find strong evidence that collateralization affects swap rates. The impact of collateralization is time varying.

Figure 2. Time series of 5-year CDS spreads

This diagram shows the time series of the daily CDS spreads for maturity of 5 years for a generic AA index from January 1, 2007 to December 30, 2011

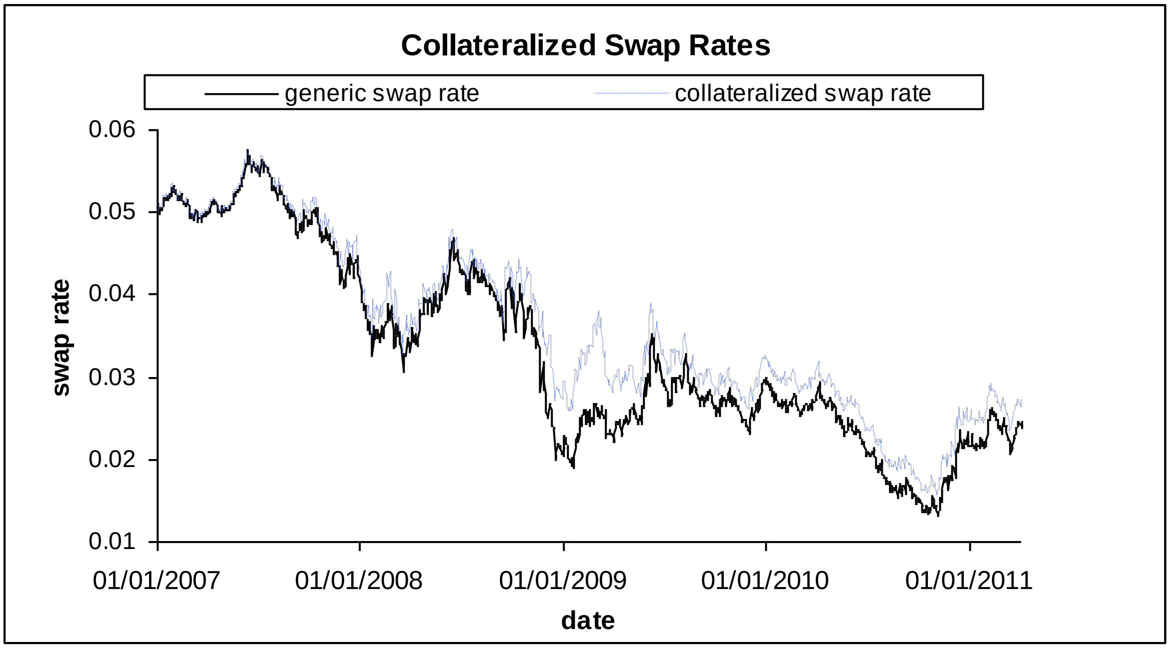


Prior to July 2007, the CDS spreads are at low levels. Consequently the partially collateralized swap rate and the mid-market swap rate are almost coincident (<3.8 bps). The difference between the partially collateralized swap rate and the risk-free swap rate reflects the cost of bearing unsecured credit risk or the cost of hedging unsecured credit risk. The cost should increase as the counterparty risk would deteriorate.

After July 2007, the CDS spreads rise dramatically. The cost of taking unsecured credit risk soars accordingly. The difference between the partially collateralized swap rate and the mid-market swap rate reaches a peak of 117 bps during the financial crisis. These results are in line with economic intuition that the cost of hedging unsecured credit risk increases considerably during periods of market stress. The empirical results lend support to economic implications and corroborate our theoretical analysis.

Figure 3. Time series of collateralized swap rates

This diagram illustrates the time series of the collateralized swap rates. The daily data are from January 1, 2007 to April 5, 2011. The partial-collateralization corresponds to the case where $H_B = 500000$.



5. Conclusion

This article addresses a very important topic of the impact of collateralization on asset prices and risk management. The economical effects and implications of collateral arrangements are essentially a matter of theoretical justification and empirical verification.

We present a comprehensive theoretical framework for pricing financial contracts under different collateral arrangements. The model shows that if a contract is over-collateralized, it is equivalent to a risk free contract. This is consistent with the market practice in which market participants commonly assume that repos and cleared contracts are virtually free of default risk. If, however, a contract is partially collateralized, its CSA value is less than the risk-free value but greater than the non-CSA value.

Empirically, we measure the effect of collateralization on pricing and compare it with model-implied prices. This calls for data on financial contracts that have different collateral arrangements but are similar otherwise. The empirical results show that the model-implied prices are quite close to the market-quoted prices, which suggests the model is fairly accurate in pricing collateralized contracts.

We find strong evidence that financial institutions are sprinting to comply with the Dodd-Frank Act. In the new practice, contracts are continuously negotiated over-the-counter as usual, but cleared and settled via clearinghouses, as clearinghouses claim that cleared contracts would be economically equivalent to their OTC counterparts. As a result, swap premia in cleared market are determined in a similar way to those in OTC markets. We argue that this practice may not be appropriate because we notice that the clearing mechanics of a clearinghouse changes the risk structure and thereby the asset prices. In fact, we find that cleared derivatives are not economically equivalent to their OTC counterparts.

Our theoretical and empirical analysis shows that collateralization affects asset prices. The effects are time varying. These findings may be of interest to regulators, academics and practitioners.

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