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Necessity of budget deficit in a growing economy where people hold money and leave a bequest

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Abstract

The main result is that if people want to hold money, then budget deficits cannot be avoided to achieve full employment when the economy grows at a positive rate under constant prices. We examine the existence of budget deficits in a growing economy in which consumers derive utility not only from consumption but also from money holding and bequest. We use an overlapping generations model in which consumers leave the bequests to descendants and hold money, and we use a Barro-type utility function, where people include the utility of their children in their own utility. We will show that budget deficit is necessary to maintain full employment under constant prices in a growing economy. Thus, the balanced budget cannot achieve full employment under constant prices. Moreover, we show that if the actual budget deficit is smaller (larger) than that which is necessary and sufficient for full employment under constant prices, a recession (inflation) occurs.

Keywords: Budget deficis, Growing economy, Money holding, Bequest, Overlapping generations model, J.M.Keynes

1 Introduction

Using an overlapping generations model in which consumers leave a bequest to descendants and hold money, we examine the existence of budget deficits in a growing economy. The significance of government debt and budget deficit, and intergenerational burden have been analyzed by Jyunpei Tanaka. References are J.Tanaka(2010,2011a,2011b,2013). Junpei Tanaka focuses on the intergenerational economic welfare gap due to the presence or absence of government debt, but his main model does not include growth and assumes that all government debt is redeemed by taxes. The interest of this paper lies elsewhere. We are interested in proving that we need budget deficits in a growing economy where consumers hold money. In our model, people save their income primarily through capital, but they are also willing to hold money other than capital for reasons such as liquidity. We use a Barro-type utility function, where people include the utility of their children in their own utility, according to Barro(1974) and Carmichael(1982).

In the next section, we will present our model and prove that a budget deficit is necessary to maintain full employment under constant prices in a growing economy. If the actual budget deficit is smaller (larger) than that which is necessary and sufficient for full employment under constant prices, a recession (inflation) occurs.

In another paper, we will analyze a similar problem in an overlapping generations endogenous growth model according to Grossman and Yanagawa(1993) and Maebayashi and Tanaka, J. (2021) incorporating money holding. Moreover, in another paper, we analyze a similar problem in a traditional neoclassical overlapping generations model with exogenous economic growth without bequest according to Diamond(1965) incorporating money holding. In a previous study about the existence of involuntary unemployment, we used another overlapping generations model without bequest according to Otaki(2007,2009,2015).

This paper is an example of an analysis using a very simple model of the following statement by J.M.Keynes.

"Unemployment develops, that is to say, because people want the moon; — men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control." (Keynes(1936), Chap.17)

In the appendix we show that if money as well as goods are produced by capital and labor, a budget deficit is not necessary for full employment under constant prices. However, if money circulates with a value greater than the cost of production, a budget deficit will be necessary. The difference between the value of money and its production cost is the so-called *seigniorage*. A moderate seigniorage to economic growth is necessary unless the production of money is quite costly.

The Barro-type utility function is often used to prove the neutrality of government debt or budget deficits, i.e., that increasing government spending due to budget deficits has no effect when future increases in the tax burden are taken into account (Barro(1974),Carmichael(1982)). This paper has a very different aim. Using a similar utility function, we prove that budget deficits are essential in a growing economy.

In our model, money is supplied by wage payments, which consumers use to pay taxes and to pay for consumption and investment in capital. What remains after that is money holding. Net money supply equals an increase in money holding.

In this paper, we will assume that the budget deficit will be financed by seigniorage and does not consider government bonds. In terms of liquidity, government bonds are considered to be somewhere between money and capital (stocks), and it would be possible to introduce them into the model, but it would complicate the discussion.

2 Budget deficit in a growing economy with money holding

We introduce the demand for the bequest of the next generation consumers and money demand or money holding as well as the consumption demand of consumers into an overlapping generations model.

2.1 Consumers' behavior

According to Barro(1974), Carmichael(1982), and J.Tanaka(2010), the utilities of consumers include the discounted value of the utilities of their descendants. Consumers live over two periods, younger (working) and older (retired) periods. In the older period, they leave the bequest to the next generation consumers as well as consume goods. The younger consumers in Period t maximize

$$V_t = U_t + \frac{V_{t+1}}{1+\delta},$$

where

$$U_t = \alpha \ln c_t^y + \beta \ln c_{t+1}^o + (1-\alpha-\beta)\ln\left(\frac{m_t}{p_t}\right),\ \alpha > 0,\ \beta > 0,\ 1-\alpha-\beta > 0.$$

V_{t+1} is the indirect utility function of the next generation consumers perceived by the consumers in Period t. n is the rate of population growth. $\delta > 0$ is the discount rate. c_t^y is consumption in the younger period, c_{t+1}^o is consumption in the older period, m_t is money holding of a younger consumer in Period t. p_t is the price in Period t. Let b_t and s_t be the bequest and savings of a consumer in Period t, then

$$b_t = (1+r_t)(s_t - m_t) + m_t - p_{t+1}c_{t+1}^o = (1+r_t)s_t - r_t m_t - p_{t+1}c_{t+1}^o.$$

r_t is the rate of return on capital received in Period $t+1$. p_{t+1} is the price in Period $t+1$. From this

$$s_t = \frac{1}{1+r_t}(b_t + p_{t+1}c_{t+1}^o) + \frac{r_t}{1+r_t}m_t. \tag{1}$$

We assume

$$s_t - m_t > 0.$$

$s_t - m_t$ is the investment to capital in Period t.

The budget constraint for younger consumers in Period t is

$$p_t c_t + s_t = (1-\tau)w_t l_t + \frac{b_{t-1}}{1+n} = (1-\tau)w_t l_t + (1+r_{t-1})\frac{s_{t-1}}{1+n} - r_{t-1}\frac{m_{t-1}}{1+n} - p_t\frac{c_t^o}{1+n}.$$

c_t^o is consumption in the older period of a consumer in the previous generation. b_{t-1} is the bequest left by previous generation consumers which is equally distributed to consumers in Period t. w_t is the nominal wage rate, and τ is the tax rate. r_{t-1}, s_{t-1} and m_{t-1} are the values of the variables in Period $t-1$. If there exists unemployment in Period $t-1$, b_{t-1}, s_{t-1}, c_t^o and m_{t-1} are the average values across employed and unemployed consumers. l_t is the indicator of whether the consumer is employed or not. If he is employed (unemployed), $l_t = 1$ ($l_t = 0$).

The budget constraint is rewritten as follows.

$$p_t c_t + p_{t+1}\frac{c_{t+1}^o}{1+r_t} + \frac{r_t}{1+r_t}m_t = (1-\tau)w_t l_t + \frac{b_{t-1}}{1+n} - \frac{1}{1+r_t}b_t.$$

The Lagrange function is

$$\mathcal{L} = \alpha \ln c_t^y + \beta \ln c_{t+1}^o + (1-\alpha-\beta)\ln\left(\frac{m_t}{p_t}\right) + \frac{V_{t+1}}{1+\delta}$$
$$+\lambda_t\left[p_t c_t + p_{t+1}\frac{c_{t+1}^o}{1+r_t} + \frac{r_t}{1+r_t}m_t - (1-\tau)w_t l_t - \frac{b_{t-1}}{1+n} + \frac{1}{1+r_t}b_t\right].$$

λ_t is the Lagrange multiplier. The first-order conditions for utility maximization are

$$\lambda_t p_t c_t^y = \alpha, \tag{2}$$

$$\lambda_t \frac{p_{t+1}}{1+r_t} c_{t+1}^o = \beta, \tag{3}$$

$$\lambda_t \frac{r_t}{1+r_t} m_t = 1-\alpha-\beta, \tag{4}$$

and

$$\frac{1}{1+\delta}\frac{\partial V_{t+1}}{\partial b_t} = \frac{\lambda_t}{1+r_t} = \frac{\alpha}{p_t(1+r_t)c_t^y}. \tag{5}$$

From the descendant's budget constraint,

$$\frac{\partial V_{t+1}}{\partial b_t} = \frac{1}{p_{t+1}(1+n)}\frac{\alpha}{c_{t+1}^y}. \tag{6}$$

From (2), (3), and (4), we get

$$c_t^y = \frac{1}{p_t}\alpha\left[(1-\tau)w_t l_t + \frac{b_{t-1}}{1+n} - \frac{1}{1+r_t}b_t\right],$$

$$c_{t+1}^o = \frac{1+r_t}{p_{t+1}}\beta\left[(1-\tau)w_t l_t + \frac{b_{t-1}}{1+n} - \frac{1}{1+r_t}b_t\right],$$

and

$$m_t = (1-\alpha-\beta)\frac{1+r_t}{r_t}\left[(1-\tau)w_t l_t + \frac{b_{t-1}}{1+n} - \frac{1}{1+r_t}b_t\right].$$

From (1), the savings are

$$s_t = \frac{1}{1+r_t}b_t + (1-\alpha)\left[(1-\tau)w_t l_t + \frac{b_{t-1}}{1+n} - \frac{1}{1+r_t}b_t\right]$$
$$= \frac{\alpha}{1+r_t}b_t + (1-\alpha)\left[(1-\tau)w_t l_t + \frac{b_{t-1}}{1+n}\right].$$

Let L_t be the employment, and L_t^f be the population in Period t. Assume that the population increases at the rate of $n > 0$, that is,

$$L_t^f = (1+n)L_{t-1}^f.$$

We assume that upto Period $t-1$ full employment is achieved and the prices are constant. The total consumption of the younger consumers in Period t is

$$C_t^y = \frac{1}{p_t}\alpha\left[(1-\tau)w_t L_t + \frac{b_{t-1}}{1+n}L_t^f - \frac{1}{1+r_t}B_t\right].$$

B_t is the total bequest in Period t. The total money holding in Period t is

$$M_t = (1-\alpha-\beta)\frac{1+r_t}{r_t}\left[(1-\tau)w_t L_t + \frac{b_{t-1}}{1+n}L_t^f - \frac{1}{1+r_t}B_t\right].$$

The total consumption of the younger consumers in Period $t+1$ is

$$C_{t+1}^o = \frac{1+r_t}{p_{t+1}}\beta\left[(1-\tau)w_t L_t + \frac{b_{t-1}}{1+n}L_t^f - \frac{1}{1+r_t}B_t\right].$$

On the other hand, the total consumption of the older consumers in Period t is

$$C_t^o = \frac{1+r_{t-1}}{p_t}\beta\left[(1-\tau)w_{t-1}L_{t-1} + \frac{b_{t-2}}{1+n}L_{t-1}^f - \frac{1}{1+r_{t-1}}B_{t-1}\right].$$

The total savings in Period t is

$$S_t = \frac{\alpha}{1+r_t}B_t + (1-\alpha)\left[(1-\tau)w_t L_t + \frac{b_{t-1}}{1+n}L_t^f\right]. \tag{7}$$

The real value of the capital in Period $t+1$ is

$$K_{t+1} = \frac{1}{p_t}(S_t - M_t) = \frac{1}{p_t(1+r_t)}(B_t - M_t + p_{t+1}C_{t+1}^o). \tag{8}$$

The savings of the consumers other than the money holding is invested to the capital.

Similarly, for Period $t-1$,

$$K_t = \frac{1}{p_{t-1}}(S_{t-1} - M_{t-1}) = \frac{1}{p_{t-1}(1+r_{t-1})}(B_{t-1} - M_{t-1} + p_t C_t^o). \tag{9}$$

The total bequest and the total money holding in Period $t-1$ are also written as

$$B_{t-1} = b_{t-1}L_{t-1}^f, \tag{10}$$

and

$$M_{t-1} = m_{t-1}L_{t-1}^f. \tag{11}$$

2.2 Firms' behavior

About production we suppose

$$y_t = F(K_t, L_t) = L_t f(k_t) = L_t F(k_t, 1).$$

y_t is the real GDP in Period t. $k_t = \frac{K_t}{L_t}$. F is the production function. We assume constant returns to scale technology. The profit of a firm is

$$\pi = p_t y_t - r_t K_t - w_t L_t = p_t L_t f(k_t) - p_t r_t K_t - w_t L_t.$$

We normalize the number of firms to one. The conditions for profit maximization are

$$p_t r_t = p_t \frac{\partial F}{\partial K_t} = p_t f'(k_t),$$

$$w_t = p_t \frac{\partial F}{\partial L_t} = p_t[f(k_t) - f'(k_t)k_t].$$

From them

$$p_t r_t K_t + w_t L_t = p_t f'(k_t)K_t + p_t[f(k_t)L_t - f'(k_t)K_t] = p_t f(k_t)L_t = p_t y_t.$$

f' is decreasing in $k_t = \frac{K_t}{L_t}$.

2.3 Steady state and budget deficit in a growing economy

Now consider a steady state where full employment is achieved with constant prices, consumption per capita, interest rate, wage rate, and bequest per capita. Therefore, we have

$$c_t^y = c_{t+1}^y,\ c_{t+1}^o = c_t^o,\ r_t = r_{t+1} = r_{t-1},\ w_t = w_{t+1} = w_{t-1},\ b_t = b_{t+1} = b_{t-1}.$$

Thus,

$$B_t = (1+n)B_{t-1},\ B_{t-2} = \frac{1}{1+n}B_{t-1}.$$

From (5) and (6) with $c_{t+1}^y = c_t^y$ and $p_{t+1} = p_t$, we need in the steady state

$$1 + r_t = (1+n)(1+\delta).$$

The steady state value of k_t satisfies

$$f'(k_t) = (1+n)(1+\delta) - 1 = n + \delta + n\delta.$$

The total nominal supply in Period t is

$$p_t y_t = w_t L_t^f + p_t r_t K_t.$$

The total nominal consumption in Period t is

$$p_t C_t^y + p_t C_t^o = \alpha\left[(1-\tau)w_t L_t^f + B_{t-1} - \frac{1}{1+r_t}B_t\right] + p_t C_t^o$$
$$= \alpha\left[(1-\tau)w_t L_t^f + B_{t-1} - \frac{1+n}{1+r_t}B_{t-1}\right] + p_t C_t^o.$$

Let G_t be the nominal fiscal expenditure in Period t. Then, the total nominal demand is

$$G_t + \alpha\left[(1-\tau)w_t L_t^f + B_{t-1} - \frac{1+n}{1+r_t}B_{t-1}\right] + p_t C_t^o + p_t(K_{t+1} - K_t).$$

$p_t(K_{t+1} - K_t)$ is the investment, which is the required cost to increase the capital from K_t to K_{t+1}. The condition for the market equilibrium is

$$G_t + \alpha\left[(1-\tau)w_t L_t^f + B_{t-1} - \frac{1+n}{1+r_t}B_{t-1}\right] + p_t C_t^o + p_t(K_{t+1} - K_t) = w_t L_t^f + p_t r_t K_t.$$

From this

$$G_t - \tau w_t L_t^f + \alpha\left[(1-\tau)w_t L_t^f + B_{t-1} - \frac{1+n}{1+r_t}B_{t-1}\right] + p_t(K_{t+1} - K_t) + p_t C_t^o$$
$$= (1-\tau)w_t L_t^f + p_t r_t K_t,$$

and so

$$G_t - \tau w_t L_t^f + \alpha\left[(1-\tau)w_t L_t^f + B_{t-1} - \frac{1+n}{1+r_t}B_{t-1}\right] + p_t K_{t+1} + p_t C_t^o$$
$$= (1-\tau)w_t L_t^f + p_t(1+r_t)K_t.$$

By (8), (9), (10) and (11),

$$G_t - \tau w_t L_t^f + \alpha\left[(1-\tau)w_t L_t^f + B_{t-1} - \frac{1+n}{1+r_t}B_{t-1}\right] + S_t - M_t + p_t C_t^o$$
$$= (1-\tau)w_t L_t^f + B_{t-1} - M_{t-1} + p_t C_t^o.$$

Since, by (7)

$$S_t = \frac{\alpha}{1+r_t}B_t + (1-\alpha)\left[(1-\tau)w_t L_t^f + \frac{b_{t-1}}{1+n}L_t^f\right]$$
$$= \frac{\alpha(1+n)}{1+r_t}B_{t-1} + (1-\alpha)\left[(1-\tau)w_t L_t^f + B_{t-1}\right],$$

we obtain

$$G_t - \tau w_t L_t^f = M_t - M_{t-1} = m_t L_t^f - m_{t-1}L_{t-1}^f = n m_t L_{t-1}^f. \tag{12}$$

This is positive so long as $m_t > 0$, that is, $1 - \alpha - \beta > 0$ and $n > 0$. We get the following result.

Proposition *If the economy grows at a positive rate, and the consumers derive positive utility from holding money, we need a positive budget deficit to maintain full employment under constant prices.*

Inflation and recession

If the actual budget deficit is smaller than that in (12), M_t is smaller than $(1+n)M_{t-1}$. Since a consumer divides his budget into consumption, bequests and money holding, his consumption is also smaller than that when (12) is satisfied. Then, a recession (with involuntary unemployment) occurs. On the other hand, if the actual budget deficit is larger than that in (12), M_t is larger than $(1+n)M_{t-1}$, and the nominal consumption also increases. However, under full employment, production can not further increase. Thus, an inflation is triggered.

3. Conclusion

We have mainly proved the following results by incorporating consumers' desire to hold money into an overlapping generations model with Barro-type bequest motives.

1. The budget deficit is necessary and inevitable to maintain full employment under constant prices

when people derive utility from money holding as well as consumption and the economy grows at a positive rate.
2. If the budget deficit is insufficient (excessive), a recession (inflation) occurs.

Thus, if money is held in reserve by the people, then the idea of a balanced budget is inconsistent with economic growth.
Although we considered only growth due to population growth, we expect to obtain similar results for growth due to technological progress.

Appendix: When Money is Produced

We suppose that money is produced by capital and labor. The production function is

$$y_t + \frac{\widehat{M}}{p_t} = F(K_t, L_t) = L_t f(k_t) = L_t F(k_t, 1).$$

$\widehat{M}$ is the supply of money in Period t. It is the total cost to produce money in Period t. The market equilibrium condition for the goods is

$$G_t + \alpha\left[(1-\tau)w_t L_t^f + B_{t-1} - \frac{1+n}{1+r_t}B_{t-1}\right] + p_t C_t^o + p_t(K_{t+1} - K_t) = w_t L_t^f + p_t r_t K_t - \widehat{M}.$$

The condition for the money market equilibrium is that the supply of money equals an increase in money holding. Therefore,

$$\widehat{M} = M_t - M_{t-1}.$$

Then, in the steady state, (12) is reduced to

$$G_t - \tau w_t L_t^f = 0.$$

Therefore, if money as well as goods are produced by capital and labor, a budget deficit is not necessary for full employment under constant prices.
However, if money circulates with a value greater than the cost of production, a budget deficit will be necessary. In that case we have

$$\widehat{M} < M_t - M_{t-1}.$$

Then,

$$M_t - M_{t-1} - \widehat{M}$$

is the *seigniorage*. A moderate seigniorage to economic growth is necessary unless the production of money is quite costly.

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