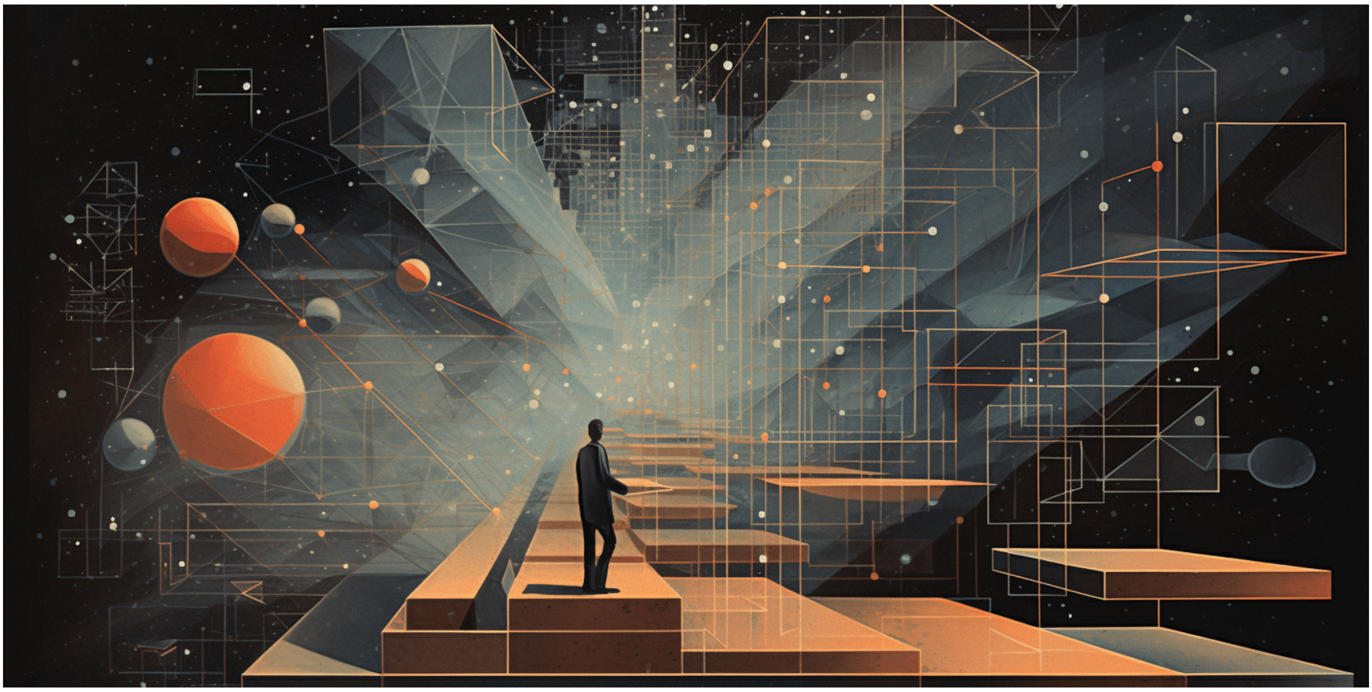
A Study on the Absolute Stationary Inertial Frame and the Relative Velocity, Inertia Mass, Momentum and Kinetic Energy in the Inertial Frame moving relative to it

JiniL Mok



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YouTube of this paper https://www.youtube.com/watch?v=gSAlaDPVbcE&t=155s

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Abstract

Assuming that an absolute stationary inertial frame exists in the universe and the speed of light is constant only in the absolute stationary inertial frame, new equations for inertial mass, momentum and kinetic energy in a moving inertial frame are derived.

In the process of deriving the new equations, an experiment was presented to obtain the velocity of the inertial frame moving relative to the absolute stationary inertial frame. If this experiment is successful, we could find out how fast and in which direction our Earth is moving in space.

1.0 Introduction

Albert Einstein derived the equations for time dilation, mass increase, and kinetic energy in the theory of special relativity published in 1905, assuming that light speed is constant in all inertial frames and the absolute stationary inertial frame does not exist in the universe [R-1].

In this study, new equations for inertial mass, momentum, and kinetic energy in an inertial frame moving relative to the absolute stationary inertial frame are derived by assuming that an absolute stationary inertial frame exists and using the two assumptions for the speed of light below.

- Assumption 1: An absolute stationary inertial frame exists in the universe
- Assumption 2: The speed of light observed in an absolute stationary inertia frame is constant regardless of the movement of the light source. That is, light emitted from both a moving and a stationary object have the same speed in an absolute stationary inertial frame.
- Assumption 3: Based on Michelson-Morley's experiment [R-2], in a moving inertial frame, the round-trip speed of light in all directions is the same and is equal to the speed of light in the absolute stationary inertial frame.

2.0 Conversion Factors to Absolute Stationary Inertial Frame

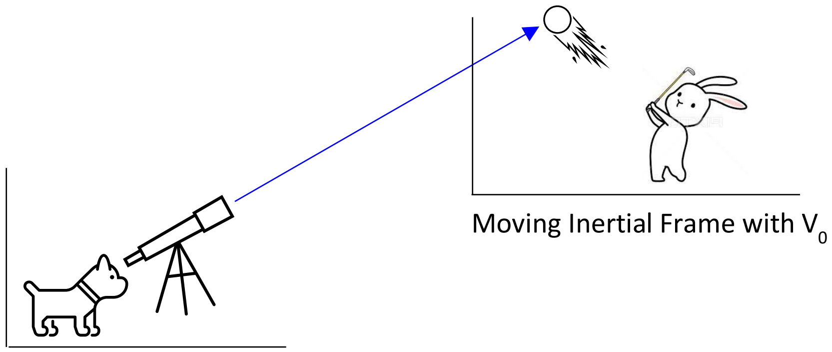

Figure 1. Observe a golf ball hit in an inertial frame flying with velocity v_0 in an absolute stationary inertial frame

As shown in Figure 1, in order to calculate the kinetic energy of an object thrown from a moving inertial frame in terms of an absolute stationary inertial frame, the following four conversion factors are used.

- Length Conversion Factor
- Time Conversion Factor
- Velocity Conversion Factor
- Mass Conversion Factor

In the process of deriving this equation, an experiment is presented to determine the velocity of an inertial frame moving relative to the absolute stationary inertial frame and the new equation for the relative velocity of an object observed in a moving inertial frame to the absolute stationary inertial frame.

2.1 Factor that converts length observed in the moving inertial frame to length observed in the absolute stationary inertial frame

As shown in Figure 2, consider that a spaceship with a semicircular reflector flies at v_0 speed and generates light at the center of the spaceship.

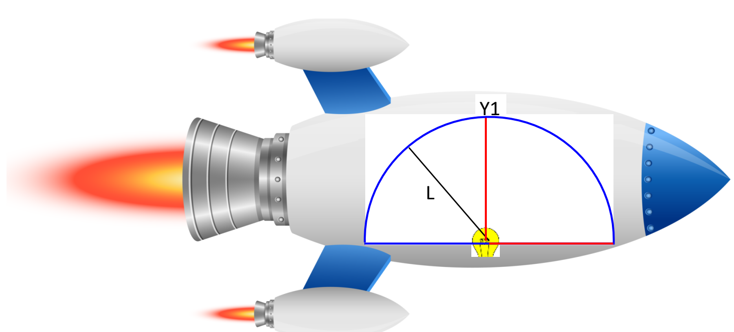

Figure 2. A spaceship flying at velocity v_0 relative to the absolute stationary inertia frame

The path of light emitted from the source at the center of the spaceship hits point Y1. Point Y1 reflects the light back to the source. The motion of this light path is observed from the absolute stationary inertial frame as follows:

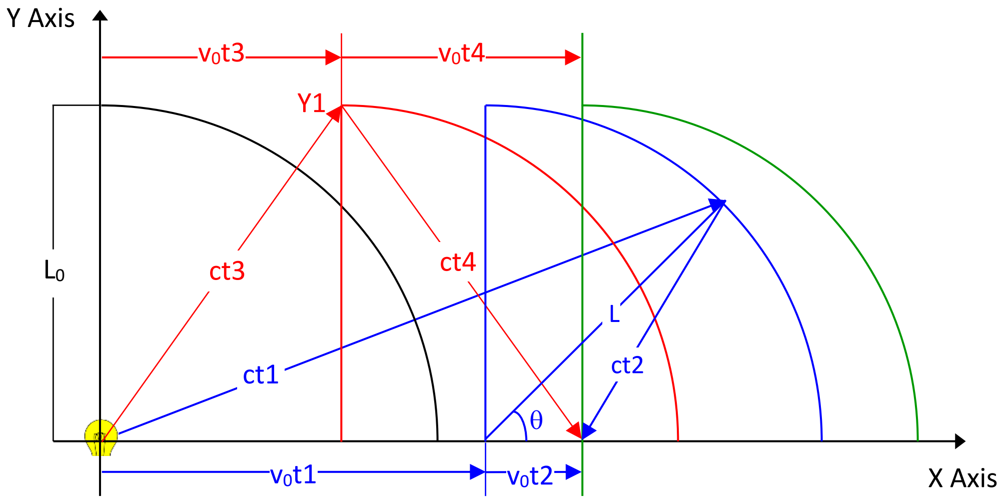

Figure 3. The path of light in a spaceship observed in an absolute stationary inertial frame

<u>The path of light reflected at point Y1 and returned to the center of the spaceship</u>

Assuming no length contraction in the Y axis, the time t3 + t4, represents the time it takes for the emitted light to hit Y1 point and reflect back to the source. The time t3 + t4 can be calculated as follows in the absolute stationary inertial frame.

t3 : The time it takes for the light emitted from the source at center of the spaceship to reach point Y1 of the reflector.

$$(L_0)^2 + (v_0 t3)^2 = (ct3)^2$$

t4 : The time it takes for the light reflected from point Y1 back to the center of the spaceship.

$$(L_0)^2 + (v_0 t4)^2 = (ct4)^2$$

$$t3 + t4 = \frac{2L_0}{\sqrt{c^2 - {v_0}^2}} \qquad (1)$$

<u>The path of light reflected from a point on the reflector and returned to the center of the spaceship</u>

The time t1 + t2, represents the time it takes for the emitted light to travel from the source at center of the spaceship to any point on the reflector, then back to the source. This can be calculated for the absolute stationary inertial frame as follows:

$$(ct1)^2 = (L\,sin\theta)^2 + (L\,cos\theta + v_0 t1)^2 = L^2 + 2Lcos\theta v_0 t1 + (v_0 t1)^2$$

$$t1 = \frac{Lv_0cos\theta + \sqrt{L^2(cos\theta)^2{v_0}^2 + L^2(c^2 - {v_0}^2)}}{c^2 - {v_0}^2} \tag{2}$$

$$(ct2)^2 = (L\ sin\theta)^2 + (L\ cos\theta - v_0t2)^2 = L^2 - 2Lcos\theta v_0t2 + (v_0t2)^2$$

$$t2 = \frac{-Lv_0cos\theta + \sqrt{L^2(cos\theta)^2{v_0}^2 + L^2(c^2 - {v_0}^2)}}{c^2 - {v_0}^2} \tag{3}$$

$$t1 + t2 = \frac{2L\sqrt{(cos\theta)^2{v_0}^2 + (c^2 - {v_0}^2)}}{c^2 - {v_0}^2} \tag{4}$$

Based on Assumption 3, the speed of light is the same in all directions, so the time for the round-trip t1+t2 (4) for any one point, and time for the round-trip t3+t4 (1) for Y1 must be the same as follows:

$$\frac{2L\sqrt{{v_0}^2(cos\theta)^2 + (c^2 - {v_0}^2)}}{c^2 - {v_0}^2} = \frac{2L0}{\sqrt{c^2 - {v_0}^2}}$$

The length L_0 in a moving inertial frame is observed to be contracted compared to L in the absolute stationary inertial frame as follows:

$$L = \frac{L_0\sqrt{c^2 - {v_0}^2}}{\sqrt{c^2 - {v_0}^2 + {v_0}^2(cos\theta)^2}} = \frac{L_0}{\sqrt{1 + \frac{{v_0}^2(cos\theta)^2}{c^2 - {v_0}^2}}} \tag{5}$$

The length contraction factor R_L that converts length in a moving inertial frame to length in the absolute stationary inertial frame is defined as follows:

$$R_L = \frac{L}{L_0} = \frac{1}{\sqrt{1 + \frac{{v_0}^2(cos\theta)^2}{c^2 - {v_0}^2}}} \tag{6}$$

2.2 Factor that converts time observed in the moving inertial frame to time observed in the absolute stationary inertial frame

In a moving inertial frame, unit time is defined as the time for light to travel a distance L_0 back and forth as shown in Figure 4.

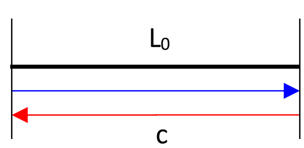

Figure 4. Unit time in an inertial frame

Unit time in an inertial frame is expressed as follows:

$$t0 = \frac{2L_0}{c} \tag{7}$$

Since the absolute stationary inertial frame observes the round-trip time of the moving inertial frame as (1) that is always greater than time of (7), it can be inferred that the unit time in the moving inertial frame is slower than the unit time in the absolute stationary inertial frame.

Using the time t in absolute stationary inertial frame time (1) and the time t_0 in a moving inertial frame time (7), the factor R_T that converts the moving inertial frame time to the absolute stationary inertial frame time is defined as follows;

$$R_T = \frac{t}{t_0} = \frac{\dfrac{2L_0}{c\sqrt{1-\dfrac{{v_0}^2}{c^2}}}}{\dfrac{2L_0}{c}} = \frac{1}{\sqrt{1-\dfrac{v_0}{c^2}}} \tag{8}$$

<u>How to measure the velocity, V_0, of a moving inertial frame</u>

Based on (2) and (3), the difference between the time t1 to reach a surface point and the time t2 to return to the source can be expressed as follows in the absolute stationary inertial frame:

$$\Delta t_a = t1 - t2 = \frac{2Lv_0 cos\theta}{c^2 - {v_0}^2} = \frac{2L_0 v_0 cos\theta}{(c^2 - {v_0}^2)\sqrt{1 + \dfrac{{v_0}^2 (cos\theta)^2}{c^2 - {v_0}^2}}} = \frac{2L_0 v_0 cos\theta}{c^2\sqrt{1-\dfrac{{v_0}^2}{c^2}}\sqrt{1-\dfrac{{v_0}^2(sin\theta)^2}{c^2}}} \tag{9}$$

Considering time dilation, the time difference in a moving inertial frame is then

$$\Delta t_s = \Delta t_a \sqrt{1-\frac{{v_0}^2}{c^2}} = \frac{2L_0 v_0 cos\theta}{c^2\sqrt{1-\dfrac{{v_0}^2(sin\theta)^2}{c^2}}} \tag{10}$$

Max Δt_s occurs at θ=0, so

$$\Delta t_{s\_max} = \frac{2L_0 v_0}{c^2} \tag{11}$$

If the maximum Δt_s_max can be measured by experiment, the velocity of the moving inertial frame can be obtained as follows:

$$v_0 = \Delta t_{s\_max} \frac{c^2}{2L_0} \tag{12}$$

If this experiment is successful, one can determine the direction and velocity of the inertial frame moving in space.

2.3 Factor that converts the velocity observed in the moving inertial frame to the velocity observed in the absolute stationary inertial frame

Using the length conversion factor R_L (6), and the time conversion factor R_T (8), the velocity conversion factor R_v can be obtained. Using this factor R_v , the velocity observed in a moving inertial frame can be converted to the velocity observed in an absolute stationary inertial frame.

$$R_v = \frac{R_L}{R_T} = \frac{1}{\sqrt{1 + \frac{{v_0}^2(cos\theta)^2}{c^2 - {v_0}^2}}} \cdot \sqrt{1 - \frac{{v_0}^2}{c^2}} = \left(1 - \frac{{v_0}^2}{c^2}\right) \cdot \frac{1}{\sqrt{1 - \frac{{v_0}^2(sin\theta)^2}{c^2}}} \tag{13}$$

2.4 Mass Increase Factor, γ due to a Moving Inertial Velocity

When a bomb with at-rest mass m_0 in the absolute stationary inertial frame flies at a speed of v_0 and explodes at a speed v_1, that is relatively much slower than the speed of light in all directions as shown in Figure 5.

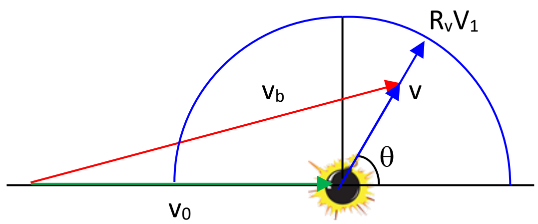

Figure 5. A bomb with at-rest mass m0 explodes with velocity v1 while flying with velocity v_0

As shown in Figure 5, the velocity v_b of the fragment observed in the absolute stationary inertial frame can be expressed as follows:

$$v_b = \sqrt{{v_0}^2 + 2v_0vcos(\theta) + v^2} \tag{14}$$

The mass increase factor γ is inferred using the following two approaches

2.4.1 Approach 1

Assume that the mass increase factor γ due to the velocity v of a moving inertial frame is as follows:

$$\gamma = \left(\frac{c^2}{c^2 - v^2}\right)^n \tag{15}$$

Assuming that the bomb mass increases by a factor of γ_0 in an inertial frame moving at v_0 , let us assume the explosive kinetic energy as follows.

$$E_{k\_bomb\_m} = \frac{1}{2} m_0 {v_1}^2 \gamma_0 = \frac{1}{2} m_0 {v_1}^2 \left(\frac{c^2}{c^2 - {v_0}^2}\right)^n \tag{16}$$

Where γ_0 : the mass increase factor γ at the velocity of the moving inertial frame, v_0

If v_1 is relatively close to 0 compared to the speed of light, the fragment kinetic energy in an arbitrary direction can be calculated as follows, using the mass increase factor γ assumed in (15).

$$\begin{aligned} E_{k\_frag} &= \int_0^{R_v v_1} v_b d(P) = \int_0^{R_v v_1} v_b d\left(\frac{m_0}{4\pi} \gamma v_b\right) = \frac{m_0}{4\pi} \int_0^{R_v v_1} v_b d\left[\left(\frac{c^2}{c^2 - {v_b}^2}\right)^n v_b\right] \\ &= \frac{m_0}{4\pi} c^{2n} \int_0^{R_v v_1} \frac{v_0 cos(\theta) + v}{[c^2 - ({v_0}^2 + 2 v_0 v cos(\theta) + v^2)]^{n+1}} [c^2 \\ &+ (2n-1)({v_0}^2 + 2 v_0 v cos(\theta) + v^2)]\, dv \\ &= \frac{m_0 c^{2n}}{8\pi(n-1)} \left[\frac{(2n-1)({v_0}^2 + 2 v_0 v_1 cos(\theta) + {v_1}^2) - c^2}{[c^2 - ({v_0}^2 + 2 v_0 v_1 cos(\theta) + {v_1}^2)]^n} - \frac{(2n-1){v_0}^2 - c^2}{(c^2 - {v_0}^2)^n}\right] \end{aligned} \tag{17}$$

The total explosive energy of the bomb for n=1/2 can be calculated by integrating Equation (17) over all three dimensions.

$$\begin{aligned} E_{k_{bomb}} &= \int_0^{\pi} E_{k_{frag}} \cdot 2\pi \cdot \sin(\theta)\, d\theta \\ &= \int_0^{\pi} \frac{m_0 c^3}{2} \left[\frac{1}{\sqrt{c^2 - \left({v_0}^2 + 2 v_0 R_v v_1 cos(\theta) + {R_v}^2 {v_1}^2\right)}} - \frac{1}{\sqrt{c^2 - {v_0}^2}}\right] \cdot \sin(\theta)\, d\theta \\ &\approx \int_0^{\pi} \frac{m_0 c^3}{2\sqrt{c^2 - {v_0}^2}} \left[\frac{2 v_0 R_v v_1 cos(\theta) + {R_v}^2 {v_1}^2}{2(c^2 - {v_0}^2)}\right] \cdot \sin(\theta)\, d\theta \\ &\approx \frac{m_0 c^3}{2\sqrt{c^2 - {v_0}^2}} \left[\frac{{v_1}^2}{2(c^2 - {v_0}^2)} 2 \frac{(c^2 - {v_0}^2)}{c^2}\right] \approx \frac{m_0 {v_1}^2}{2} \frac{c}{\sqrt{c^2 - {v_0}^2}} \\ &= \frac{m_0 {v_1}^2}{2} \left(\frac{c^2}{c^2 - {v_0}^2}\right)^{1/2} \end{aligned} \tag{18}$$

Since Equation (18) satisfies Equation (16) for n=1/2, we can infer the mass increase factor as follows.

$$\gamma = \left(\frac{c^2}{c^2 - v^2}\right)^{1/2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19}$$

2.4.2 Approach 2

Assuming that the bomb mass increases by a factor of γ_1 when it explodes at v_1 in an inertial frame moving at v_0 , let us assume the explosive kinetic energy as follows.

$$E_{k\_bomb} = \frac{1}{2} m_0 {v_1}^2 \gamma_1 \tag{20}$$

Where γ_1 : the mass increase factor γ at the velocity of square root of average ${v_b}^2$ for the fragment

$$\text{Average of } {v_b}^2 = \int_0^\pi \left[{v_0}^2 + 2v_0 R_v v_1 cos(\theta) + {R_v}^2 v_1^2\right] \frac{\sin(\theta)}{2} d\theta$$
$$= {v_0}^2 + {v_1}^2 \frac{\left[c^2 - {v_0}^2\right]^{\frac{3}{2}}}{c^2 v_0} tan^{-1}\left(\frac{v_0}{\sqrt{c^2 - {v_0}^2}}\right) \approx {v_0}^2 + {v_1}^2 \frac{\left(c^2 - {v_0}^2\right)}{c^2}$$
$$\approx {v_0}^2 + {v_1}^2 - \frac{{v_0}^2 {v_1}^2}{c^2}$$

If v_1 is relatively close to 0 compared to the speed of light, the kinetic energy of the fragments in an arbitrary direction cab be approximately expressed using the mass increase factors γ_0 and γ_1 as follows:

$$E_{k\_frag} = \int_0^{R_v v_1} v_b d(P) = \int_0^{R_v v_1} v_b d\left(\frac{m_0}{4\pi}\gamma v_b\right) = \frac{m_0}{4\pi}\int_0^{R_v v_1} \{{v_b}^2 d\gamma + \gamma[v_0 cos(\theta) + v_1]dv\}$$
$$\approx \frac{m_0}{4\pi}\left\{\frac{\left[2{v_0}^2 + 2v_0 R_v v_1 cos(\theta) + {R_v}^2 v_1^2\right]}{2}(\gamma_1 - \gamma_0) + \frac{[\gamma_1 v_0 cos(\theta) + \gamma_1 R_v v_1 + \gamma_0 v_0 cos(\theta)]}{2} R_v v_1\right\} \tag{21}$$

Where γ_0 : the mass increase factor γ at the velocity of the moving inertial frame, v_0

By integrating Equation (21) over all three-dimensions, the total explosive energy of the bomb can be obtained as follows:

$$E_{k\_bomb} = \int_0^\pi \lim_{v_1 \to 0} E_{k\_frag} \cdot 2\pi \sin(\theta)\, d\theta$$
$$= \frac{1}{2} m_0 \left\{\left(2{v_0}^2 + 2{v_1}^2 - \frac{2{v_0}^2 {v_1}^2}{c^2}\right)\gamma_1 - \left(2{v_0}^2 + {v_1}^2 - \frac{{v_0}^2 {v_1}^2}{c^2}\right)\gamma_0\right\} \tag{22}$$

Since the explosive energy calculated in (22) from the perspective of an absolute stationary inertial frame should be equal to the assumed explosive energy in (20), $E_{k\_bomb} = \frac{1}{2} m_0 {v_1}^2 \gamma_1$ from the perspective of a moving inertial frame, $\frac{\gamma_0}{\gamma_1}$ can be obtained as follows:

$$\left(2{v_0}^2 + {v_1}^2 - \frac{2{v_0}^2 {v_1}^2}{c^2}\right)\gamma_1 = \left(2{v_0}^2 + {v_1}^2 - \frac{{v_0}^2 {v_1}^2}{c^2}\right)\gamma_0$$

$$\frac{\gamma_0}{\gamma_1} = 1 - \frac{{v_0}^2{v_1}^2}{c^2\left(2{v_0}^2 + {v_1}^2 + \frac{{v_0}^2{v_1}^2}{c^2}\right)} \approx 1 - \frac{{v_0}^2{v_1}^2}{c^2\left(2{v_0}^2 + {v_1}^2\right)} \approx 1 - \frac{{v_1}^2}{2c^2} \approx \sqrt{1 - \frac{{v_1}^2}{c^2}} \tag{23}$$

Based on (23), assuming $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and directly calculating $\frac{\gamma_0}{\gamma_1}$ as (24) , we get the same result as (23).

Therefore, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ can be inferred

$$\frac{\gamma_0}{\gamma_1} = \frac{\sqrt{1 - \frac{{v_0}^2 + {v_1}^2 - \frac{{v_0}^2{v_1}^2}{c^2}}{c^2}}}{\sqrt{1 - \frac{{v_0}^2}{c^2}}} = \frac{\sqrt{c^2 - {v_0}^2 + {v_1}^2 - \frac{{v_0}^2{v_1}^2}{c^2}}}{\sqrt{c^2 - {v_0}^2}} = \sqrt{1 - \frac{{v_1}^2 - \frac{{v_0}^2{v_1}^2}{c^2}}{c^2 - {v_0}^2}}$$
$$= \sqrt{1 - \frac{{v_1}^2(c^2 - {v_0}^2)}{c^2(c^2 - {v_0}^2)}} = \sqrt{1 - \frac{{v_1}^2}{c^2}} \tag{24}$$

3.0 Kinetic Energy, Momentum and Inertial Mass in an Inertial Frame Moving Relative to an Absolutely Stationary Inertial Frame

3.1 Kinetic Energy Observed in Absolute Stationary Inertial Frame

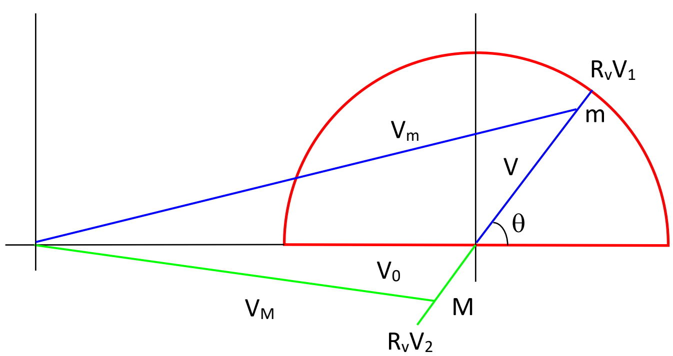

Figure 6. The velocity of an object m thrown with a velocity V_1 in an inertial frame M moving at a velocity V_0 and the velocity change of the inertial frame M, as observed in an absolute stationary frame

As shown in Figure 6, in an inertial frame with mass M moving with velocity v_0 , an object with mass m is thrown at angle θ with a velocity v_1.

Observing this in the absolute frame, the object with mass m moves as $R_v v_1$ by the velocity conversion factor R_v (13) , and in order to conserve momentum, the inertial frame with mass M moves in the opposite direction at $v_2 = \frac{m_0 v_1}{M_0}$ velocity.

Therefore, the velocities of m and M observed in the absolute stationary inertial frame can be expressed as:

$$v_m = \sqrt{{v_0}^2 + 2v_0 v cos(\theta) + v^2} \tag{25}$$

$$v_M = \sqrt{{v_0}^2 - 2v_0 \frac{m_0 v}{M_0} cos(\theta) + \left(\frac{m_0 v}{M_0}\right)^2} \tag{26}$$

Using the mass increase factor γ from (19), m and M are expressed as follows:

$$m = m_0\gamma = \frac{m_0}{\sqrt{1 - \frac{{v_m}^2}{c^2}}} \tag{27}$$

$$M = M_0\gamma = \frac{M_0}{\sqrt{1 - \frac{{v_M}^2}{c^2}}} \tag{28}$$

Using Equations (25) to (28), the kinetic energy observed in the absolute stationary inertial frame can be calculated as follows:

$$\begin{aligned} E_{ka}(v_1) &= \int_0^{R_v v_1} v_m d(m v_m) + \int_0^{R_v v_2} v_M d(M v_M) \\ &= \int_0^{R_v v_1} v_m d(\frac{m_0}{\sqrt{1 - \frac{{v_m}^2}{c^2}}} v_m) + \int_0^{R_v v_2} v_M d(\frac{M_0}{\sqrt{1 - \frac{{v_M}^2}{c^2}}} v_M) \\ &= m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{\left({v_0}^2 + 2v_0 R_v v_1 \cos(\theta) + {R_v}^2 {v_1}^2\right)}{c^2}}} - \frac{1}{\sqrt{1 - \frac{{v_0}^2}{c^2}}} \right] - \frac{m_0 v_0 R_v v_1 \cos(\theta)}{\left(1 - \frac{{v_0}^2}{c^2}\right)^{\frac{3}{2}}} \end{aligned} \tag{29}$$

3.2 Momentum observed in Moving Inertial Frame

The moving inertial kinetic energy E_{KS} can be calculated with the momentum P_s, and velocity v as follows:

$$E_{ks}(v_1) = \int_0^{v_1} v d(P_S) \tag{30}$$

Since the kinetic energy E_{Ka} calculated in the absolute stationary inertial frame, and the kinetic energy E_{KS} calculated in the moving inertial frame must be the same, $d(Ps)$ can be obtained by differentiating (30) and (29) as follows:

$$d\big(E_{Ks}(v)\big) = vd(P_S) = d(E_{Ka}(v))$$

$$d(P_S) = \frac{d\big(E_{Ka}(v)\big)}{v} = \frac{m_0 c^3}{v} d\left\{\frac{1}{\sqrt{c^2 - \big(v_0{}^2 + 2v_0 R_v v\cos(\theta) + R_v{}^2 v^2\big)}} - \frac{1}{\sqrt{c^2 - v_0{}^2}} - \frac{v_0 R_v v_1 \cos(\theta)}{(c^2 - v_0{}^2)^{\frac{3}{2}}}\right\} \quad (31)$$

The momentum in the moving inertial frame can be obtained by integrating $d(Ps)$ (31) as follows:

$$Ps(v_1) = \int_0^{v_1} d(Ps) = m_0 c^3 R_v \int_0^{v_1} \frac{1}{v}\left\{\frac{v_0\cos(\theta) + R_v v}{\big[c^2 - \big(v_0{}^2 + 2R_v v_0 v\cos(\theta) + R_v{}^2 v^2\big)\big]^{3/2}} - \frac{v_0\cos(\theta)}{(c^2 - v_0{}^2)^{3/2}}\right\} dv$$

$$= \frac{m_0 R_v}{\left(1 - \frac{v_0{}^2}{c^2}\right)}\left\{\frac{R_v v_1 + 2v_0\cos(\theta)}{\sqrt{1 - \frac{\big(v_0{}^2 + 2R_v v_0 v_1\cos(\theta) + R_v{}^2 v_1{}^2\big)}{c^2}}} - \frac{v_0\cos(\theta)}{\sqrt{1 - \frac{v_0{}^2}{c^2}}}\left[2 - ln(2) + ln\left(\sqrt{1 - \frac{2R_v v_0 v_1\cos(\theta) + R_v{}^2 v_1{}^2}{c^2 - v_0{}^2}} - \frac{v_0 R_v v_1\cos(\theta)}{c^2 - v_0{}^2} + 1\right)\right]\right\} \quad (32)$$

3.3 Inertial Mass in a Moving Inertial Frame

Using the momentum Equation (32) in the moving inertial frame, the inertial mass in the moving inertial frame can be obtained as follows:

$$m = \lim_{v_1 \to 0} \frac{Ps(v_1)}{v_1} = \frac{m_0}{\sqrt{1 - \frac{v_0{}^2}{c^2}}}\left(1 + \frac{2v_0{}^2(cos\theta)^2}{c^2 - v_0{}^2(sin\theta)^2}\right) \quad (33)$$

Based on (33), the inertial mass is a function of the velocity of the inertial frame and direction.

4.0 Analysis in 3-Dimensional Inertial Frame

4.1 3-Dimensional Velocity Conversion Factor

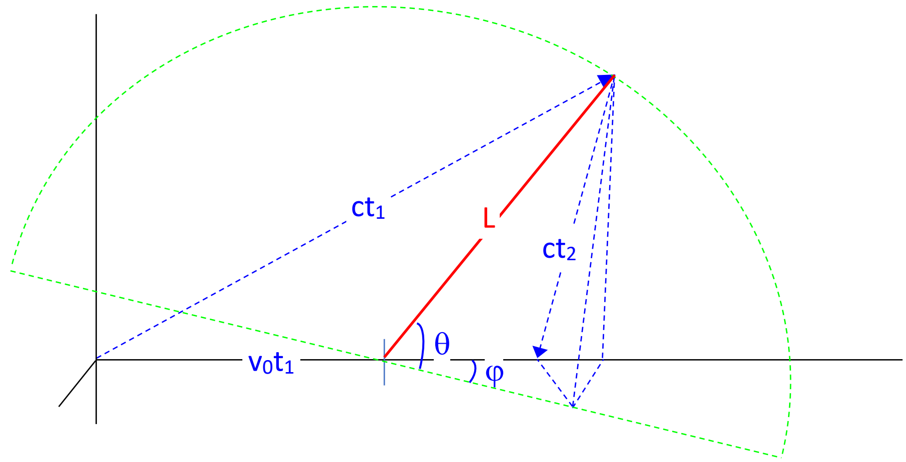

Figure 7. The path of light in a spaceship, as observed from a 3-dimensional absolute stationary inertial frame

Based on the method in Section 2.1, the 3D length conversion factor R_{L_3D} can be obtained as follows.

$$(ct1)^2 = (v_0t1)^2 + 2v_0t1L\cos\theta\cos\varphi + (L\cos\theta)^2 + (L\sin\theta)^2 = (v_0t1)^2 + 2v_0t1L\cos\theta\cos\varphi + L^2$$

$$(ct2)^2 = (v_0t2)^2 - 2v_0t2L\cos\theta\cos\varphi + (L\cos\theta)^2 + (L\sin\theta)^2 = (v_0t2)^2 - 2v_0t2L\cos\theta\cos\varphi + L^2$$

$$t1 + t2 = \frac{2L\sqrt{(cos\theta cos\varphi)^2{v_0}^2 + (c^2 - {v_0}^2)}}{c^2 - {v_0}^2} \quad (34)$$

Based on assumption 3, t1+t2 (34) must be the same as t3+t4 (1).

$$R_{L\_3D} = \frac{L}{L_0} = \frac{\sqrt{c^2 - {v_0}^2}}{\sqrt{(cos\theta cos\varphi)^2{v_0}^2 + (c^2 - {v_0}^2)}} = \frac{1}{\sqrt{1 + \frac{{v_0}^2(cos\theta cos\varphi)^2}{c^2 - {v_0}^2}}} \quad (35)$$

The 3D velocity conversion factor R_{v_3D} can be obtained using the time conversion factor R_T (8) and the length conversion factor R_{L_3D} (35) as follows:

$$R_{v_{3D}} = \frac{R_{L_{3D}}}{R_T} = \left(\frac{c^2 - {v_0}^2}{c}\right) \cdot \frac{1}{\sqrt{c^2 - {v_0}^2(1 - cos^2\varphi cos^2\theta)}}$$
$$= \left(1 - \frac{{v_0}^2}{c^2}\right) \cdot \frac{1}{\sqrt{1 - \frac{{v_0}^2(1 - cos^2\varphi cos^2\theta)}{c^2}}} = \left(\frac{c^2 - {v_0}^2}{c}\right) \cdot \frac{1}{\sqrt{c^2 - {v_0}^2 sin^2\varphi - {v_0}^2 cos^2\varphi sin^2\theta}} \quad (36)$$

4.2 Maximum Velocity in a Moving Inertial Frame

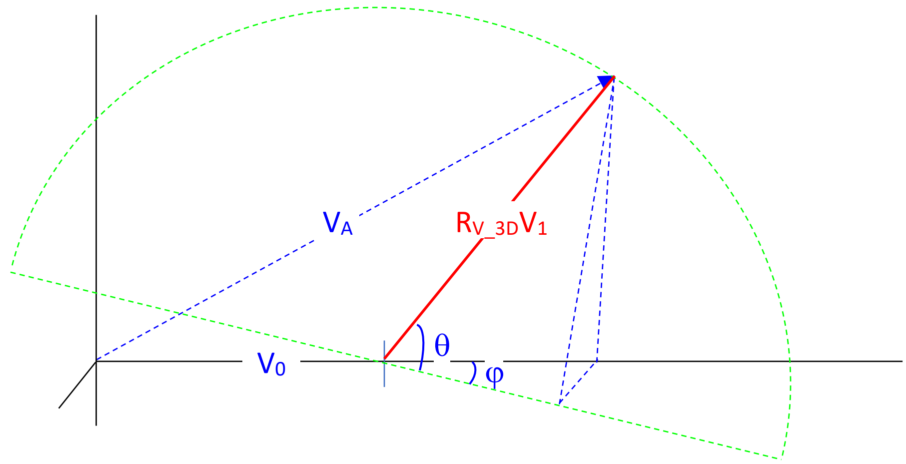

Figure 8. Velocity observed in absolute stationary inertial frame for an object thrown by V_1 in an inertial frame moving at velocity V_0

When an object is thrown at v_1 in an inertial frame moving at v_0 , it is observed as $R_v v_1$ in an absolute stationary inertial frame, so the object velocity observed in the absolute stationary inertial frame can be expressed as follow:

$$v_A = \sqrt{{v_0}^2 + 2v_0 R_{v\_3D} v_1 \cos\varphi\cos\theta + {R_{v\_3D}}^2 {v_1}^2}$$

$$= \sqrt{{v_0}^2 + 2v_0 v_1 \frac{c^2 - {v_0}^2}{c\sqrt{c^2 - {v_0}^2(1 - cos^2\varphi cos^2\theta)}}\cos\varphi\cos\theta + \left(\frac{c^2 - {v_0}^2}{c\sqrt{c^2 - {v_0}^2(1 - cos^2\varphi cos^2\theta)}}\right)^2 {v_1}^2} \quad (37)$$

Since the velocity v_A observed in an absolute stationary inertial frame cannot exceed the speed of light, c, the maximum value of v_1 can be obtained by substituting $v_A = c$ in (37) as follows:

$$v_{1\_max} = \frac{c}{c^2 - {v_0}^2}\sqrt{c^2 - {v_0}^2(1 - cos^2\varphi cos^2\theta)} \cdot \left[-v_0 cos\varphi cos\theta + \sqrt{c^2 - {v_0}^2(1 - cos^2\varphi cos^2\theta)}\right] \quad (38)$$

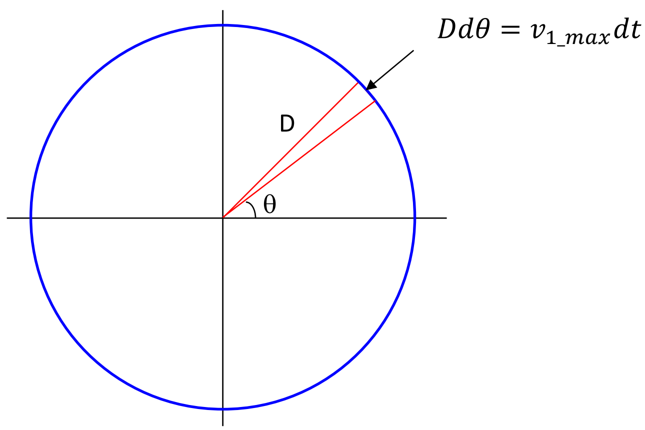

Figure 9. Circular motion at the maximum velocity of the inertial system

At any φ, the average velocity of $v_{1\_\max}$ in 0 < θ < 2π can be obtained by dividing the circumference of a circle with radius D, that is, 2πD by the time required for circular motion. As shown in Figure 9, the time it takes to go around the circle, $T_{2\pi D}$ is obtained by integrating the time to travel $Dd\theta$ with $v_{1\_\max}$, that is, $\frac{Dd\theta}{v_{1\_\max}}$ over 0 < θ < 2π.

$$T_{2\pi D} = \int_0^{2\pi} \frac{Dd\theta}{v_{1\_\max}} = \frac{D}{c}\int_0^{2\pi}\left[1+\frac{v_0 cos\varphi cos\theta}{\sqrt{c^2 - {v_0}^2 sin^2\varphi - {v_0}^2 sin^2\theta}}\right] d\theta = \frac{2\pi D}{c} \tag{39}$$

Since the time for circular motion at the maximum velocity is constant regardless of the direction, the average maximum velocity, $v_{1\_ave\_\max}$ is always c. This means that the circular motion of an object in a moving inertial frame cannot exceed the speed of light, c in any direction

$$v_{1\_ave\_max} = \frac{2\pi D}{T_{2\pi D}} = c \tag{40}$$

Suppose we observe the forward light, that is, the light in the direction of $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$ and $\frac{-\pi}{2} < \varphi < \frac{\pi}{2}$ from a spaceship moving at the speed of light.

When v_0 is close to c, Equation (38) converges to c/2 for the direction of $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$ and $\frac{-\pi}{2} < \varphi < \frac{\pi}{2}$ as follows:

$$\begin{aligned} v_{1\_\max\ \_c\_\text{front}} &= \lim_{v_0 \to c} v_{1\_max} \\ &= \lim_{v_0\to c}\frac{c}{-2v_0}\left\{\frac{-v_0(1-cos^2\varphi cos^2\theta)}{\sqrt{c^2-{v_0}^2(1-cos^2\varphi cos^2\theta)}}\Big[-v_0 cos\varphi cos\theta \right. \\ &+ \sqrt{c^2-{v_0}^2(1-cos^2\varphi cos^2\theta)}\Big] \\ &\left. + \sqrt{c^2-{v_0}^2(1-cos^2\varphi cos^2\theta)}\left[-cos\varphi cos\theta - \frac{v_0(1-cos^2\varphi cos^2\theta)}{\sqrt{c^2-{v_0}^2(1-cos^2\varphi cos^2\theta)}}\right]\right\} \\ &= \frac{c}{2} \end{aligned} \tag{41}$$

4.3 3-Dimensional Kinetic Energy, Momentum and Inertial Mass in a Moving Inertial Frame

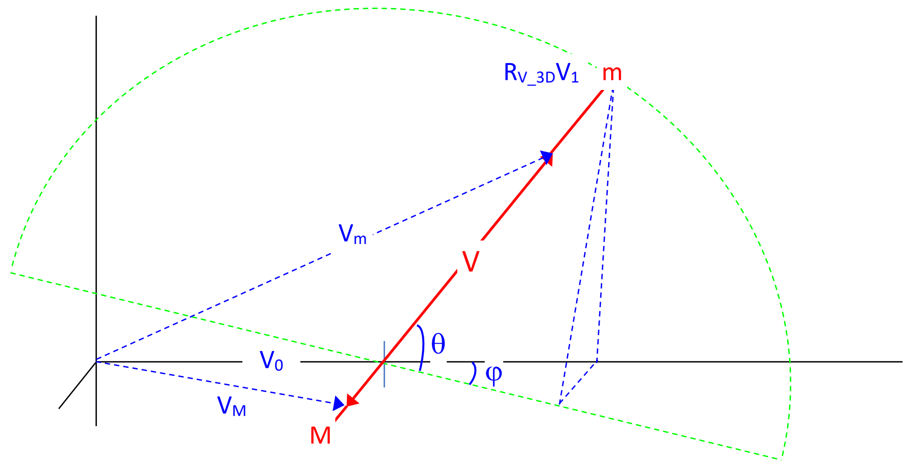

Figure 10. The velocity of an object m thrown with a velocity V_1 in an inertial frame M moving at a velocity V_0 and the velocity change in the inertial frame M, as observed from an 3D absolute stationary inertial frame

The velocity of an object m thrown in a moving inertial frame and the velocity of the inertial frame M can be obtained as follows.

$$v_m = \sqrt{{v_0}^2 + 2v_0 v\cos\theta\cos\varphi + v^2}$$

$$v_M = \sqrt{{v_0}^2 - 2v_0\frac{m_0 v}{M_0}\cos\theta\cos\varphi + \left(\frac{m_0 v}{M_0}\right)^2}$$

Based on the method used in Section 3.1, the 3D kinetic energy in an inertial frame can be obtained as follows:

$$E_{ka}(v_1) = \int_0^{R_{v\_3D}v_1} v_m d(mv_m) + \int_0^{R_{v\_3D}v_2} v_M d(Mv_M)$$

$$= m_0c^2\left[\frac{1}{\sqrt{1-\frac{\left({v_0}^2 + 2v_0R_vv_1\cos\theta\cos\varphi + {R_v}^2{v_1}^2\right)}{c^2}}} - \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}}\right] - \frac{m_0v_0R_vv_1\cos\theta\cos\varphi}{\left(1-\frac{{v_0}^2}{c^2}\right)^{\frac{3}{2}}} \quad (42)$$

Based on the method used in Section 3.2, the 3D momentum in an inertial frame can be obtained as follows.

$$P_{S\_3D}(\theta,\varphi,v_1)=\frac{m_0R_v}{\left(1-\frac{v_0{}^2}{c^2}\right)}\left\{\frac{R_vv_1+2v_0\cos\theta\cos\varphi}{\sqrt{1-\frac{\left(v_0{}^2+2R_vv_0v_1\cos\theta\cos\varphi+R_v{}^2v_1{}^2\right)}{c^2}}}-\frac{v_0\cos\theta\cos\varphi}{\sqrt{1-\frac{v_0{}^2}{c^2}}}\left[2-ln(2)+ln\left(\sqrt{1-\frac{2R_vv_0v_1\cos\theta\cos\varphi+R_v{}^2v_1{}^2}{c^2-v_0{}^2}}-\frac{v_0R_vv_1\cos\theta\cos\varphi}{c^2-v_0{}^2}+1\right)\right]\right\} \quad (43)$$

Based on the method used in Section 3.3, the 3D inertial in an inertial frame can be obtained as follows.

$$m_{3D}=\lim_{v_1\to 0}\frac{P_{S\_3D}(v_1)}{v_1}=\lim_{v_1\to 0}\frac{dP_{S\_3D}(v_1)}{dv_1}=\frac{m_0}{\sqrt{1-\frac{v_0{}^2}{c^2}}}\left(1+\frac{2v_0{}^2cos^2\theta cos^2\varphi}{c^2-v_0{}^2(1-cos^2\theta cos^2\varphi)}\right) \quad (44)$$

4.4 Hafele-Keating Experiment

The Hafele-Keating experiments [R-3] and [R-4] were conducted in 1971 to verify special relativity. This experiment was carried out with a cesium atomic clock. Four atomic clocks were placed in an airport, and four atomic clocks were loaded on an airplane to circle the earth in the east and west directions.

The results of the experiment are as follows:

	nanoseconds, predicted			Experiment Result
	gravitational (general relativity)	kinematic (special relativity)	Total	nanoseconds, measured
Eastward	+144 ±14	−184 ±18	−40 ±23	−59 ±10
Westward	+179 ±18	+96 ±10	+275 ±21	+273 ±7

Considering the predicted time gain by general relativity, the experimental results showed the time gain by special relativity in the eastward plane is -0.59x10^{-7} – 1.44x10^{-7} = -2.03x10^{-7} and the time gain in the westward plane is 2.73x10^{-7} – 1.79x10^{-7} = 9.4x10^{-8}. According to special relativity, since all motion is relative, the time gain experienced in an airplane flying in both east and west directions should be negative.

The result of this experiment is interpreted as follows according to the theory presented in this paper.

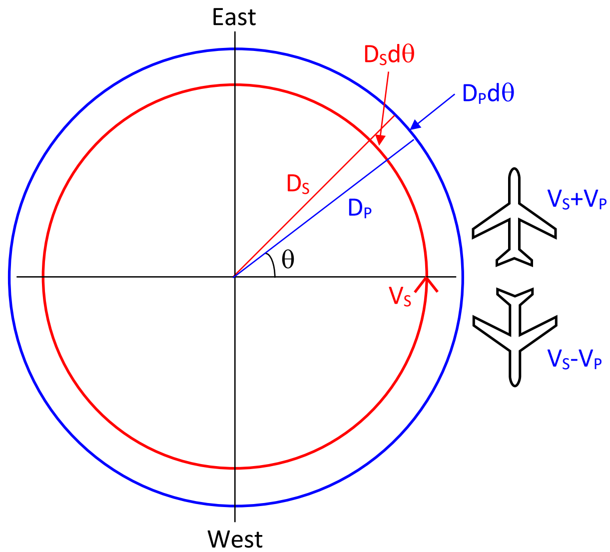

Figure 11. Airplane travel around the Earth, which rotates in the east

Assuming the following conditions in Figure 11

- Earth Radius $D_S = 6371\ km$
- Earth's Rotation Velocity and Direction $v_S = 1670\ km/hr,\ East$
- Airplane Altitude from the center of the Earth $D_P = 6381\ km$
- Airplane Velocity relative to Earth's surface $v_P = 1000\ km/hr$

The inner circle (radius D_S) is the Earth's surface and is rotating at a velocity v_S and the outer circle (radius D_P) is the airplane's path.

The time at the center of the Earth (no Earth's rotation) that it takes for the Earth surface to pass $D_S d\theta$ at the Earth's rotation velocity v_S is:

$$dt_{EC} = \frac{D_S d\theta}{v_S} \tag{45}$$

This time can be expressed as follows in the absolute inertia frame.

$$dt_{Abs\_EC} = dt_{EC}\frac{c}{\sqrt{c^2 - {v_0}^2}} = \frac{D_S d\theta}{v_S}\frac{c}{\sqrt{c^2 - {v_0}^2}} \tag{46}$$

The time at the Earth's surface, dt_{ES} , is expressed as (47) using the time in absolute stationary inertial frame (46) and the Earth's surface velocity v_{Abs_ES} obtained based on (37).

$$dt_{ES} = \frac{D_S d\theta}{v_S} \frac{c}{\sqrt{c^2 - {v_0}^2}} \sqrt{1 - \frac{{v_{Abs\_ES}}^2}{c^2}} \\ = \frac{D_S d\theta}{v_S} \frac{c}{\sqrt{c^2 - {v_0}^2}} \sqrt{1 - \frac{{v_0}^2 + 2v_0 R_{v_{3D}} v_S \cos\varphi \cos\theta + {R_{v\_3D}}^2 {v_S}^2}{c^2}} \tag{47}$$

Since an airplane traveling eastward is in the same direction as the Earth's rotation, the time in this airplane can be expressed as (48) using the time and airplane velocity v_{Abs_PE} from the absolute stationary inertial frame.

$$dt_{PE} = \frac{D_P d\theta}{v_S + v_P} \frac{c}{\sqrt{c^2 - {v_0}^2}} \sqrt{1 - \frac{{v_{Abs\_PE}}^2}{c^2}} \\ = \frac{D_P d\theta}{v_S + v_P} \frac{c}{\sqrt{c^2 - {v_0}^2}} \sqrt{1 - \frac{{v_0}^2 + 2v_0 R_{v_{3D}} (v_S + v_P) \cos\varphi \cos\theta + {R_{v\_3D}}^2 (v_S + v_P)^2}{c^2}} \tag{48}$$

Since an airplane traveling westward is in the opposite direction to the Earth's rotation, the time in this airplane can be expressed as (49) using the time and airplane velocity v_{Abs_PW} from the absolute stationary inertial frame.

$$dt_{PW} = \frac{D_P d\theta}{v_S - v_P} \frac{c}{\sqrt{c^2 - {v_0}^2}} \sqrt{1 - \frac{{v_{Abs\_PW}}^2}{c^2}} \\ = \frac{D_P d\theta}{v_S - v_P} \frac{c}{\sqrt{c^2 - {v_0}^2}} \sqrt{1 - \frac{{v_0}^2 + 2v_0 R_{v_{3D}} (v_S - v_P) \cos\varphi \cos\theta + {R_{v\_3D}}^2 (v_S - v_P)^2}{c^2}} \tag{49}$$

Since the Earth rotates, the airplane departing in the eastward direction circles the Earth twice and meets the departing point (Airport) at θ_E, which circles the Earth once.

$$\theta_E = \frac{2\pi[2D_P v_S - D_S(v_S + v_P)]}{D_S(v_S + v_P) - D_P v_S} = 243.73 \tag{50}$$

Assuming φ =0 and $v_0 = 300km/sec$, the time passed at the airport and in the plane traveling eastward can be calculated as follows:

$$T_{Plane\_East} = \int_0^{4\pi+\theta_E} dt_{PE} \\ = \frac{D_P}{(v_S + v_P)} \frac{c}{\sqrt{c^2 - {v_0}^2}} \int_0^{4\pi+\theta_E} \sqrt{1 - \frac{{v_0}^2 + 2v_0 R_{v_{3D}} (v_S + v_P) \cos\varphi \cos\theta + {R_{v\_3D}}^2 (v_S + v_P)^2}{c^2}} d\theta \\ = 40.19837554470179\ hour \tag{51}$$

$$T_{Earth\_East} = \int_0^{2\pi+\theta_E} dt_{ES}$$

$$= \frac{D_S}{v_S} \frac{c}{\sqrt{c^2 - {v_0}^2}} \int_0^{2\pi+\theta_E} \sqrt{1 - \frac{{v_0}^2 + 2v_0 R_{v_{3D}} v_S \cos\varphi \cos\theta + {R_{v\_3D}}^2 {v_S}^2}{c^2}} d\theta \quad (52)$$

The time gain in the plane travelling eastward is

$T_{Plane\_East} - T_{Earth\_East} = -2.39 \cdot 10^{-7}$ (Time gain measured during eastward travel, $-2.03 \cdot 10^{-7}$)

Since the Earth rotates, the airplane departing in the westward direction meets the departing point (Airport) at θ_W , which circles the earth once.

$$\theta_W = \frac{2\pi[0 \cdot D_P v_S - D_S(v_S - v_P)]}{D_S(v_S - v_P) - D_P v_S} = 240.57 \quad (53)$$

The time passed at the airport and in the plane traveling westward can be calculated as follows:

$$T_{Plane\_West} = \int_0^{\theta_W} dt_{PW}$$

$$= \frac{D_P}{(v_S - v_P)} \frac{c}{\sqrt{c^2 - {v_0}^2}} \int_0^{\theta_W} \sqrt{1 - \frac{{v_0}^2 + 2v_0 R_{v_{3D}} (v_S - v_P) \cos\varphi \cos\theta + {R_{v\_3D}}^2 (v_S - v_P)^2}{c^2}} d\theta \quad (54)$$

$$= 39.98818631489202\ hour$$

$$T_{Earth\_West} = \int_0^{2\pi+\theta_W} dt_{ES}$$

$$= \frac{D_S}{v_S} \frac{c}{\sqrt{c^2 - {v_0}^2}} \int_0^{2\pi+\theta_W} \sqrt{1 - \frac{{v_0}^2 + 2v_0 R_{v_{3D}} v_S \cos\varphi \cos\theta + {R_{v\_3D}}^2 {v_S}^2}{c^2}} d\theta \quad (55)$$

$$= 39.98818631484384\ hour$$

The time gain in the plane travelling westward is

$T_{Plane\_West} - T_{Earth\_West} = 1.74 \cdot 10^{-7}$ (Time gain measured during westward travel, $9.4 \cdot 10^{-8}$)

In this way, it can be predicted that time goes slower in the eastward travel than on the ground and faster in the westward travel, which is consistent with the results of the Hafele-Keating experiment to some extent.

4.5 Twin Paradox

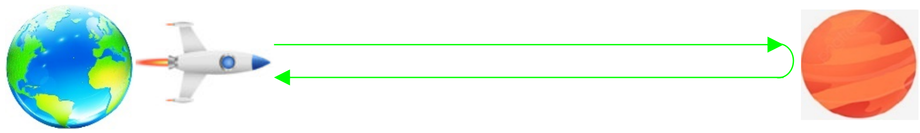

Figure 12. A twin brother travels in a spaceship to an orange star

Let's imagine that the younger twin remains on Earth, and the older twin flies to an orange star in a spaceship and returns. The time that has passed for the younger twin on Earth is T_{twin_earth}, this time is $T_{twin\_abs} = T_{twin\_earth}\frac{\sqrt{c^2-v_A{}^2}}{c}$ in the absolute stationary inertial frame, so the time that has passed for the older twin in the spaceship on the way to the star is

$$T_{twin\_space\_1} = \frac{T_{twin\_earth}}{2}\frac{c}{\sqrt{c^2-v_0{}^2}}\sqrt{1-\frac{v_A{}^2}{c^2}} = \frac{T_{twin\_earth}}{2}\frac{\sqrt{c^2-\left(v_0{}^2+2v_0R_{v_{3D}}v_1\cos\varphi\cos\theta+R_{v\_3D}{}^2v_1{}^2\right)}}{\sqrt{c^2-v_0{}^2}}$$

The time elapsed in the spaceship while returning from the star to Earth can be calculated as follows, since it flies in the opposite direction.

$$T_{twin\_space\_2} = \frac{T_{twin\_earth}}{2}\frac{\sqrt{c^2-\left(v_0{}^2-2v_0R_{v_{3D}}v_1\cos\varphi\cos\theta+R_{v\_3D}{}^2v_1{}^2\right)}}{\sqrt{c^2-v_0{}^2}}$$

Therefore, the time thas has passed for the older twin in the spaceship during the round trip to the orange star is estimated as follows:

$$T_{twin\_space} = \frac{T_{twin\_earth}}{2\sqrt{c^2-v_0{}^2}}\left(\sqrt{c^2-\left(v_0{}^2+2v_0R_{v_{3D}}v_1\cos\varphi\cos\theta+R_{v\_3D}{}^2v_1{}^2\right)} + \sqrt{c^2-\left(v_0{}^2-2v_0R_{v_{3D}}v_1\cos\varphi\cos\theta+R_{v\_3D}{}^2v_1{}^2\right)}\right) \tag{56}$$

For example, assuming $v_0 = 0.01c, \theta = 0, \varphi = 0$, when the twin brother returns from a star 7 light years away from Earth in a spaceship flying at $v_1 = 0.7c$, 20 years have passed for the younger twin on Earth and 14 years have passed for the older twin in the spaceship as per (56).

5.0 Conclusion

The conclusion of this study and the difference from the theory of special relativity are as follows:

	Theory of Special Relativity	Conclusion of this study
Absolute Stationary Inertial Frame	None	Exists
Constancy of Light Speed	Constant in all inertial frames	Constant only in the absolute stationary inertial frame. In a moving inertial frame, the round-trip speed of light is constant.
Velocity of a moving inertial frame	No way to measure	Measurable by experiment $v_0 = \Delta t_{s\_max} \frac{c^2}{2L_0}$
The Speed of light observed in a spaceship flying at the speed of light	c	$\frac{c}{2}$
Twin Paradox	Not explained clearly	$\frac{T_{twin\_earth}}{2\sqrt{c^2 - v_0^2}} \left(\sqrt{c^2 - \left(v_0^2 + 2v_0 R_{v_{3D}} v_1 \cos\varphi \cos\theta + R_{v\_3D}^2 v_1^2\right)} + \sqrt{c^2 - \left(v_0^2 - 2v_0 R_{v_{3D}} v_1 \cos\varphi \cos\theta + R_{v\_3D}^2 v_1^2\right)} \right)$
Hafele-Keating Experiment	Not explained for westward plane clock	Explained why the westward plane clock is faster than the clock in earth
Momentum	$\frac{m_0 v_1}{\sqrt{1 - \frac{v_1^2}{c^2}}}$	$\frac{m_0 R_v}{\left(1 - \frac{v_0^2}{c^2}\right)} \left\{ \frac{R_v v_1 + 2v_0 \cos(\theta)}{\sqrt{1 - \frac{\left(v_0^2 + 2R_v v_0 v_1 \cos(\theta) + R_v^2 v_1^2\right)}{c^2}}} - \frac{v_0 \cos(\theta)}{\sqrt{1 - \frac{v_0^2}{c^2}}} \left[2 - ln(2) + ln\left(\sqrt{1 - \frac{2R_v v_0 v_1 \cos(\theta) + R_v^2 v_1^2}{c^2 - v_0^2}} - \frac{v_0 R_v v_1 \cos(\theta)}{c^2 - v_0^2} + 1 \right) \right] \right\}$
Kinetic Energy	$m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{v_1^2}{c^2}}} - 1 \right]$	$m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{\left(v_0^2 + 2v_0 R_v v_1 \cos(\theta) + R_v^2 v_1^2\right)}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \right] - \frac{m_0 v_0 R_v v_1 \cos(\theta)}{\left(1 - \frac{v_0^2}{c^2}\right)^{\frac{3}{2}}}$
Inertial Mass	$\frac{m_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}$	$\frac{m_0}{\sqrt{1 - \frac{v_0^2}{c^2}}} \left(1 + \frac{2v_0^2 (cos\theta)^2}{c^2 - v_0^2 (sin\theta)^2} \right)$

6.0 References

[R-1] A. Einstein (1905) On The Electrodynamics of moving bodies, https://www.fourmilab.ch/etexts/einstein/specrel/specrel.pdf

[R-2] Michelson-Morley Experiments, https://en.wikipedia.org/wiki/Michelson%E2%80%93Morley_experiment

[R-3] Hafele, J. C.; Keating, R. E. (July 14, 1972). "Around-the-World Atomic Clocks: Predicted Relativistic Time Gains". Science. 177 (4044): 166–168. doi:10.1126/science.177.4044.166.

[R-4] Hafele, J. C.; Keating, R. E. (July 14, 1972). "Around-the-World Atomic Clocks: Observed Relativistic Time Gains". Science. 177 (4044): 168–170. doi:10.1126/science.177.4044.168.