Antineutrinos produced from β decays of neutrons cannot be in coherent superpositions of different mass eigenstates

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The entire wavefunction of the antineutrino-proton-electron system, produced by the β decay of a neutron is analyzed. It is proven that the antineutrino cannot be in coherent superpositions of different mass eigenstates, irrespective of the initial momentum distribution of the neutron.

The discovery of neutrino oscillations represents one of the most remarkable physical advancements achieved during the past three decades [1-6]. These phenomena cannot be interpreted within the framework of the standard model [7]. According to the standard model, there are three generations of leptons, each composed of a charged lepton and a neutrino. Charged leptons gain their mass by the Yukawa interaction, where the left-handed and right-handed leptons are coupled by the Higss scalar field. Each lepton generation has its own flavor. Charged leptons with different flavors have very different masses, e.g, the muon is heavier than the electron by two orders of magnitude. In distinct contrast, all neutrinos are left-handed so that they cannot get mass from the Higss field. The leptons belonging to the same family can be transformed into each other by exchanging a W boson. However, the Lagrangian of the lepton model does not contain any term that can change the flavor of a lepton.

To interpret neutrino oscillations, it was postulated that each neutrino flavor corresponds to a linear superposition of three distinct mass eigenstates [8],

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle , \tag{1}$$

where $\alpha = e, \mu, \tau$ denotes the neutrino flavor, while $j = 1$ to 3 labels the mass eigenstate. When the momentum of the neutrino (p) is much larger than the mass m_j, the energy of the mass component $|\nu_j\rangle$ can be well approximated by $E_j \simeq p + m_j^2/(2p)$. After a propagation time t, $|\nu_j\rangle$ accumulates a phase $\phi_j = -E_j t$. Consequently, the initial flavor eigenstate $|\nu_\alpha\rangle$ evolves to $\sum_j U_{\alpha j} e^{i\phi_j} |\nu_j\rangle$. The probability for the neutrino to remain in the flavor eigenstate $|\nu_\alpha\rangle$ is given by

$$P_{|\nu_\alpha\rangle \to |\nu_\alpha\rangle} = \left| \sum_j U_{\alpha j} U_{\alpha j}^* e^{i\phi_j} \right|^2 . \tag{2}$$

Due to the phase difference accumulated by different neutrino mass eigenstates, $P_{|\nu_\alpha\rangle \to |\nu_\alpha\rangle}$ exhibits oscillatory behaviors. The quantum coherence between the mass eigenstates is responsible for these behaviors.

This interpretation is valid only when the neutrino can be in the superposition state of Eq. (1) at the production. Previously, it was realized that the neutrino emitted by an unstable particle with a definite momentum is necessarily entangled with the particles accompanying the neutrino [9-11]. This entanglement would destroy the coherence between the neutrino's mass eigenstates. To overcome this problem, it was argued that the neutrino can be disentangled with the accompanying particles when the momentum uncertainty of the unstable particle is sufficiently large [9-14]. However, I find that different mass eigenstates of such a neutrino, if they exist, are necessarily correlated with different joint momentum states of the entire system, including the neutrino and the accompanying particles, which prohibits occurrence of interference effects between the mass eigenstates. I will illustrate this point with the β decay of a neutron.

Lemma 1: For the β decay, when there is no mass-momentum entanglement, the produced electron antineutrino has a definite mass.

The wavefunction of the neutron, before undergoing the β decay, can be expanded as

$$|\psi_n\rangle = \int \varphi(\mathbf{P}_n) d^3\mathbf{P}_n |\mathbf{P}_n\rangle , \tag{3}$$

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where $|\mathbf{P}_n\rangle$ denotes the momentum eigenstate of the neutron with the eigenvalue $\mathbf{P}_n$. Suppose that the electron antineutrino produced by the β decay possesses three different mass eigenstates, which are not entangled with different momentum eigenstates of the antineutrino, proton, and electron. Then the state of the composite system can be written as

$$|\psi_{\nu+p+e}\rangle = \int d^3\mathbf{P}_\nu d^3\mathbf{P}_p d^3\mathbf{P}_e F(\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e) |\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e\rangle (\sum_j C_j \left|\bar{\nu}_j\right\rangle). \tag{4}$$

Here $|\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e\rangle$ represents joint momentum eigenstate of the entire system, where the antineutrino, proton, and electron are all in their momentum eigenstates with the eigenvalues $\mathbf{P}_\nu$, $\mathbf{P}_p$, and $\mathbf{P}_e$ respectively. The joint probability amplitude distribution $F(\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e)$ satisfies the normalization condition

$$\int d^3\mathbf{P}_\nu d^3\mathbf{P}_p d^3\mathbf{P}_e \left|F(\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e)\right|^2 = 1. \tag{5}$$

The joint momentum of the entire antineutrino-proton-electron system is essentially in a superposition of infinitely many components, which implies that both the total momentum and energy are undeterministic. Despite these uncertainties, the energy and momentum conservation laws are still satisfied for each momentum component of the wave function, as correctly pointed out in Ref. [9].

We here consider a specific component, denoted as $\left|\mathbf{P}_\nu^0, \mathbf{P}_p^0, \mathbf{P}_e^0\right\rangle$. The momentum conservation law implies that this momentum component originates from the neutron momentum component $\left|\mathbf{P}_n^0\right\rangle$, with

$$\mathbf{P}_n^0 = \mathbf{P}_\nu^0 + \mathbf{P}_p^0 + \mathbf{P}_e^0. \tag{6}$$

The energies of the neutron, proton, and electron associated with the component $\left|\mathbf{P}_\nu^0, \mathbf{P}_p^0, \mathbf{P}_e^0\right\rangle$ are given by

$$\begin{aligned} E_n^0 &= \sqrt{m_n^2 + (P_n^0)^2}, \\ E_p^0 &= \sqrt{m_p^2 + (P_p^0)^2}, \\ E_e^0 &= \sqrt{m_e^2 + (P_e^0)^2}, \end{aligned} \tag{7}$$

where m_n, m_p, and m_e are the masses of the neutron, proton, and electron, respectively. According to the energy conservation law, the antineutrino's energy associated with the component $\left|\mathbf{P}_\nu^0, \mathbf{P}_p^0, \mathbf{P}_e^0\right\rangle$ is definite, given by $E_\nu^0 = E_n^0 - E_p^0 - E_e^0$. Consequently, the antineutrino's mass is also definite, which is equal to $m_\nu = \sqrt{(E_\nu^0)^2 - (P_\nu^0)^2}$. This leads to $m_j = m_\nu$ when $C_j \neq 0$. This result is inconsistent with the postulation that each flavor eigenstate is a linear superposition of three different mass eigenstates. If the flavor oscillations are caused by nonzero mass differences, such a state cannot exhibit any oscillatory behavior.

lemma 2: For the β decay, different mass eigenstates of the produced electron antineutrino are necessarily correlated with different joint antineutrino-proton-electron momentum states.

Generally, the wavefunction of the entire system produced by the β decay can be written in the form of

$$|\psi\rangle = \sum_j \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} d^3\mathbf{P}_{p,j} d^3\mathbf{P}_{e,j} G(\mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j}) |\mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j}\rangle \left|\bar{\nu}_j\right\rangle, \tag{8}$$

where σ_j denotes the distribution region of the joint antineutrino-proton-electron momentum associated with the antineutrino mass eigenstate $\left|\bar{\nu}_j\right\rangle$. In order to satisfy the condition $m_1 \neq m_2 \neq m_3$, there should not be any overlapping between the momentum distribution regions associated with different mass eigenstates, i.e., $\sigma_j \cap \sigma_k = \oslash$ for $j \neq k$. This can be interpreted as follows. Suppose that there is an overlapping between the regions σ_j and σ_k with $j \neq k$. Then, according to the aforementioned analysis, both m_j and m_k can be uniquely determined by a specific joint momentum component $\left|\mathbf{P}_\nu^0, \mathbf{P}_p^0, \mathbf{P}_e^0\right\rangle$ that falls within the overlapping regime. This implies $m_j = m_k$ when $\sigma_j \cap \sigma_k \neq \oslash$.

For the entangled state of Eq. (8), when the momentum states are traced out, the mass degree of freedom is left in a classical mixture, described by the density operator

$$\begin{aligned} \rho_\nu &= Tr_{\mathbf{P}_\nu, \mathbf{P}_n, \mathbf{P}_e} |\psi\rangle\langle\psi| \\ &= \int d^3\mathbf{P}_\nu d^3\mathbf{P}_p d^3\mathbf{P}_e \langle \mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e | \psi\rangle \langle\psi| \mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e\rangle \\ &= \sum_{j,k} D_{j,k} \left|\bar{\nu}_j\right\rangle \left\langle\bar{\nu}_k\right|, \end{aligned} \tag{9}$$

with

$$D_{j,k} = \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} d^3\mathbf{P}_{p,j} d^3\mathbf{P}_{e,j} \int_{\sigma_k} d^3\mathbf{P}_{\nu,k} d^3\mathbf{P}_{p,k} d^3\mathbf{P}_{e,k}$$
$$G(\mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j}) G^*(\mathbf{P}_{\nu,k}, \mathbf{P}_{p,k}, \mathbf{P}_{e,k})$$
$$\langle \mathbf{P}_{\nu,k}, \mathbf{P}_{p,k}, \mathbf{P}_{e,k} | \, \mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j} \rangle \, . \tag{10}$$

Since $\sigma_j \cap \sigma_k = \oslash$ for $j \neq k$, each of the joint momentum eigenstates in the region σ_j is orthogonal to all the momentum eigenstates in σ_k. This implies $\langle \mathbf{P}_{\nu,k}, \mathbf{P}_{p,k}, \mathbf{P}_{e,k} | \, \mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j} \rangle = 0$ throughout the integral region $\sigma_j \otimes \sigma_k$. Therefore, we have

$$\rho_\nu = \sum_j D_{j,j} \left| \bar{\nu}_j \right\rangle \left\langle \bar{\nu}_j \right| , \tag{11}$$

where

$$D_{j,j} = \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} d^3\mathbf{P}_{p,j} d^3\mathbf{P}_{e,j} \left| G(\mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j}) \right|^2 . \tag{12}$$

In other words, the quantum coherence among the mass eigenstates is destroyed by their quantum entanglement with different joint momentum states.

The entanglement-induced loss of coherence can also be understood in terms of complementarity, according to which the interference between state components of one freedom degree would be destroyed if the information about it is stored in another freedom degree [15-24]. Here the information about the mass of the antineutrino is encoded in the joint antineutrino-proton-electron momentum. This information could be extracted in principle, which is sufficient to destroy the interference between the mass eigenstates. It does not matter whether or not the information is read out. The loss of interference due to entanglement has been demonstrated in a number of experiments [18-24].

If antineutrino flavor eigenstates are defined as superpositions of mass eigenstates, for such a classical mixture ρ_ν, the population of the flavor eigenstate $\left| \bar{\nu}_\alpha \right\rangle$ is given by

$$P_\alpha = \sum_j \left| D_{j,j} U_{j\alpha} \right|^2 ,$$

where

$$\left| \bar{\nu}_j \right\rangle = \sum_j U^\dagger_{j\alpha} \left| \bar{\nu}_\alpha \right\rangle .$$

Consequently, the probability for detecting the antineutrino in each flavor has a nonzero probability, which is time-independent. This is inconsistent with the well-known β decay experiments [4], where the produced antineutrino is initially of e-type, and then undergoes flavor oscillations.

In summary, we have shown that the electron antineutrino created by the β decay of a neutron cannot be in a coherent superposition of different mass eigenstates. If the entire antineutrino-proton-electron wavefunction emerging from the β decay involve different mass eigenstates, they are necessarily entangled with different momentum states, as a consequence of the momentum and energy conservation laws. Such mass-momentum entanglement would destroy the quantum coherence between the mass eigenstates. This conclusion holds for any neutrino or antineutrino produced by the decay of an unstable particle. The result enforces the claim that neutrino oscillations originate from virtual excitation of the Z bosonic field [25].

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