Review of: "Autonomous Second-Order ODEs: A Geometric Approach"

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Potential competing interests: No potential competing interests to declare.

The manuscript titled "Autonomous second-order ODEs: a geometric approach" written by A. J. Pan-Collantes and J. A. Alvarez-Garcia is a remarkable work in the research field of differential geometry and represents an interplay between the systems of autonomous second order ordinary differential equations and the Riemannian geometry of the first-order jet bundle. More exactly, starting with a SODE (system of second order differential equations), a Riemannian metric was associated, and then the whole geometry was associated with the system of second order ordinary differential equations. Particularly, the components of the corresponding Riemann curvature tensor and the intrinsic Gaussian curvature were computed, including for a few examples.

Moreover, a relationship between the solutions of the ODE and the geodesic curves associated with the metric was established. Also, the notion of energy foliation and its connection to the classical energy concept was introduced. Finally, by studying the geometry of the leaves of the foliation, the obtained results were applied to the Lagrangian mechanical systems, and an autonomous Lagrangian for the damped harmonic oscillator was presented.

It will be very interesting to compare this approach with the geometric Kosambi-Cartan-Chern (KCC) theory, which has developed a lot in the last decades. Taking into account that the framework for KCC geometric theory is the tangent bundle of a manifold, I recommend this approach because it is closer to the study of the nonlinear behaviour of the dynamics of SODE, e.g., Jacobi stability.

In conclusion, I really appreciate this manuscript, and I recommend the publication in any valuable specialized journal.