# Candidate Potential for Studies of A ≥ 1600 Superheavy Nuclei

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## Abstract

Uncertainties in the nuclear interaction for A ≥ 1600 superheavy nuclei preclude absolute theoretical predictions of nuclear properties including single-particle energies, half-lives and Q-values. However, a model potential can be developed to predict trends in these properties and suggest islands of stability in A ≥ 1600 nuclei. The Rost-1600 interaction is derived from the Rost interaction by increasing its potential strength by 15%. This interaction will be used in subsequent studies of A ≥ 1600 nuclei.

## 1.0 Introduction

Superheavy nuclei and their stability are of continuing experimental and theoretical interest[1-33]. Theoretical investigations of the A = 500-800 mass region[21-23] were performed using the Rost interaction[2], that was noted to be appropriate for superheavy studies in Ref. 5. Studies of superheavy systems were extended to the 800 ≤ A < 1200 mass region using the modified Rost interaction[24] that accounted for uncertainties in A ≈ 400 – 500 superheavy potentials[3]. References 25 – 28 provided theoretical estimates for bound systems in the 800 ≤ A < 1200 mass region using the modified Rost interation[24], but this potential produced no bound states for A >1130 nuclear systems[28].

To address A ≥ 1200 systems, the adjusted Rost interaction was developed[29]. The adjusted Rost interaction was successfully utilized in 1600 > A ≥ 1200 systems[30-33]. However, it failed to yield any bound systems for A > 1540[33]. This paper develops a potential for use in A ≥ 1600 systems.

Recent work concludes that a reasonable approach for investigation of superheavy systems is to provide a set of calculations that search for regions of stability[21-33]. These calculations predict essential nuclear properties including level structures and decay energies, but their main value is the determination of stability trends. Although Q-values and half-lives are calculated, their relative values have more meaning than their predicted values. Without experimental data, A ≥ 1600 calculations provide qualitative trends and predictions of possible stability islands rather than specific half-lives or Q-values.

This paper adopts a similar philosophy to develop a characteristic interaction to investigate qualitative trends in A ≥ 1600 nuclei. The interaction is based on previously utilized potentials, their associated uncertainty, and capability to produce known properties of existing nuclei[2,3,5,21-33].

The use of single-particle energy levels to investigate nuclear stability is well established[3,5]. Extrapolations to the superheavy mass regions are speculative, and using a more sophisticated method is not required in view of the uncertainties encountered in these calculations. Methods that are more sophisticated are appropriate when data is available to examine fine model details and interaction characteristics. As demonstrated in Refs. 3 and 5, single-particle energy level calculations are appropriate for initial calculations into a superheavy mass region where there is no experimental data to guide the calculations. Moreover, theoretical calculations are currently the only way to investigate the A ≥ 1600 mass region because an experimental investigation is not currently feasible. As part of the theoretical basis, this paper suggests a nuclear interaction that provides an approach to investigate these systems and characterize their properties.

## 2.0 Existence of A ≥ 1600 Nuclear Systems

The proposed theoretical study of the stability of extremely heavy, hypothetical nuclei with the mass number A ≥ 1600 can be challenged as speculative and difficult to test. Although these contentions have some merit, they do not preclude the existence of these systems or negate the importance of their investigation. The existence of A ≥ 1600 systems can only be resolved by experimental observation. Until then, theoretical efforts can suggest mass regions where experimental observations would be most likely to succeed. This paper provides a portion of the theoretical basis for planning these future experiments.

Experimental investigations will certainly be challenging, but there are accelerator methods that could be developed to reach the A ≥ 1600 mass region. In addition, the decay modes of these systems offer a unique signature regarding their investigation. These approaches are addressed in more detail in Section 8.0.

## 3.0 Calculational Methodology

Nuclear stability with respect to alpha and beta decay is addressed using the method previously published by the author[21-33] and coworkers[5] that is similar to the approach of Ref. 3. The single-particle level spectrum is generated using a Woods-Saxon potential with parameters optimized to permit extrapolation into the superheavy region.

Since the method for calculating single-particle energies in a spherically symmetric potential is well established, it will not be repeated. The methodology as applied to A ≥ 570 systems was provided in detail in Ref. 21, which extended the approach of Petrovich et al.[5] to 570 ≤ A ≤ 620 mass region. Refs. 22 and 23 investigated the 620<A<700 and 700 ≤ A < 800 mass region, respectively using the Rost interaction[2]. 800 ≤ A < 900[25] , 900 ≤ A < 1000[26], 1000 ≤ A < 1100[27], and 1100 ≤ A < 1200[28] systems were investigated using the modified Rost interaction[24]. The adjusted Rost interaction[29] was utilized for 1200 ≤ A < 1300[30] , 1300 ≤ A < 1400[31], 1400 ≤ A < 1500[32], and 1500 ≤ A < 1600[33]calculations. Specific details of the numerical method and convergence criteria are provided in Refs. 5 and 21-36.

## 4.0 Nuclear Interaction

The unmodified Rost interaction[2] and the pairing interaction of Blomqvist and Wahlborn[36] were used to investigate A = 570 – 800 systems[21-23], and its parameters are provided in Refs. 5 and 21. The lack of binding in A ≥ 800 systems produced by the Rost interaction required the introduction of the modified Rost interaction[24]. In a similar manner, the lack of binding in A ≥ 1200 systems produced by the Rost interaction required the introduction of the adjusted Rost interaction[29]. Ref. 33 encountered similar binding energy issues for 1500 ≤ A < 1600[33] systems.

These issues now require the development of the Rost-1600 interaction that will be utilized in A ≥ 1600 systems. In developing the Rost-1600 interaction, alterations to the magnitude of the potential strength, pairing interaction, and spin-orbit interaction[37-39] were considered. Based on the development of the adjusted Rost interaction[29], only the magnitude of the potential strength was incorporated into the Rost-1600 interaction.

Accordingly, the Rost-1600 interaction has the form:

$$V_0 = 51.6\lambda\left[1 \pm 0.73\frac{N-Z}{A}\right] \quad (1)$$

where λ = 1.15. For comparison, λ = 1.0 corresponds to the Rost interaction[2], λ = 1.05 corresponds to the modified Rost interaction[24], and λ = 1.10 corresponds to the adjusted Rost interaction[29]. The only alteration included in the Rost-1600 interaction was a 15% increase in the potential strength[2]. Modifications in the pairing interaction were not included in the Rost-1600 interaction because they would distort the single-particle level energies and decay characteristics derived from the Rost interaction[2,24,29].

### 4.1 Potential Strength Considerations

A potential applicable to A ≥ 1600 systems must be constructed in a manner consistent with the general uncertainties in the nuclear interaction. Table 1 lists a representative sample of these uncertainties in order to guide the determination of the strength of an interaction applicable for use in A ≥ 1600 systems. The calculations summarized in Table 1 span a wide range of nuclear systems (i.e., light nuclei through nuclear matter).

Table 1 Uncertainty in the Nuclear Potential Strength

| Reference | Calculation and Mass Region | Uncertainty |
|---|---|---|
| Bevelacqua[40] | R-Matrix Structure Model in A = 2 - 4 Nuclei | 5 – 20% |
| Bevelacqua and Philpott[41] | R-Matrix Structure Model in $^{4}$He Energy LevelsHe | 17% |
| Wiringa, Stoks, and Schiavilla[42] | Nuclear Potential with 40 adjustable parameters developed using for 4301 pp and np data in the energy range 0 - 350 MeV for a number of nuclear systems. | <20% (System dependent) |
| Blomqvist and Wahlborn[36] | Shell Model in Lead Region | 10% |
| Lukasiak and Sobiczewski[3] | Single-particle Levels in A = 400 - 500 Systems | 5% |
| Gad and Mansour[43] | Single-Particle Spectrum and Binding Energy of Pure Neutron Matter | < 30% (Varied with the Fermi momentum) |

Blomqvist and Wahlborn[36] performed calculations in the lead region and acknowledged potential strength uncertainties of about 10%. Lukasiak and Sobiczewski[3] noted a 5% variation in potential strength for A≈ 400 - 500 systems. This uncertainty was used as the basis for establishing the modified Rost interaction[24].

These values are consistent with structure calculations of light nuclear systems that indicated potential strength uncertainties of 5 – 20% for A = 2 – 4 systems[40] and 17% for the $^{4}$He binding energy[41]. Wiringa et al.[42] investigated a nuclear potential with 40 adjustable parameters developed using for a set of 4301 pp and np scattering data points in the energy range 0—350 MeV. This broad study provided an uncertainty in the potential strength that is less than 20% for a wide range of nuclear systems. Calculations for nuclear matter including the single-particle spectrum, and binding energy of pure neutron matter by Gad and Mansour[43] suggested uncertainties less than 30%, and these uncertainties varied with the Fermi momentum.

These results for a range of calculations from light nuclei to nuclear matter suggest potential strength uncertainties could be as large as 30%. A crude average supports the general impression that nuclear structure calculations are generally 10 – 15% accurate which is reflected in uncertainties in the nuclear interaction. Following the discussion in Ref. 24 and 29, the Rost-1600 interaction for A ≥ 1600 systems will focus on modifications of the potential strength using the results of Table 1 as a guide. Moreover, the potential will use Eq. 1, but will determine an appropriate λ value for use in A ≥ 1600 systems. Based on the aforementioned discussion, it is reasonable to utilize λ = 1.15 to investigate the bounding characteristics of A ≥ 1600 superheavy nuclear systems.

## 5.0 Overview of Potential Selection

The effects of the interaction uncertainties noted in Section 4.0 are investigated to establish an interaction for the A ≥ 1600 mass region. Tables 2 and 3 summarize the effects of strengthening the Rost interaction. Table 2 summarizes the results of the methodology of Ref. 29, and the impact of modifications of the potential on the characteristics of the resulting (1128, 330) nuclear system. Table 3 reviews the impacts of the model potential strength in the (1600, 440) system.

Table 2 Sensitivity of (1128, 330) Half-Lives to the Modified and Adjusted Rost Interactions[a]

| Interaction | $Q_\alpha$ (MeV) | $T^\beta_{1/2}$ | $T^\alpha_{1/2}$ [b] | $T^{eff}_{1/2}$ |
|---|---|---|---|---|
| Modified Rost (λ = 1.05) + BW Pairing[c] | 26.3 | 45 s[d] | 1.3 d | 45 s |
| Adjusted Rost (λ = 1.10) + BW Pairing[c] | 17.7 | 0.82 s[d] | stable | 0.82 s |
| Adjusted Rost (λ = 1.11) + BW Pairing[c] | 15.9 | 47 ms[e] | stable | 47 ms |
| Adjusted Rost (λ = 1.14) + BW Pairing[c] | 10.6 | 2.2 ms[e] | stable | 2.2 ms |
| Adjusted Rost (λ = 1.15) + BW Pairing[c] | 8.85 | 1.3 ms[e] | stable | 1.3 ms |

[a] Eq. 1 and Ref. 29.

[b] Half-lives greater than $10^{20}$ y are listed as stable.

[c] Blomqvist and Wahlborn (BW)[36].

[d] First-forbidden $2m_{21/2}$ (n) to $1l_{17/2}$ (p) beta decay transition.

[e] Allowed $2l_{17/2}$ (n) to $1l_{17/2}$ (p) beta decay transition.

Table 3 Sensitivity of (1600, 440) Energy Levels to the Rost-1600 Interactions with the Blomqvist and Wahlborn Pairing Interaction

| λ[a] | Last Bound Neutron Level | Neutron Binding Energy (MeV) | Total Number of Bound Neutrons | Last Bound Proton Level | Proton Binding Energy (MeV) | Total Number of Bound Protons |
|---|---|---|---|---|---|---|
| 1.13 | $1u_{33/2}$ | 0.458 | 1198 | $4p_{1/2}$ | 0.564 | 420 |
| 1.14 | $4j_{15/2}$ | 0.318 | 1214 | $3g_{9/2}$ | 0.465 | 430 |
| 1.15 | $4j_{15/2}$ | 0.570 | 1214 | $3g_{7/2}$ | 0.163 | 464 |

[a] Eq. 1.

As expected from previous publications[3,5,21-33], the calculations are sensitive to the potential magnitude. As the potential strength increases, the single-particle levels become more tightly bound. The alpha decay half-life increases and the $Q_\alpha$ value decreases which is attributed to the increased potential strength. Beta decay half-lives are more complex and depend on the neutron and proton level energies involved in the transition and their associated quantum numbers.

It is desirable that changes to the model interaction[2,24,29] preserve the characteristics derived from the Rost interaction. For example, the mode of decay (e.g., specific beta transition) derived from the various Rost interactions[5,21-33] should be consistent.

In (1128, 330), the λ = 1.05 and 1.10 values, summarized in Table 2, leads to a beta decay that occurs through a first-forbidden $2m_{21/2}$(n) to $1l_{17/2}$(p) proton transition[29]. The λ = 1.11, 1.14, and 1.15 values suggest the (1128, 330) beta decay occurs through an allowed $2l_{17/2}$ (n) to $1l_{17/2}$ (p) transition. Accordingly, Ref. 29 notes that a λ value exceeding 1.10 does not meet the goal of preserving the beta decay transition characteristics noted previously.

As noted in Ref. 29, potential changes of a few percent have a significant impact on the calculated binding energies, half-lives, and Q values[3,5,21-33]. As the potential strength increases, the levels become more tightly bound and the total number of bound single-particle levels increases. This effect is also illustrated in Table 3 for (1600, 440). The increased binding permits more bound systems to exist, which enhances the probability of establishing trends and locating possibly islands of stability in superheavy systems.

As noted in Ref. 29 and Table 2, increasing λ from 5 to 10% decreases the $Q_\alpha$ value by 8.6 MeV. This is a significant change that increases the alpha decay half-life ($T^{\alpha}_{1/2}$) from about 1.3 d (λ =1.05) to $1.6\,10^{20}$ y (λ =1.10)[29]. Beta decay half-lives ($T^{\beta}_{1/2}$) decrease from 45 s (λ =1.05) to 0.82 s (λ =1.10)[29]. These wide variations are expected from the sensitivity to the potential strength noted in previous calculations[3,5,21-33]. In investigating these trends, the most relevant quantity is the effective half-life ($T^{eff}_{1/2}$) , which tends to smooth variations in its component half-lives

$$T^{eff}_{1/2} = [T^{\alpha}_{1/2} T^{\beta}_{1/2}]/[T^{\alpha}_{1/2} + T^{\beta}_{1/2}] \quad (2)$$

$T^{eff}_{1/2}$ represents the net response of the system to its various decay modes. The trend in effective half-life as a function of λ is less dramatic than changes in the alpha decay half-life and similar to the beta decay half-life changes[5,21-33].

The methodology of Ref. 29 serves as a guide to the selection of the Rost-1600 interaction. In particular only the potential strength merits adjustment to avoid perturbing the inherent properties of the A > 1600 nuclear systems. As noted in Ref.29, altering the pairing interaction Blomqvist and Wahlborn[36] disrupts the expected trends and is inconsistent with the desire to maintain continuity of the previous calculations[5,21-33].

## 6.0. Selection of Bounding Potential

The results summarized in Table 3 suggest that Rost-1600 potential strength increases smaller than 15% do not lead to a bound (1600, 440) system. λ values less than 1.15 do not lead to a sufficient number of bound protons single particle energy levels to successfully create (1600, 440).

Utilizing the results of Table 1, which considered published variations in the potential strength for a wide range of nuclear systems, the bounding A ≥ 1600 interaction is limited to a 15% increase (λ = 1.15) in the Rost interaction[2]. This limitation is projected to not disrupt the investigated systematic trends and permits bounding nuclear properties of A ≥

1600 systems to be determined.

The Rost-1600 interaction with λ = 1.15 provides more deeply bound systems than the Rost (λ = 1.00)[2] modified Rost interaction (λ = 1.05)[24], and adjusted Rost interactions (λ = 1.10)[29]. Considering previous calculations[5,21-33] and the results summarized in Table 1, an adjustment of 15% falls within the expected uncertainty in the nuclear interaction. Accordingly, the Rost-1600 interaction with λ = 1.15 and the pairing interaction of Blomqvist and Wahlborn[36] will be utilized in subsequent calculations in the A ≥ 1600 systems. Utilizing the interaction of Ref. 36 maintains the consistency with Refs. 5 and 21-33.

The use of the Rost-1600 interaction is analogous to lowering the level of water covering the surface of the earth. As the water level is lowered (i.e., potential strength increases), additional land areas are revealed (i.e., islands of stability). The position of the island remains the same, but its relative importance and size (i.e., degree of binding) is made more apparent by strengthening the interaction (i.e., lowering the water level). Although a more quantitative result is desirable, the paucity of data in the A ≥ 1600 mass region precludes a more definitive option. However, it does offer the potential to locate possible islands of stability and to ascertain their relative importance.

The reader should note that increasing deviations from the original Rost interaction[2] make the calculational predictions more uncertain. Accordingly, the results of the calculated binding energies, Q-values, and half-lives are not definitive. These results provide a relative comparison of the various systems utilizing the Rost-1600 interaction.

## 7.0. Model Weaknesses

The Rost-1600 model interaction is extrapolated from Z ≤ 82 data without the benefit of experimental benchmarks in the A ≥ 1600 mass region. Although this is a necessity due to the lack of experimental data, it must be acknowledged as a weakness in the present approach. This weakness will be applicable for any theoretical investigation in the A ≥ 1600 mass region.

Another weakness of the approach[5,21-33] is treating all evaluated nuclei as spherically symmetric systems. Many of these systems are likely deformed and these deformations should be included in subsequent investigations. These calculations have been initiated.

The aforementioned weaknesses are difficult to assess, but the model prediction of A ≥ 1600 stability can be partially evaluated by comparing the (A, Z) values of these systems to the predictions of Adler's relationship[44,45] that provides the most stable nucleus Z value for a given A:

$$Z = \frac{0.487A}{1 + A^{2/3}/166} \quad (3)$$

For example, this relationship suggests that the A = 1600, 1700, and 1800 systems should be most stable for Z values of 427, 446, and 463, respectively. Although qualitative, the Adler relationship provides a benchmark for subsequent calculations. Based on previous calculations[5,21-33], reasonable comparison between the model and predictions of the

Adler relationship of Eq. 3 was noted to place a portion of the model weakness issues into perspective.

## 8.0. Experimental Verification

Any experimental investigation of A ≥ 1600 nuclear systems presents a considerable challenge. Conventional binary collision processes involving heavy ion beams and techniques are not capable of reaching this mass region. Experimental investigation of the A ≥ 1600 mass region requires a novel approach. For example, simultaneously colliding seven $^{238}U$ ions theoretically reaches the A = 1600 mass region, but this approach is not yet viable. In the interim, the author hopes that other theoretical work will challenge and refine the conclusions of this paper, and experimentalists will develop accelerator techniques to collide multiple beams to reach the A ≥ 1600 mass region.

A possible measurement approach is offered by the high alpha particle energies emitted by the postulated superheavy systems. Based on Refs. 5 and 21-33, the alpha particle energies of these theoretical superheavy nuclei are about 100% larger than the measured Z = 114-118 values[46]. This substantial increase in alpha particle energies offers a possible avenue for their experimental verification.

Compared to Z = 114 - 118 nuclei[46], the higher alpha particle energies from the A ≥ 1600 nuclei are expected to have a longer range in a material medium. This range manifests itself as a longer track length as the alpha particle is attenuated by a medium. Measuring alpha track lengths is a well-established approach in applied physics including the measurement of the $^{222}Rn$ air concentration[45.47]. Since the track length is related to the alpha particle energy, it provides a possible method to verify the existence of an A ≥ 1600 superheavy system.

## 9.0 Conclusions

The Rost-1600 interaction with λ = 1.15 and the pairing interaction of Blomqvist and Wahlborn[36] provide an interaction to investigate single-particle levels in A ≥ 1600 nuclear systems, but is projected to not disrupt trends in nuclear properties including the beta decay mode. Calculating absolute nuclear properties in the superheavy mass region is fraught with uncertainty without specific experimental data. The Rost-1600 interaction and the pairing interaction of Blomqvist and Wahlborn[36] permit the investigation of trends in A ≥ 1600 superheavy nuclei properties and the location of possible islands of stability. The validity of the proposed interaction methodology will not be known until A ≥ 1600 data become available.

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