Review of: "Horizon and curvature"

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Potential competing interests: No potential competing interests to declare.

I enumerate below some suggestions and detected typos in order to improve the presentation of the paper:

- The planet "Earth" should be written with a first capital letter in the whole paper.

- Page 1: the three first centered formulas need a point, a comma and a point in the end, respectively.

- Page 1, bottom: better if a point is used instead of a comma for the decimal numbers. There is also a missing space between the first two numbers and the "m".

- Page 1, line 4 from below: remove "If". In the previous line and two lines below there are two instances of "a plane". To avoid misunderstanding with the mathematical object it is better if it is said "an airplane".

- Page 2, title Sec. 3: remove the point in the end.

- page 2, first line: "ìnverse" should be "inverse".

- Page 2, line above Prop. 3.1, remove the space between "property" and ":".

- Page 2, proof of Prop. 3.1:

  -first line: ``points" and ``as a consequence"

  - 4th line: $(0, R)$ (mathematical format).

- Page 3: the last part of the proof is blurry. In particular, "since the curve might be a spiral an infinite number of steps may be necessary for the proof to be complete". A proof in Mathematics is always done in a finite number of steps, so this argument is not convincing. I suggest restricting Prop. 3.1 to the strictly convex case, which is what is considered in the rest of the paper. In addition, for strictly convex curves, I am thinking that the proof can maybe be simplified, because the idea is to take the point of minimum curvature of the strictly convex curve and consider the osculating circle there (which will contain the whole curve). Any circle centered there with a bigger radius will contain the curve as well.

- Prop. 3.2: define H(h, M). In general it may be needed to define two horizons (although in the proof are the same): a right H^{+}(h,M) and a left one, H^{-}(h,M). In the proof:

  - Delete the ")" before "]".

  - N positive integer. Say "N > 1$.

  - Define $L>0$.

  - Clarify what you mean by "slope", "slope jumps", "face" and "corners/smoothing corners".

  - 3rd line from below: "lengths", not "lengthes".

  - Why the maximum is less than $2H_0$? I would say $2NH_0$, but it may be a matter that I did not understand the notation that has not been introduced.

- Thm. 3.3:
    - Where do the two "basic formulas" come from? The second one seems to come from appplying Taylor's formula to $f'(x)$. And what about the first one?
    - The notation o(x^n) must be explained. It seems that you refer to terms of degree greater than n. There may exists both notations, but I would say it is more common to use o(x^n) for the terms of degree n and greater.
    - Specify clearly that what tends to zero is $H^2(h)$ by hypothesis. That implies that the limit below is ≥ 1. Why is it $=1$?

- Somewhere in the paper it should be made clear that it suffices to study strictly convex planar curves because we make sections of a strictly convex body through planes to study their horizons in some direction. Also, a brief explanation of the main results for the non-specialized reader may be helpful.
- The addition of figures in Sec. 2 and Sec. 3.3 would be helpful as well.