Research Article

Indeterminism, Non-Maximality, and Discontinuity in Everettian Quantum Mechanics

Conor Husbands[1]

1. Independent researcher

Whilst several of the merits of Everettian Quantum Mechanics (EQM) have been subject to dispute, its status as a deterministic theory has been granted as more or less axiomatic. Accordingly, literature in the field of quantum foundations is replete with strident professions of EQM's deterministic credentials. This is surprising in view of the scant independent justification for them, as well as the diversity of characterisations of determinism, filling out a spectrum which might more typically elicit scrutiny of claims of compliance. This article distinguishes and analyses three separate characterisations of determinism, including a Laplacian variant, a branching-style variant (DBRN) inspired by recent commentary, and a variant couched in terms of the continuity of subsystem identity. Applying these to the case of EQM shows that the latter qualifies as deterministic only under the weakest of these definitions. It is concluded that EQM's violation of these stronger standards constitutes a meaningful limitation on the sense in which the theory can be considered genuinely deterministic, and that claims of this nature must be qualified carefully.

Correspondence: papers@team.qeios.com — Qeios will forward to the authors

Introduction

Whilst several of the merits of Everettian Quantum Mechanics (EQM) have been subject to dispute, its status as a deterministic theory has been granted as more or less axiomatic. Proponents of EQM identify its compliance with notions of locality and realism as its distinguishing features, defending these notions at length against the objections of competing interpretations and critics. However, the basis for its satisfaction of determinism, a principle violated in most reconstructions of "orthodox"[1] or Copenhagen-

 doi.org/10.32388/3RE2VT

inspired QM, is rarely contested by either party. This is to some extent a surprising state of affairs given (i) the lack of a rigorous demonstration of this satisfaction and (ii) the variety of definitions of determinism which have emerged from modern research in the philosophy of science. This article proposes to consider the sense in which EQM conforms to notions of determinism. It argues that only a heavily qualified opinion can be justified in this regard, since weak notions of determinism are fulfilled, but stronger notions are violated in principle. Consequently, the attractions of EQM are somewhat diminished: it fails to meet stronger standards of determinism exhibited by many other otherwise comparable theories, including the branching-style determinism of Müller and Placek ("DBRN") and subsystem continuity ("SC").[1]

Literature in the field of quantum foundations is replete with strident professions of EQM's deterministic credentials. Dewitt and Graham consider the interpretation "rigorously deterministic" by virtue of the absence of any postulate of collapse.[2][3] Greaves takes this absence, along with the existence of "no necessary existence of initial conditions" as indicators of a "straightforwardly deterministic" approach.[4] Vaidman considers EQM "deterministic, local, free of paradoxes, and fully consistent with our experience," and dismisses the appearance of probability in the theory as illusory.[2] Deutsch claims that the Everettian vantage point yields the insight that "the laws of quantum physics are perfectly deterministic," at least at the level of the universal wavefunction.[5] Wallace exhibits similar inclinations in a number of places.[6][7][8] Timpson and Brown state that the "appropriate correlations" which crystallise between interacting subsystems in the course of the state vector's evolution "follow deterministically, given the interactions that the systems undergo."[3] These and other contributors give EQM credit for many other merits besides this – almost all of which have met with fierce debate in quantum foundations – yet its qualification as a deterministic approach is rarely challenged. Nor, indeed, is the sense in which it so qualifies elaborated. This is a surprising fact given that competing definitions of determinism abound, making it a doctrine sufficiently opaque and diffuse that a theory's satisfaction or violation of it is hardly an immediate, non-inferential fact.

There is therefore a lacuna in these discussions. This lacuna is the focus of this article. The following section considers a canonical definition of determinism provided originally by Laplace and reconstructed in contemporary terms by the likes of Earman, Müller and Placek, along with others. It argues that these formulations are limited in two main ways: (i) they feature serious ambiguities which make them unreliable as litmus tests for the determinism or indeterminism of theories and interpretations; (ii) they are weak, and are passed by interpretations of quantum mechanics which advocates of EQM would consider to be less deterministic than theirs. Subsequently, two other definitions of determinism are examined, including Müller and Placek's notion "DBRN" and subsystem continuity. It is demonstrated that EQM fails to meet these standards, in contrast to many other physical theories with which, in some cases, Everettians profess a kinship. It is concluded that EQM's violation of these standards constitutes a meaningful limitation on the sense in which the theory can be considered genuinely deterministic, and that claims of this nature must be qualified carefully.

Laplacian Determinism

It is quite evident that any judgement passed on EQM's status as a deterministic or indeterministic interpretation presupposes a sufficiently clear definition of these terms. A traditional point of departure is the authoritative expression of determinism provided by Laplace[9] in his foundational essay on probability, which has been developed and refined in recent literature in the philosophy of science. Whilst the intuition behind this expression remains present in many discussions, the reliance of its original presentation on hypothetical and epistemological notions, appealing to the subjunctive and to the mental properties of an imaginary demon, constitutes significant limitations. Earman has provided an influential account of Laplacian determinism which overcomes many of these defects, and serves as a useful point of departure:

> The world $w \in W$ is Laplacian deterministic just in case for any $w' \in W$, if w and w' agree at any time, then they agree for all times.[10]

Of course, the notion of a world must be defined more clearly for this to serve as a viable test, that is, for it to constitute a function which takes a scientific theory as its input and labels it deterministic or indeterministic as an output.[1] Earman's work provides that W is the "collection of all physically possible worlds, that is, possible worlds which satisfy the natural laws obtaining in the actual world."[10] A world is further defined as a "four-dimensional space-time world, the actual world being the collection of all events that have ever happened, are now happening, or ever will happen, and a possible world being a collection of possible events representing possible alternative histories to that of the actual world."

A noteworthy feature of this presentation is its nomological content. It involves nomological content in that the definition of worlds refers to those sets of events which are consistent with natural laws. These aspects of Earman's presentation are common to many Laplace-inspired characterisations of determinism. Hoefer, for instance, provides a criterion for determinism in terms of the fixity of future events by virtue of natural law given an initial state:

> Determinism is true of the *world* if and only if, given a specified *way things are at a time t*, the way things go *thereafter* is *fixed* as a matter of *natural law*.[11]

Several of the founders of quantum mechanics provided similar characterisations, albeit less rigorously formulated. Born, for example, offers an implicit criterion for determinism as follows: “the laws…are so constructed that, if the variables in a closed system are given at some initial point in time, they can be calculated for any other instant.”[12] The violation of this principle by quantum theory he considers responsible for the rejection of determinism by some (though not all) of his contemporaries. Whilst couched in terms of calculability which echoes the Laplacian notion of prediction, this definition requires, as does Earman’s, the subjection of elements of the theory in question to natural laws. The elements of a theory which mark it as deterministic or indeterministic are, on these accounts, events which obey natural laws.

In turn, it is natural to demand an explanation of the notion of natural law. This can be elaborated by means of a precise, modern formulation of determinism provided by Müller and Placek in their incisive 2018 account. This formulation, “DEQN,” is largely aligned with the definitions of authors quoted in this section, and is one of three they examine in their paper. The test of determinism is passed or failed based on the behaviour of its defining differential equations, the mathematical representation of its natural laws, in relation to solutions to these equations, the mathematical representation of its events which comprise worlds or histories.[1] Given a set of solutions to these equations at a given time, Müller and Placek specify that a unique future or past set of solutions is implied for a deterministic theory, just as Earman requires the identification of w and $w^{'}$. Determinism under DEQN “boils down to the existence of a unique solution for each appropriate initial value.”[1] With this reading, the fixity, predictability or calculability occurring in the writings of Hoefer, Born and Laplace becomes the equivalence of sets of solutions. This provides a clearer representation of the two aspects of LD most in need of explication, namely those of worlds and laws. Embedding these insights, the following version of LD can be defined:

> LD: Given a solution S_i of the governing equations $f_1, f_2, \ldots, f_n$ of a theory T_1 at a time t_i there is a unique solution set S_j at a time t_j which is a solution of $f_1, f_2, \ldots, f_n$.

 doi.org/10.32388/3RE2VT

What are the strengths and weaknesses of this model, as applied to EQM? Do they warrant the conclusions of the authors cited in the introduction, for whom this interpretation amounts to a paragon of determinism set apart from its competitors? Indeed, prevalent strains of Everettianism broadly satisfy the requirements set out in the above paragraphs. In contrast to collapse-dependent theories such as GRW,[13] no supplementary dynamics are posited in order to overcome the so-called measurement problem, and to reconcile it with quotidian macroscopic experience. The evolution of the quantum state obeys the Schrödinger equation at all times, and a defined solution at a given time has a determinate time-evolved counterpart.

However, even against this apparently plain backdrop, two problems remain which scupper any attempt to license the qualification of EQM as a distinctively deterministic interpretation and establish an advantage relative to others. First of all, the mere existence of solutions – even unique solutions – to a set $f_1, f_2, \ldots, f_n$ as required by LD is too weak a definition to warrant this conclusion, in the detrimental sense that other quantum interpretations also clear this bar. Second of all, serious ambiguities remain in this formulation, even in the case of the apparently formal rendering of LD. In respect of the first of these two difficulties, even a standard Copenhagen-inspired approach shorn of the postulation of collapse accomplishes the *desiderata* in question. The Schrödinger evolution of the state provides for any number of time-evolved solutions to the guiding equation. These solutions will, in typical cases, constitute probability distributions by virtue of the complex amplitudes associated with a multiplicity of incompatible measurement outcomes. Such an evolution is manifestly indeterministic despite its compliance with the stipulations of LD, undermining its utility as a criterion. The possibility of probabilistic solutions to $f_1, f_2, \ldots, f_n$ which are symptomatic of indeterminism therefore exposes the limitations of the definition; were it adopted, there would be no explanatory gain in the affirmation of EQM relative to less heterodox interpretations. This highlights the fact that the existence of solutions to equations with no constraints on what these represent is not sufficient to capture the essence of determinism, and does not rule out equations which capture the behaviour of systems with objectively probabilistic features: solutions which are probabilistic could comprise a unique set, but embed aleatory behaviour which is patently indeterministic.

Relatedly, in respect of the second of the two difficulties presented in the previous paragraph, despite its clean appearance and clearly greater robustness than the colloquial formulations found throughout the literature, LD is seriously ambiguous, primarily owing to the very few constraints it imposes on the solutions of the governing equations. In order to qualify as a faithful definition of determinism,

doi.org/10.32388/3RE2VT

constraints on admissible solutions would have to be set up in a way that reflects the modal content of the doctrine. Earman himself notes that this is a crucial and distinctive aspect of the Laplacian doctrine, insofar as qualifying or disqualifying any candidate theory requires making a comparison between different possible worlds and their identification or differentiation at different times.[10] However, *prima facie,* this requirement is not met by LD. If the kind of object that can count as deterministic is not a world that could be possible or necessary, but an abstract mathematical object such as a solution, the modal content is no longer expressed effectively. The absence of constraints of this sort constitutes a defect in LD and in many discussions that develop characterisations tantamount to it. Further, there is a clear need, unmet by LD, to draw a distinction between local and global solutions. The existence of unique local solutions does not guarantee the existence of unique global solutions for certain systems such as singularities, yet it is the latter that is required to ensure determinism is respected. Müller and Placek highlight the importance of this distinction in "Defining Determinism," as does Earman at length throughout *A Primer on Determinism*.[4] Therefore, the intuitions LD is designed to capture and express are poorly served by this formulation, despite its prevalence and the prevalence of similar expressions. LD accordingly amounts to a necessary but not sufficient condition to qualify theories as deterministic.

Maximality and branching

The insufficient weight accorded to the uniqueness or exclusivity of solutions in the Laplacian framing of determinism can be remedied by bringing a more stringent standard into the discussion. This standard, which Müller and Placek consider to be the prevailing contemporary view in the philosophy of science, is topologically richer than LD and involves the decomposition of systems' histories into segments whose convergence and divergence signal determinism or indeterminism respectively.[1] These segments are time-stamped collections of physical variables that solve the defining differential equations of the theory, such as a particle's trajectory through space. The test Müller and Placek administer involves the following steps:

> If there are two realizations that can be identified at one time, but whose future segments after that time cannot be identified, this signals indeterminism. If the test fails, that is, if all realizations that can be identified at one time can also be identified at all future times, then the theory is deterministic.[1]

doi.org/10.32388/3RE2VT

They explicitly identify the consistent histories of recent EQM as examples of these segments.[5] A similar presentation is given by List and Pivato in their research into dynamic and stochastic systems:

> A system is called *deterministic* if, in that system, the past always determines the future. Formally, for any history h and any point in time t, let h_t be the *initial segment* of that history up to t. This is the function h restricted to the points in time up to t. History h is *deterministic* if, at any time t in T, the initial segment h_t admits only one possible continuation in Ω, where a *continuation* of h_t is a history h such that $h'_t = h_t$. History h is *indeterministic* if, for some time t, h_t has more than one possible continuation in Ω. The system as a whole is called *deterministic* if all histories in Ω are deterministic, and *indeterministic* if some histories in Ω are indeterministic.[14]

Here, the primary contrast with the LD formulation is a distinction of two registers in the interpretation in question. Whereas LD refers simply to solutions with little elaboration of what counts as solutions, these are explicitly distinguished in the Müller-Placek and List-Pivato papers. Whilst the test of LD is evaluated at the global level, these tests are evaluated at the local level of the subsystems tracing out partial segments. Candidates for compliance with LD are entire, universal states; candidates for compliance in this instance are its decompositions. This distinction is widely acknowledged in analyses of deterministic EQM. List and Pivato themselves attribute to systems distinct "higher-level" and "lower-level" dynamics and claim that these two may produce inconsistent outcomes to the above tests, with the status of each with respect to the doctrine in question becoming an emergent feature.[6] Similarly, Wallace's careful statements on the matter espouse nuanced views on determinism within EQM. He relativises its status as a deterministic theory to the fundamental (*cf.* emergent) level, that is, the level of the universal, unitarily evolving quantum state as opposed to that of decoherent quasi-classical histories.[6] Deutsch makes related observations in his *The Fabric of Reality*.[5]

Clearly, therefore, EQM's satisfaction or violation of this definition will depend on which of its objects are identified as segments or realisations. If taken to quantify over subsystems, an obvious problem arises: decoherence-induced branching generates approximately dynamically independent future segments from identical segments in their causal past. Following the presentation of Saunders, the strings of projectors below illustrate this divergence:[7]

$$C_\alpha = P_{\alpha_n}(t_n)\,P_{\alpha_{n-1}}(t_{n-1})\ldots P_{\alpha_1}(t_1)$$

doi.org/10.32388/3RE2VT

$$C_{\alpha'} = P_{\alpha'_n}(t_n) P_{\alpha'_{n-1}}(t_{n-1}) \ldots P_{\alpha'_1}(t_1)$$

where $\alpha = \alpha'$ for $t < t_i$ and $\alpha \neq \alpha'$ otherwise. Thus, orthogonality between states subsequent to t_i is assured:

$$t \geq t_i \rightarrow \langle C_\alpha \varphi | C_{\alpha'} \varphi \rangle \approx 0$$

A simple case is given by the histories below:

$$C_\alpha = P_0(t_j) P_\uparrow(t_i)$$

$$C_{\alpha'} = P_0(t_j) P_\downarrow(t_i)$$

where $j < i$, P_0 is the projector corresponding to a resting, pre-measurement apparatus state and $P_\uparrow, P_\downarrow$ the post-measurement states of this apparatus for the measurement of spin along a given axis. Letting $C_\alpha = h, C_{\alpha'} = h'$, the List-Pivato criterion is violated by the existence of dual orthogonal continuations. In the Müller-Placek presentation, however, more care is needed, since two different variants of the criterion are defined: DMAP and DBRN. Whereas the former leaves open the means to undermine this counter-example, the latter explicitly blocks any formalism which accommodates branching histories. As above, DMAP is subject to a laconic definition which is equivocal with respect to the definition of "segment." A defence of EQM can therefore be mounted by restricting its interpretation: the only admissible definition of this term is the global state evolving in accordance with the Schrödinger dynamics. This prohibits the equalities $C_\alpha = h, C_{\alpha'} = h'$ which compare segments with decoherent histories. From the perspective of the fundamental global state vector, no branching occurs, unlike at the emergent level of subsystems. This entails a difficulty in conforming the Müller-Placek notion of a segment to the Everettian ontology.

In order to overcome this difficulty, the definition of DBRN (or "Branching-Style Determinism") in Müller-Placek[1] incorporates a distinctive feature. It accommodates *modal thickness* by means of which branching orthogonal outcomes can be considered as testable segments. A segment is represented by means of a "model" of the relevant theory which constitutes a collection of histories which may branch. Decoherent histories are therefore internal to models in the sense that a given model may stand in a one-to-many relation with its histories. Formally, this model is represented by means of a 4-tuple $\langle M, <, S, f \rangle$ where $\langle M, < \rangle$ defines a "possibly branching, tree-like partial ordering" and f a function which takes elements of $\langle M, < \rangle$ to states S in accordance with the theory's dynamics. Chains are then defined as subsets of $\langle M, \; <\rangle$ which can represent individuated decoherent branches. The Müller-Placek DBRN criterion is then given as:

> DBRN: A model, $\langle M, \; <, S, f\rangle$, is indeterministic if and only if $\langle M, \; <\rangle$ contains more than one maximal chain. The model is deterministic if and only if $\langle M, \; <\rangle$ contains just one maximal chain.[8]

The additional concept of maximality which appears in this criterion is crucial in applying this framework to actual cases, which Müller and Placek adopt from Butterfield's work.[15] The intended function of maximality is to disqualify from the assessment those cases in which the future agreement between diverging evolutions is secured by imparting sufficiently vague or weak properties to them at the outset that the requirements of determinism are trivially satisfied. Consider, for instance, an initial state defined to be the disjunctive totality of all logically conceivable experimental results a physical state could manifest after a given time. In this case, there is only one chain in the tuple $\langle M, \; <\rangle$, namely, the set of all logically conceivable states. Absent the insistence that the chain be maximal, this chain would count as deterministic insofar as the initial state has one unique continuation, namely all its conceivable future states. Maximality is designed to disqualify absurdly trivial cases such as this.

For the sake of precision, the official characterisation of maximality is this: a state is maximal just in case its description includes "the logically strongest consistent properties the theory can express."[15] Importantly, maximal logical strength is not equivalent to maximal mathematical fine-graining but measures the state-description which is consistent with the fewest states allowable by the theory. Accordingly, EQM cannot submit disjunctive descriptions such as superpositions as chains eligible for the DBRN label; these are logically weaker in Butterfield's sense when compared with descriptions such as $P_0\left(t_j\right) P_{\uparrow}\left(t_i\right)$ or $P_0\left(t_j\right) P_{\downarrow}\left(t_i\right)$. The logically strongest descriptions admitted by EQM being these descriptions, the question as to whether these qualify or disqualify EQM as consistent with DBRN can be settled by asking whether these comprise just one maximal chain. Clearly they do not, by virtue of the fact that these chains branch via decoherence. Therefore, EQM violates DBRN.

How impactful is this conclusion? If DBRN is accepted as a plausible characterisation of determinism, EQM is indeterministic in this sense. Müller and Placek themselves regard DBRN as the "most useful general definition of determinism for physical theories," having examined several others.[1] They consider the move to restrict the test of determinism to models featuring maximal chains to be a procedure common to "any good analysis of determinism," branching structure to be "a natural representation of a theory's indeterminism" and their analysis to prove "the greater simplicity and conceptual primacy of the DBRN approach."[1] Everettians, naturally, could dispute this and argue for the adoption of a weaker criterion. Their counter-argument may well involve dismissing the violation as just another addition to the cemetery of folk intuitions built up by the trailing decades of research into quantum foundations. Perhaps, indeed, if a successful theory refutes hitherto conceived metaphysical notions such as DBRN-determinism, they themselves should be deemed refuted by the empirical success of the former.

In response to this, it is worth observing that (i) other interpretations of QM, notably Bohmian mechanics, and (ii) many other physical theories – in some cases those that are taken to support analogies to EQM by its advocates – satisfy these desiderata. In respect of (i), Bohmian mechanics, for instance, seems *prima facie* compatible with DBRN. The central object of this theory that is postulated in addition to the standard formalism, the guiding equation, fixes a single individual outcome and record given a specified initial state, and is defined as follows:[16]

$$\frac{d\mathbf{Q}_k}{dt} = \frac{\hbar}{m_k} Im \frac{\varphi * \partial_k \varphi}{\varphi * \varphi} (\mathbf{Q}_1 \ldots \mathbf{Q}_n)$$

Models for Bohmian mechanics accordingly contain just one maximal chain and meet DBRN. In respect of (ii), several examples of traditionally deterministic theories that also meet DBRN can be provided, which reflects adversely on EQM. Consider, for instance, the example of hydrodynamics.[9] Direct comparisons between this theory and EQM have been made by Everettians, with the behaviour of decoherent histories being compared to systems described by hydrodynamic equations of motion. A classical Euler or Navier-Stokes framing of this theory with a velocity and density field and suitable boundary conditions determines a unique global solution, and a single unique evolution without branching. This therefore contrasts with the Everettian treatment. By consequence of (i) and (ii), even if EQM's violation of DBRN were taken as grounds to reject the conclusions of Müller and Placek, it would accumulate defects relative to competing interpretations of QM as well as to archetypal physical theories. This violation must therefore be considered a result that weakens and restricts EQM's claim to cohere with determinism. The next section reinforces this conclusion by appeal to a related characterisation of this doctrine.

doi.org/10.32388/3RE2VT

Subsystem continuity

Although stronger for the reasons outlined above than DEQN and LD, DBRN is by no means the strongest plausible rendering of determinism or the only aspect of the principle that is contravened by EQM. The branching structure of decoherent histories within EQM significantly disrupts the theory's ability to identify discrete future continuants at the level of subsystems. More specifically, it rules out a partitioning or decomposition of the global state that is consistent with a unique mapping between its elements and a given future subsystem state. This section notes the inconsistency of EQM with a closely related principle, namely subsystem continuity which, it suggests, has no more scant a precedent in deterministic physical theories than does DBRN.

The first notion, subsystem continuity, can be explained in terms of decoherent histories by means of strings of projectors. Consider a Hilbert space $H = H_S \otimes H_E$ partitioned into subsystem and environment subspaces together with a state vector $|\varphi(t)\rangle$. Consider further an ordered sequence of times $t_0, t_1, \ldots, t_n$. As above, an operator C_α can be defined in terms of subsystem projectors as follows:

$$C_\alpha = P_{\alpha_n}(t_n)\, P_{\alpha_{n-1}}(t_{n-1}) \ldots P_{\alpha_1}(t_1)$$

Assuming decoherence, this fulfils $\langle C_\alpha \varphi | C_{\alpha'} \varphi \rangle \approx 0$ for $\alpha \neq \alpha'$. Now, an interpretation I counts as subsystem continuous just in case there is a set of projectors $\{P_{\gamma_i}\}$ such that, for this C_α and for any C_β such that

$C_\beta = P_{\beta_m}(t_m)\, P_{\beta_{m-1}}(t_{m-1}) \ldots P_{\beta_{n+1}}(t_{n+1})\, C_\alpha$ and $\left\langle \varphi(t) | C_\beta{}^\dagger C_\beta | \varphi(t) \right\rangle \neq 0$,

this C_β is unique. In paraphrase: an interpretation I counts as subsystem continuous just in case its subsystem projectors possess a unique continuation. This captures the preclusion by Subsystem Continuity (SC) of (approximately) post-decoherence evolutions with non-zero associated probabilities. This principle is closely related to the Earmanian notion of local determinism elaborated in various contexts in *A Primer on Determinism*.[10] As noted above, it is also closely related to DBRN, albeit with an emphasis on the preservation of subsystem identity under time-evolution as opposed to the uniqueness of entire chains in the model.

The natural next step involves applying this framework to EQM. It is straightforward to show that SC is thereby violated by means of a derivation along the same lines as the test of EQM against DBRN in the previous section. A basic case of a Stern-Gerlach spin-measurement performed on an incident electron along an axis suffices for this purpose. First, take the terms t_n and t_{n+1} describing the pre- and post-measurement times in the laboratory rest frame. Next, define H_S such that, $\forall t\,(t \leq t_n)$, the subsystem comprises the familiar setup of incident electron and observer or apparatus: $|\varphi\,(t \leq t_n)\rangle \;=\; \frac{1}{\sqrt{2}}(|\;\uparrow\rangle_S + |\;\downarrow\rangle_S)\,|\;\uparrow\rangle_O$. In this case, the relevant projector is $C_\beta = P_{\beta_n}(t_n)$ which undergoes the transition $P_{\beta_n}(t_n) \rightarrow\; P_{\uparrow_m}(t_m)\,,\; P_{\downarrow_m}(t_m)$ corresponding to distinct post-measurement states $|\;\uparrow\rangle_S, |\;\downarrow\rangle_S$. As a result, there are two continuations $C_\uparrow, C_\downarrow$ such that $\left\langle \varphi(t)|{C_\uparrow}^\dagger C_\uparrow|\varphi(t)\right\rangle \neq 0$ and $\left\langle \varphi(t)|{C_\downarrow}^\dagger C_\downarrow|\varphi(t)\right\rangle \neq 0$. Subsystem continuity is violated. This eventuality amounts to a formal expression of the impossibility of assigning a unique time-evolved trajectory to a given subsystem.

An additional stipulation is necessary as to which tensor products of the overall Hilbert space are admissible, given that the formalism itself enables any number of decompositions: when applying the criteria for subsystem continuity, the factorisation chosen must conform to observable outcomes in a simple and natural way. For instance, a simple and natural decomposition of H in the case described in the previous paragraph partitions the space into up and down pre- and post-measurement states of the electron and neutral, up and down pre- and post-measurement states of the apparatus. This kind of decomposition features in most discussions of such experiments. An inadmissible choice, by contrast, would be a time-dependent decomposition such as the following: First, take times t_{pre} and t_{post} describing the pre- and post-measurement times in the laboratory rest frame. Next, define H_S such that $\forall t\,(t \leq t_{pre})$ the subsystem comprises the familiar setup of incident electron and apparatus: $|\varphi\,(t \leq t_{pre})\rangle \;=\; \frac{1}{\sqrt{2}}(|\;\uparrow\rangle_S + |\;\downarrow\rangle_S)\,|\;\uparrow\rangle_O$. Let for $\forall t\,(t \geq t_{post})$ $H_S \rightarrow S \otimes R$ so that $H = S \otimes R \otimes H_E$ where R is the state of a distant, non-interacting particle. Since this particle is uncorrelated with the interacting subsystem and lies outside its past light cone, the overall state becomes $|\varphi\,(t \geq t_{post})\rangle = \frac{1}{2}(|\;\uparrow\rangle_S|\;\uparrow\rangle_O + |\;\downarrow\rangle_S|\;\downarrow\rangle_O)\,(|\;\uparrow\rangle_R + |\;\downarrow\rangle_R)$. Accordingly, the particle and apparatus do not possess unique time-evolved counterparts by dint of the fact that their mathematical representation branches into states $|\;\uparrow\rangle_R, |\;\downarrow\rangle_R$. Given no constraint on admissible factorisations, subsystem continuity is violated by virtue of the lack of a unique continuation. With the stipulation of naturalness and simplicity, perverse counter-examples such as this are disqualified, seeing as this factorisation is clearly physically unnatural. However, without the stipulation of simplicity and naturalness constraining the factorisation, the fact that EQM violates subsystem continuity could be explained away by contriving cases in which any theory violates it – even those the consensus considers deterministic in the stronger senses treated here. This result would trivialise the violation and vitiate the criterion altogether. Whilst this formulation, relying on notions of simplicity and naturalness, is inevitably vague given that a comprehensive elaboration of these terms is beyond the scope of this section, this is analogous to the way in which constraints are applied to the mathematical formalism of many other physical theories in order to map them onto physically relevant systems, such as natural choices of coordinate systems in general relativity. As Maudlin argues:

> Any empirical physical theory must have implications for observable events at the scale of everyday life, even though that scale plays no special role in the basic ontology of the theory itself. The fundamental physical scales are microscopic for the "local beables" of the theory and universal scale for the non-local beables.[11]

This stipulation does not appear to be present in the framework of Müller and Placek, making SC a more exacting standard for the classification of deterministic theories.

It may be wondered why the fact that EQM rules out SC in principle is surprising or objectionable. Indeed, the argumentative tactic often deployed by those who defend the interpretation attempts to dissolve rather than refute its counter-intuitive implications – whether the implication that, for instance, the problematic status of probability in the theory is in fact a feature of empirical probability in general, or whether the implication that the vagueness of decoherence is no different from the vagueness of many other accepted phenomena supervening on an underlying distribution with greater degrees of freedom.[12] Therefore, the following defence of EQM could arise: if subsystem continuity is violated by EQM, the former should be rejected and the latter should be preserved; if dogmas of philosophy unravel in the theory which best fits our observations, so much the worse for the dogmas. If this defence be pursued, there are two important responses.

Firstly, the fact that SC is not only violated, but violated *in principle*, is significant. No interpretation which provides a factorisation of the Hilbert space and dynamics consistent with EQM as defined can preserve uniqueness of continuation, even if these were applied at the level of the reduced density matrix defining subsystems. SC-violation is not limited in scope to a specific Schrödinger evolution $H\,|\varphi(t)\rangle$ but encompasses all those which are consistent with this Schrödinger evolution and feature the transition $P_{\beta_n}(t_n) \to\ P_{\uparrow_m}(t_m)\,,\ P_{\downarrow_m}(t_m)$. Otherwise, EQM would have a natural defence: it would be logically possible that another dynamical theory of which EQM is a special case could be found which explains this evolution. Such a possibility would enable Everettianism to escape the semblance of indeterminism by imputing a statistical character to the dynamics, or by arguing that the violation arises only from coarse-graining. By contrast, in order to test the consistency of EQM with the possibility *in principle* that subsystem continuity be upheld, only the minimum constraints required to ensure its consistency with EQM are imposed.

Secondly, there are other interpretations which fulfil SC, as well as broader scientific theories which EQM cites as inspiration, as was the case in DBRN. From a Bohmian perspective, for example, SC is satisfied, as only one branch or continuation is selected by the configuration $\mathbf{Q}_t$. Moreover, aside from the question of its status vis-à-vis other interpretations of QM and entirely outside of this context, EQM falls beneath the standards set by theories such as fluid mechanics, for instance: the fluid parcels described by this latter theory may not possess an identity map to successor parcels with a different coarse-graining but, relative to a physically natural consistent coarse-graining, subsystem continuity in the sense defined above is preserved. Just as in the case of DBRN discussed above, other theories which serve as important benchmarks for the attainment of determinism sustain a burden which EQM fails. If the test were unreasonably stringent, the different classifications accorded by SC to different candidates would prise open a serious explanatory gap.

What follows from this set of results? A plausible, albeit discriminating, conception of determinism does not classify EQM as a deterministic theory. If it were to be maintained that EQM nonetheless qualifies as deterministic in some different sense, the caveat is required that this sense is all the more weak – and all the more divergent from that of the other theories mentioned above from which it aspires to draw inspiration.

Conclusion

Everettian Quantum Mechanics' consistency with determinism is widely held to be a distinctive merit of the theory and a mark of its value proposition over against competing interpretations, many of which are avowedly indeterministic. Espoused consistently, this perspective is in evidence in Everett's original writings as well as those of contemporary champions of EQM such as Carroll, and has stimulated far less opprobrium than other, critically besieged features of the theory such as its consistency with notions of locality.[3][17] However, such a conclusion is hardly evident from mere inspection, especially in view of the complex and heterogeneous topography of modern characterisations of determinism: recent years have seen a clear intensification of the debate over the meaning of this concept, the 2018 analysis of Müller and Placek being a prime example. This article attempts to fill aspects of this lacuna. It offers several relevantly distinct definitions of determinism, from weaker, more traditional Laplacian conceptions to stronger, more modern branching-style conceptions, as follows:

Laplacian Determinism: Given a solution S_i of the governing equations $f_1, f_2, \ldots, f_n$ of a theory T_1 at a time t_i, there is a unique solution set S_j at a time t_j which is a solution of $f_1, f_2, \ldots, f_n$.

DBRN: A model, $\langle M, \ <, S, f\rangle$, is indeterministic if and only if $\langle M, \ <\rangle$ contains more than one maximal chain.

Subsystem Continuity: An interpretation I counts as subsystem continuous just in case its subsystem projectors possess a unique continuation.

Whilst EQM satisfies LD, so do a panoply of other physical theories whose deterministic credentials are considered tenuous by comparison, including competing interpretations of QM. Thus, satisfaction of LD cannot constitute an accomplishment of determinism which is sufficiently distinctive of EQM relative to these interpretations. Further, LD suffers from unhelpful ambiguities, especially as respects the denotation of "solution" which appears in the definition. Unique solutions can be calculated for many systems which would not reasonably qualify as deterministic, as is evident in the case of solutions of statistical laws which are ascribed probability weights. In the absence of further constraints on which solutions are admissible, LD is too equivocal to decide cleanly between theories competing for this label. EQM's compliance with LD is therefore no assurance of its "rigorously" or "straightforwardly deterministic" character.[3][4]

Instead, stronger and more precise notions must be adopted which overcome these limitations, the Müller-Placek DBRN formulation and SC being the examples considered herein. EQM's violation of these principles constitutes a significant limitation on the sense in which it amounts to a deterministic theory.

It is noteworthy that in many of these discussions the feature adduced in support of EQM's classification is very frequently the absence of any kind of collapse postulate in its account of measurement and decoherence, and its refusal to supplement the Schrödinger dynamics with anything additional. The implication seems to be that this refusal alone is sufficient to rescue quasi-classical deterministic intuitions from the conceptual bonfire of Copenhagen QM. Carroll, Wallace, DeWitt and others all make statements along these lines, perhaps most quintessentially the following: "This state vector never collapses, and hence reality as a whole is rigorously deterministic." [7][3][17] Unelaborated, however, this is an argument not for determinism but for unitarity. The latter is a mathematical feature of linear operators within vector spaces; the former is a metaphysical attribute. The relation of the two may be plausibly intuitive yet, without an independent argument for their identity, determinism cannot be deduced from this relation. Without such an argument, pronouncements as to EQM's deterministic credentials must be caveated commensurately, and the ambition to reconcile quantum mechanics with determinism in its strongest sense falls short of being accomplished.

Footnote

1 See [18] for a compelling account of the limitations of this widely used term.

2 [19] Here, "EQM" and the many-worlds interpretation of QM ("MWI") are taken to be coterminous.

3 [20] Page numbers herein refer to the arXiv version.

4 See [1][10].

5 See e.g. [21].

6 [14] These authors' framework appears to be conceived to accommodate global or macroscopic indeterminism's consistency with local or microscopic determinism, rather than the reverse, but the potential for asymmetry in either direction is possible in principle.

7 [22] See also [21].

8 [1] Note that an alternative definition is also provided on page 242.

9 See e.g. [21] and [23].

10 This notion of subsystem continuity is structurally analogous to a canonical principle outlined by Earman in *A Primer on Determinism* and *World Enough and Space-Time*. This principle, *local* determinism, is defined principally in the context of relativity and field theories, as follows: "Local Laplacian determinism means that the state in a region R is determined by the state on any spacelike S such that $S \subset C^-(R)$ and $R \subset D^+(S)$." Analogously to the subsystems of QM, here, the focus is not so much on solutions to dynamical equations, but the state in a given region, as the subject matter of determinism. [10][24]

11 See e.g. [25]. See also [26], in which he highlights the challenge in resolving abstract high-dimensional objects onto the localised objects in lower-dimensional spacetimes which are the subjects of experiments.

12 See e.g. [27].

doi.org/10.32388/3RE2VT

References

1. a, b, c, d, e, f, g, h, i, j, k *Müller T, Placek T (2018). "Defining Determinism." Br J Philos Sci.* ***69****(1):215–252. https://academic.oup.com/bjps/article/69/1/215/2669643.*
2. ^ *Everett H (1973). "The Theory of the Universal Wavefunction." In: DeWitt B, Graham N. The Many-Worlds Interpretation of Quantum Mechanics, pp. 3-140. Princeton: Princeton University Press.*
3. a, b, c, d *DeWitt B, Graham N (1973). The Many-Worlds Interpretation of Quantum Mechanics. Princeton: Princeton University Press.*
4. a, b *Greaves H (2004). "Understanding Deutsch's Probability in a Deterministic Multiverse." Stud Hist Philos Mod Phys.* ***35****(3):423–456. https://www.sciencedirect.com/science/article/abs/pii/S135521980400036X.*
5. a, b *Deutsch D (1997). The Fabric of Reality. London: Penguin.*
6. a, b *Wallace D (2012). The Emergent Multiverse: Quantum Theory According to the Everett Interpretation. Oxford: Oxford University Press.*
7. a, b *Wallace D (2013). "A Prolegomenon to the Ontology of the Everett Interpretation." In: Ney A, Albert D. The Wave Function: Essays on the Metaphysics of Quantum Mechanics, pp. 203-222. US: Oxford University Press.*
8. ^ *Wallace D (2013). "The Everett Interpretation." In: Batterman R, editor. The Oxford Handbook of Philosophy of Physics. Oxford: Oxford University Press. https://philsci-archive.pitt.edu/8888/1/Wallace chapter in Oxford_Handbook.pdf.*
9. ^ *Laplace P (1951). A Philosophical Essay on Probabilities. Truscott F, Emory F, translators. New York: Dover.*
10. a, b, c, d, e *Earman J (1986). A Primer on Determinism. Holland: Kluwer.*
11. ^ *Hoefer C (2024). "Causal Determinism." In: Zalta E, Nodelman U, editors. The Stanford Encyclopedia of Philosophy. The Stanford Encyclopedia of Philosophy. https://plato.stanford.edu/entries/determinism-causal/.*
12. ^ *Born M (1969). "Is Classical Mechanics in Fact Deterministic." In: Physics in My Generation. New York: Springer.*
13. ^ *Ghirardi G, Rimini A, Weber T (1986). "Unified Dynamics for Microscopic and Macroscopic Systems." Phys Rev D.* ***34****(2):470–491.*
14. a, b *List C, Pivato M (2021). "Dynamic and Stochastic Systems as a Framework for Metaphysics and the Philosophy of Science." Synthese.* ***198****(3):2551–2612. https://pmc.ncbi.nlm.nih.gov/articles/PMC8570365/.*
15. a, b *Butterfield J (2005). "Determinism and Indeterminism." In: Craig E, editor. Routledge Encyclopedia of Philosophy, Vol. 3. London: Routledge.*

16. ^Goldstein S (2025). "Bohmian Mechanics." In: Zalta E, Nodelman U, editors. The Stanford Encyclopedia of Philosophy (Fall 2025 Edition). The Stanford Encyclopedia of Philosophy. https://plato.stanford.edu/archives/fall2025/entries/qm-bohm/.
17. a, bCarroll S (2019). Something Deeply Hidden: Quantum Worlds: and the Emergence of Spacetime. Oneworld Publications.
18. ^Wallace D (2019). "What is Orthodox Quantum Mechanics." In: Cordero A. Philosophers Look at Quantum Mechanics, pp. 285-312. Cham: Springer Verlag. https://arxiv.org/abs/1604.05973.
19. ^Vaidman L (2014). "Quantum Theory and Determinism." Quantum Stud Math Found. **1**:5–38. https://link.springer.com/article/10.1007/s40509-014-0008-4.
20. ^Timpson C, Brown H (2002). "Entanglement and Relativity." In: Fano V, Lupacchini R. Understanding Physical Knowledge. University of Bologna. https://arxiv.org/pdf/quant-ph/0212140.
21. a, b, cHalliwell J (2010). "Macroscopic Superpositions, Decoherent Histories, and the Emergence of Hydrodynamic Behaviour." In: Saunders S et al., editors. Many Worlds? Everett, Quantum Theory and Reality, pp. 99-117. Oxford: Oxford University Press.
22. ^Saunders S (2010). "Many Worlds? An Introduction." In: Saunders S et al., editors. Many Worlds? Everett, Quantum Theory and Reality, pp. 1-49. Oxford: Oxford University Press.
23. ^Hartle J (2010). "Quasiclassical Realms." In: Saunders S et al., editors. Many Worlds? Everett, Quantum Theory and Reality, pp. 73-98. Oxford: Oxford University Press.
24. ^Earman J (1989). World Enough and Space-Time: Absolute versus Relational Theories of Space and Time. Cambridge: The MIT Press.
25. ^Maudlin T (2015). "The Universal and The Local in Quantum Theory." Topoi. **34**(2):349–358.
26. ^Maudlin T (2010). "Can the World be Only Wavefunction." In: Saunders S et al., editors. Many Worlds? Everett, Quantum Theory and Reality, pp. 121-143. Oxford: Oxford University Press.
27. ^Wallace D (2012). "Decoherence and Ontology." In: Saunders S, Barrett J, Kent A, Wallace D, editors. Many Worlds?, pp. 53-72. Oxford: Oxford University Press. https://arxiv.org/abs/1111.2189.

Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.