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Qeios

Peer Review

Review of: "Visualizing Generalizations of the Pythagorean Theorem"

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The Article by **Dirk Huylebrouck "Visualizing Generalizations of the Pythagorean Theorem"** deals with an exhaustive account of the proofs given over the years of the Pythagorean theorem and of its generalizations.

The "theorem" attributed to Pythagoras, a mathematician of Greek culture, was known even before the concept of theorem (namely the derivation of a proposition starting from previously established conditions) was known. It was the result of thousands of years of studies by various mathematical cultures (Indian, Assyro-Babylonian, Egyptian ...)

The theorem was included in Euclid's Elements, which represented the first work providing a systematic (axiomatic) exposition of the Greek conception of geometry.

The Article is interesting for various reasons, including the vast bibliography. It therefore deserves careful consideration.

The article presents "visual" proofs of Pythagoras's statement and of its generalizations. This procedure was probably that which inspired the very first intuitions of the ancient mathematicians, but it should be noted that the use of more modern tools like those of trigonometric/algebraic means yields an analogously simple understanding.

Regarding, e.g., the Ebsui-Nortrott theorem (Fig.1) (Fig. 3 in the article), according to which the sum of the areas of the yellow squares is five times that of the magenta square, namely ***A1+A2=5A***, the relevant proof is straightforwardly obtained using ordinary trigonometric tools.

Denoting by a, b, c the sides of the internal triangle and with β, α, $\pi/2$ the opposing angles, accordingly we find

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$$c^2 + b^2 - 2bccos(\pi - \beta) = A2$$

$$c^2 + a^2 - 2accos(\pi - \alpha) = A1$$

Summing term by term the two identities, we find

$$3c^2 + 2c[bcos(\beta) + \mathrm{acos}\ (\alpha)] = A2 + A1$$

being

$$bcos(\beta) + \mathrm{acos}(\alpha) = c$$

We eventually find

$$5c^2 = A1 + A2$$

and since

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$$A = c^2$$

, the theorem is proved.

The theorems reported in Figs. (4,5) of the article can be proved along analogous lines.

An extensive bibliography is provided, and it proposes elements of thought for a new way of teaching Euclidean Geometry at junior/high school levels.

The most adopted pedagogical attitude is that developed in the West towards Mathematics, namely a definite set of axioms useful to derive a logical connection between the various geometrical entities.

On the other side, a different attitude was developed in ancient times (the so-called Assyro-Babylonian conception) in which the development of mathematics is based on more flexible rules in which intuition plays a central role.

Within this context, the figures contained in the article are a very good visual aid to support the intuition and even to generalize the Pythagorean theorem itself. The pitfall of this pedagogical attitude is that it may hamper the ability to think in abstract terms.

However, this can be overcome by pupils with a genuine interest in Math because, as explained by pedagogists like Piaget, that abstraction comes after visual intuition.

A suggestion by this Referee is that of including in the references the following book

"Vedic Mathematics, a mathematical tale from the Ancient Veda to Modern times"

by ***Giuseppe Dattoli, Silvia Licciardi and Marcello Artioli (World Scientific 2021)***

Whose initial Chapter contains some speculations on the "ancient view" of Pythagoras and Euclid's theorems, which clarifies the role of the interplay between western and eastern views in the development of Mathematics.

The Chapter also contains different proofs of the theorem as well as the “proofs” yielding the two Euclid’s theorems using a mixed algebraic/geometrical language.

In conclusion, the opinion of this referee is that the article deserves publication (with or without the inclusion of the suggested reference).

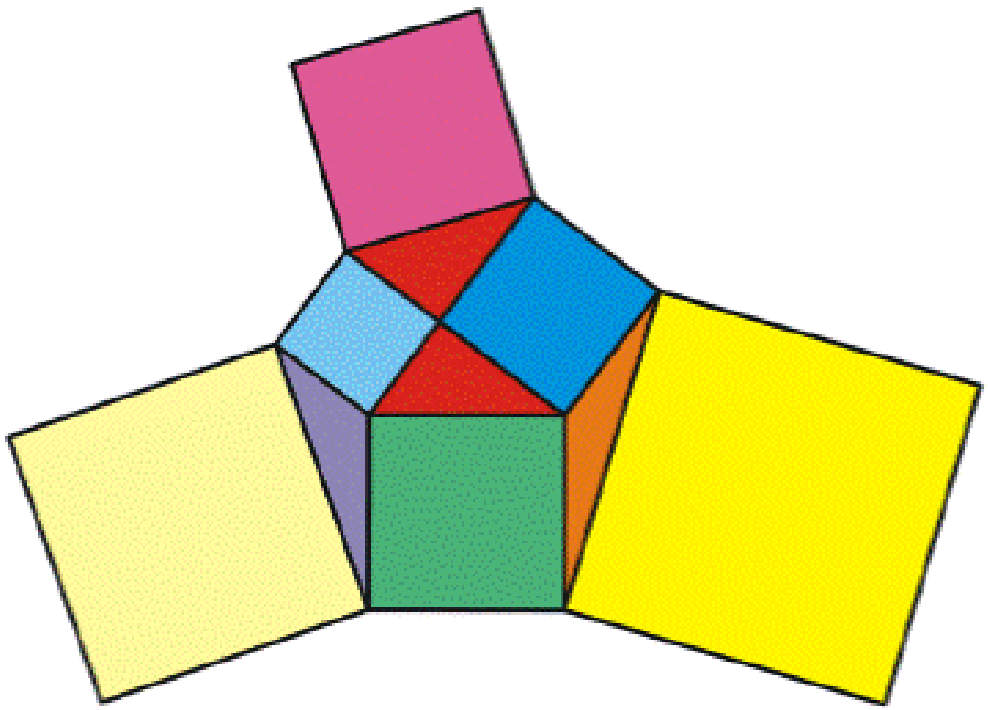

Fig. 1

1-st Ebisui - Notrott theorem

Declarations

Potential competing interests: No potential competing interests to declare.

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