## Peer Review

## Review of: "Inverting No-Hair Theorems: How Requiring General Relativity Solutions Restricts Scalar-Tensor Theories"

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In this article, the authors study scalar-tensor theories and derive sufficient conditions for the existence of stealth black hole solutions when including terms up to cubic order in second-order derivatives of the scalar. The authors restrict their search to the context of a scalar field whose kinetic term is constant and derive their main results in section 2. In particular, they provide the explicit sufficient conditions to be satisfied by the couplings/functions determining the scalar-tensor theory in order for it to allow for such stealth black holes. Afterwards, they also examine odd-parity perturbations and analyse the stability of solutions.

There are various aspects to be addressed:

- 1. The acronym HOST is not defined (DHOST is indeed defined). It is understood by the context, but it should be defined.
- 2. The restriction to profiles of the scalar field for which the kinetic term is constant is not sufficiently motivated. There is some argumentation in the introduction and section 2.2, but I think it is not sufficient. What other solutions of scalar-tensor theories have X=const.? What happens if one departs from that condition?
- 3. Below eq. (2.10), the authors write "\phi=qt+\psi(r)". What is \psi(r)? Furthermore, no coordinates have been introduced at this point.
- 4. The derivation of conditions (2.14) is not explained. How could one get those conditions? What terms of (2.8) must be set to zero? The authors should clarify this aspect in detail, as this corresponds to their main result.

- 5. Are conditions (2.14) necessary, or only sufficient? Perhaps this could be answered as point "4." is addressed.
- 6. Conditions (2.14) refer to minimally coupled matter with a cosmological constant. However, in eq. (2.8), no cosmological constant was included. This should be amended.
- 7. There is a typo in (3.1). The usual "\mathrm{d}\varphi^2" is missing.
- 8. Above (3.9), it is said that "(...) metric perturbations are usually decomposed in terms of the spherical harmonics Y\_{lm}. Note that one can set m=0 without loss of generality thanks to the spherical symmetry (...)". The authors should be more precise and justify this statement further.
- 9. In (4.1), the auxiliary field \$\chi\$ is introduced. However, the explanation of how it is introduced is somewhat short. Although some references are highlighted, how the auxiliary field \$\chi\$ appears in the quadratic Lagrangian should be carefully explained.

I only recommend the article for publication if the above points are addressed.

## **Declarations**

Potential competing interests: No potential competing interests to declare.