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Teaching Mathematics with Creativity

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Abstract

This article explores and reviews the literature about the latest research-based teaching and learning techniques or strategies that are used by some of the most passionate and enthusiastic classroom teachers, often for the purpose of enlightening, firing up, or illuminating classroom lessons in order to grab students' immediate attention, curiosity, and overall interest in the learning process. The question used to guide our exploration of the literature is: How do effective classroom teachers manage to always create a conducive, welcoming, and exciting rather than depressive learning environment? While the current research output is more often than not forward-looking, that is, it is too busy moving forward as it comes up daily with suggested new ways of teaching and learning, almost nothing or little attention is given to what is already available, discovered, or recorded. This means there is an implication that a valuable knowledge domain already discovered could remain largely un-utilized, an unused knowledge domain that is already available in the recorded literature, which can be put into practice in classroom settings. Therefore, this review article highlights some of the latest, creative, and effective teaching techniques that the authors explored and found impressive. These are effective strategies that might be otherwise overlooked, overshadowed, or blurred; but they are effective techniques that can be utilized by the most passionate and enthusiastic classroom teachers for the purpose of making mathematics attractive to all types of learners, often by the ways of generating, stimulating, and/or maintaining

students' interest in the mathematics subject matter.

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1. Introduction

The overall primary purpose of teaching in general is to facilitate, motivate, and inspire learning; and anybody can stand in front of learners in a classroom setting, but it takes a real classroom teacher to make sure that learning takes place in a meaningful way. That is why we can witness all kinds of classroom teachers teaching with all sorts of teaching styles. For example, while some classroom teachers may teach with passion, technology, or handouts, other classroom teachers may teach with creativity, music, or singing songs in the classroom setting. Still others can even teach with threats or through intimidations. All of these various and diverse teachers' teaching styles can be effective depending, however, on how each style is well mastered, implemented, or communicated; and therefore, there may be no bad or good teaching style, there may be only an effective or ineffective style, an appropriate or inappropriate one, a satisfactory or unsatisfactory style, depending on the eyes of the beholders, which in this case are the students themselves–the classroom learners. The overall effective teachers' teaching traits such as immediacy, quality, and competence are mediated, regulated, and/or glued together by the classroom teachers' interpersonal communication competence.

Our specific goal in this article is to try to teach mathematics with or through creativity; another is to highlight and recommend some research-supported classroom practices that can help generate, stimulate, and maintain students' interest in the mathematics learning experiences.

Meanwhile, our secondary–aimed objective is to advance a proposal that introduces teaching approaches which can help make mathematics attractive to all types of learners; and also to introduce a unique–creative way that can turn mathematics teaching and learning into an exciting, memorable, and enjoyable learning experience. Interest in a subject matter in general, and in particular, the student's interest in mathematics, is a multidimensional phenomenon and perhaps the single most motivating, inspiring, and driving force for all human activities, including learning. Because we are exploring a multidimensional–delicate phenomenon such as generating, stimulating, and maintaining students' varying interest in mathematics, we have also adopted a holistic, dynamic, and multidimensional approach, view, or perspective where all possible contextual features of the learning environment are considered in addressing interest-related issues, essential components, or its dynamic variables. The justification, rationale, and the reason for our declared particular purpose (the generation, stimulation, and maintenance of students' varying interest in a subject matter) may be better expressed or illustrated by the following creative story.

In the early 1940s, goes the story, the great doctor Albert Einstein was teaching at Princeton University; and one day, he gave an exam to his senior class of physics students. 'Professor Einstein,' his assistant sought his attention as they walked leisurely on a sidewalk or hallway, 'have you, by any chance, noticed that you have just given exactly the same exam you gave to the same-similar group of students a year ago?' Yes, I noticed that, it is exactly the same exam, replied Doctor Einstein. 'But prof, how in the world could you possibly do that, Professor Einstein,' his assistant wondered. 'It is because the answers have changed although the questions are still the same,' Professor Einstein replied. If this story made sense and was then true in the 1940s, it is still even truer in our present-day of today's rapidly–changing technological advancements.

This is to claim and argue that teaching as a business may not have changed very much since that time, but the teaching and learning techniques have changed over time and are changing rapidly. Therefore, there is a need to always adopt rather than try to resist these rapid changes; changes which are reflected in today's regular public opinions Gallup polls, opinions which reflect real-world life experiences; as increasingly more and more people are not only questioning the current directions of public education, especially higher education, in particular where things tend to be the opposite of what they are called, described, or expected to be, but also maybe abandoning it altogether for other alternatives or the private sector [1]. These technology-driven rapid changes imply that practicing classroom teachers must always be adaptive, versatile, flexible, keep an open mind, and maintain a growth mind-set while at the same time acquiring teachers' creative abilities in order to adapt to such rapid technological and research-driven changes.

Creativity, however, is a slippery and delicate phenomenon which is well-defined and much-developed outside the realms of the mathematics classroom settings or contexts; and it is defined in many different ways by many different sources outside mathematics classroom contexts. Some sources define *creativity* as thinking outside of the box while still in the box [2]. Others define *creativity* vaguely as an ambiguous process of coming up with something new, but which is original and has value or utility [3]. Therefore, it can be argued that *creativity* is dynamic (non-linear) and it includes the most basic–essential elements of the thinking process or the usual habits of mind such as *intuition*, *common sense*, and *imagination* (essential elements which are thinking related or habits of mind that science does not yet really know how to explain) and arguably, even a habit such as *procrastination*–the academia's most dreaded habit of mind–is an essential element of *creativity*, especially when moderately used or interpreted as a*reflection* in actions or on teaching activities.

However, for all the rapid–daily changing practical purposes in various classroom settings, we would rather rephrase *creativity* a little bit more differently because classroom teachers do not always have to come up all the time with something new or original. We therefore rephrase *creativity* somehow differently in a versatile, flexible, and dynamic fashion: either as (1) the classroom teacher's ability to *acquire*, *adapt*, and *utilise* multiple techniques or strategies to *achieve* certain learning outcomes; (2) the classroom teacher's creative ability to*perform* different-multiple functions or tasks with a single tool; and/or (3) the classroom teacher's ability to *fit together* or harmonise all the interacting–moving parts of various dynamic components of the present-day classroom contexts. Therefore, with these flexible definitions, classroom teachers in present-day classroom settings are encouraged to teach mathematics in a creative fashion so as to generate, stimulate, and maintain students' various interests in the subject matter.

2. The Present-day Context of the Classroom Setting

A well–intended but delicately context-dependent learning place such as the classroom environment of the present-day era can be so complex, so dynamic, and so diverse that it is better approximated or described in terms of metaphors such as the *animal school* fable, *chaos theory*, or *entropy* analogy and the *butterfly effect*:

The first metaphor that best approximates or reflects the general present-day classroom context or situation is an interesting parable called the *animal school*[4] said to be first narrated by George Reaves in the 1940s; and the*animal school* fable reads as follows:

> *Once upon a time, the animals decided they must do something heroic to meet the problems of the "new world," so they organized a school. They adopted an activity curriculum consisting of running, climbing, swimming, and flying. To make it easier to administer the curriculum, all the animals took all the subjects. The duck was excellent in swimming. In fact, he swam better than his instructor. But he made only passing grades in flying and was very poor in running. Since he was slow in running, he had to stay after school and also drop swimming in order to practice running. This was kept up until his webbed feet were badly worn, and he became only average in swimming. But average was acceptable in school, so nobody worried about that, except the duck. The rabbit started at the top of the class in running but had a nervous breakdown because of so much makeup work in swimming. The squirrel was excellent in climbing until he developed frustration in flying class, where his teacher made him start from the ground up instead of from the treetop down. He also developed a "Charlie horse" from overexertion and then got a C in climbing and a D in running. The eagle was a problem child and was disciplined severely. In the climbing class, he beat all the others to the top of the tree but insisted on using his own way to get there. At the end of the year, an abnormal eel that could swim exceedingly well and also run, climb, and fly a little had the highest average and was valedictorian. The prairie dogs stayed out of school and fought the tax levy because the administration would not add digging and burrowing to the curriculum. They apprenticed their children to a badger and later joined the groundhogs and gophers to start a successful private school.* [4]

This fable describes exactly what is often found in a diverse, general classroom situation of today's era, especially in a more perceived-boring mathematics classroom environment. The *animal school* is a typical case reflecting the inconvenient truth about the classroom reality: a messy mixture of varying academic abilities and limitations, various individual talents and mediocracies, and distinctive skill sets (efficiencies) and deficiencies. This fable clearly indicates that the badly desired academic abilities, the much-hyped intelligence quotients (IQ), the so-called IQ labels (a description which only suggests that some parts or essential components of the whole thing are missing); and/or the intellects (the data collections and storages), often confused, mistaken, and even equated for an actual-real intelligence–a multidimensional phenomenon which includes essential elements of *awareness*, *wisdom*, and *humility*–are *diverse*, *distinctive*, and *dynamic*. This is because an ability in one area of academia is a deficiency in another, as the fable illustrates. This fable suggests that the primary purpose of education should be rethought, revised, and reoriented toward individual growth or development while at the same time balancing the perceived deficiencies or mediocracies in other areas of academia. A broader, wider, and more holistic approach, a multidimensional, dynamic approach to teaching and learning while at the time balancing and supplementing the perceived deficiencies, limitations, or gaps, is a way to pursue in a classroom setting of our present-day era.

Another related metaphor, called complexity in the classroom setting, is a more recent application of entropy or the chaos theory in the classroom environment. Said to be first described by David Sobel in Kane's edition [5], this metaphor is an argument as it claims and argues about the inevitability, unexpectedness, and unpredictable classroom situations of the present-day learning environment:

> *Rather than assume that classrooms behave like clockwork, let us consider that they work like weather systems, one system that chaos scientists have been working to understand. Weather systems have classically eluded long-range predictability because they are multivariable systems with a "sensitive dependence on initial conditions." With so many interacting variables, slight changes at some distant point can make a major impact on how weather systems will evolve. That is why an ironclad forecast for beautiful weekend weather on Thursday can turn into intermittent showers by Saturday morning. Classrooms have the same kinds of dynamics. When you factor in 20 different personalities, unexpected fights in the hallway, cancelled band practice, the unexpected birth of baby gerbils, and eight students absent because of the flu, it is hard to guarantee that your weekly curriculum plans, written on Sunday evening, will have much resemblance to the classroom state of affairs on Thursday. It is feasible to stay on track, but sometimes only at the expense of numerous missed possibilities. Certainly, teachers need a clear vision of what's appropriate and useful and make choices about the potential productivity of any tangent. But everyone acknowledges that curriculum becomes intriguing, alive, and compelling when something out of the blue captures the imagination of a group of children. Chaos theory suggests that we should recognize the inherent unpredictability of the behaviour of such a complex system as a school classroom.* [5]

Although the above *chaos theory* metaphor may be seen as just a fantastic, imaginative, and creative way of looking hypothetically at what would be otherwise considered a normal classroom situation, it depicts quite accurately what should be expected in a more boring mathematics classroom environment. In a context such as the present-day classroom environment, characterized by metaphors such as the *animal school* and/or *chaos theory*, various distinctive skill sets are recognized while perceived deficiencies or mediocracies are acknowledged and then improved rather than denied.

2.1. Butterfly Effects in the Classroom Setting

Chaos theory, which involves entropy or complexity in the classroom setting, is often associated with and explained in terms of a *butterfly effect*, a phenomenon which explains the relationship between seemingly arbitrarily minor–random causes or changes and their probable drastic–huge effects, results, or outcomes [6]. For example, it is often joked that a butterfly flapping its wings randomly somewhere in New York is capable of causing a typhoon or a tornado elsewhere at a faraway distance such as in Tokyo. This concept of a *butterfly effect*, that is, the tinny little things or changes teachers do in order to make a difference in the classroom setting, demonstrates that a seemingly minor teaching technique or strategy in a classroom can lead or contribute to an exponential increase in students' motivation or inspiration—eventually causing probable major positive results or favourable learning outcomes in the classroom setting [6].

Alternatively stated, the *butterfly effect* stresses that minor differences or similarities at the beginning may have huge-dramatic consequences in the long run, that is, some initial minor changes or adjustments at the beginning may later on lead to huge unexpected-positive results or outcomes. This means a slight change in the initial condition of a given situation can have drastic-huge implications for future behaviours or outcomes; and in fact, the behaviour of a system with different initial conditions or situations, no matter how similar they appear, diverges exponentially as time passes [6]. Hence, this phenomenon known as the *butterfly effect*–an earlier offshoot of chaos theory or complexity in the classroom setting–demonstrates these complexity features and underscores the interdependence of all the dynamic–interactive components in any given system, situation, or setting, such as the students' feared and dreaded mathematics classroom environment [6].

2.2. External Variables that Affect the Classroom Setting

In addition to the internal complexity in the dynamics of the classroom setting, there are also external variables that can affect the classroom environment; and it is common sense, intuitively self-evident, and an open secret that present-day classroom teachers are overcriticized or less trusted, perhaps second only to politicians or car mechanics; that is, classroom teachers are over-stressed, overworked, underpaid, and underappreciated. Despite the continuous criticism from the classroom regular critics (such as students, parents, administrators, and even the least trusted politicians), criticism which is always either soaked in, brushed aside, or welcomed and more often than not appreciated by most classroom teachers as a necessary component of continuous improvement or growth, classroom teachers generally do their best through the way of teacher resilience and wellbeing [7][8][9]; and by investing appropriate time, effort, and energy in utilizing the concept of instructional best teaching practices, often in the form of carefully planned lessons, organized communication skills, and effective delivery of the planned lessons. This is in addition to the classroom teachers' passionate commitment to other related components of the business of teaching in the classroom setting. Therefore, classroom teachers may be stressed out, burned out, and overloaded with extraneous busy working schedules, but they still make sure that the primary role or purpose of teaching and learning is satisfied, that is, student learning is facilitated in the most meaningful way possible.

Being stressed or burned out by the overloaded working schedule does not, however, prevent passionate classroom teachers from often going out of their ways in order to explore new techniques or strategies; as teachers try out new ways of teaching and learning, they more often than not find little ways that help foster, nurture, or trigger students' most powerful learning tools such as the students' intrinsic motivation and an inspiration for learning, tools which are essential for mastering the course materials, no matter how difficult the subject matter might appear.

While it is generally an accepted argument that mathematics itself is one of the most difficult subjects to learn by students, there are ways (according to the literature) mathematics teachers can facilitate students' learning of the mathematics subject matter. Such ways include, but are not limited to, teachers developing students' interest in the subject matter, triggering and maintaining students' intellectual curiosity, and fulfilling students' basic physio-psychological needs, to mention but a few teaching tools, techniques, strategies, or the *butterfly effects* in the classroom setting.

3. Teaching Techniques and Strategies in the Present-day Classroom Setting

The practice–related question used as a guide in this review article was how classroom teachers actually manage to be effective in such a highly context-dependent, complex, chaotic, and unpredictable learning environment such as the present-day classroom. While courage, endurance, and comfort with complexity is a necessity, a quick–short answer (according to the literature) is that passionate classroom teachers are always aware of the concept of the *butterfly effects* discussed in the preceding section; especially those classroom teachers perceived as crafty, creative, and probably the most effective in the classroom settings, are the ones who often go the extra mile. They go ways out of the box in order to create a *butterfly effect*, a contextualised, conducive, enjoyable, and interesting learning environment for their diverse learners. Going the extra mile is in addition to always embracing the concepts of continuous–professional development or growth. This is in addition to the idea of instructional–best teaching practices in the classroom setting; usually in the forms of carefully planned lessons, sufficient and adequate communication-organizational skills, and clarity of instructions, plus motivating and inspiring students with captivating lessons during delivery.

Some literature–research suggested crafty and tinny-little ways that can help generate*butterfly effects* to maintain students' interest in mathematics in the classroom setting is quite a long list of techniques or strategies that include even the most shunned-traditional lectures, especially the typical thought-provoking, inspiring lectures that ponder and wrestle with perhaps the most ultimate why question of all times: the puzzle of existence and everything else in the universe [10]. Here in this brief review, we highlight some of these teaching and learning techniques or strategies that we (the authors of this review article) explored and found impressive; and we admit that we only scratched the surface–given that the list of techniques can be endless.

4. Debating Mathematical Concepts, Ideas, or Perceptions

Debating mathematical concepts, ideas, or perceptions is something that has not been paid careful attention, but one that has the ability to grab the audience's full attention or interest, especially when two or more knowledgeable mathematicians engage each other in a public debate, discussion, or conversation about mathematical concepts, ideas, and/or perceptions. Mathematics is a subject matter that has completely escaped scrutiny or careful examination, not only because of the perceived difficulties involved in learning the subject matter–perhaps due to the apparent pointlessness, dryness, or meaninglessness of mathematics–but also because of the diverging views of those who manage to visualise and grasp this largely symbolic language such as mathematics, a language which is largely unspoken but quite explainable. For example, most mathematicians cannot agree on what exactly the subject is: for example, is the subject an art, a science, both, or neither, an indication of a lack of full understanding of the subject matter itself among professional mathematicians. Yet, every member of the mathematics community knows that with very good-clear definitions, symbols, and notations, one can easily and almost mindlessly carry out–in an autopilot mode–all the desired mathematical operations quite correctly; as was the case soon after the introduction of Leibniz's notations, which accelerated the development of calculus, and also the introduction of zero into the Indo–Arabic numeral system, which greatly revolutionised earlier mathematical development.

In the absence of a unified, agreed definition (which is actually good for the diversity of opinions), some really hard-core mathematicians within the mathematics community would go further and equate the mathematics subject matter with reality itself (such as mathematical existence being the same thing as physical existence), which only begs more questions than it resolves: e.g., which intermediate–momentary reality or whose reality it is, the reality as we know it as suggested by everyday arbitrary, generic, and trivial different life experiences or the imaginable–creative Platonic reality, which cannot be a very accurate reality because it is far removed from an existential really-real reality, although it is short of nothingness–the minimum possible reality–as such a perfect reality sounds too special [10]; and therefore too good to be true; that is, an elegant, uniformly smooth perfect reality void of all bumps, roughness, or incongruence (e.g., absence of little pockets of chaos, entropy, or ugliness, and/or all the fair mixtures that always accompany a would-be accurate really-real reality): a messy mixture yet attractive while boring at the same time, a reality which we always try to escape, usually by finding or doing something passionate or interesting.

In fact, equating mathematics with the whole of reality would be equivalent to touching different parts of an elephant (with blindfolded eyes) and then mistakenly believing that each of those parts is the whole elephant, a conclusion that would make an elephant smaller than the sum of its parts.

Meanwhile, others in the mathematics community view mathematics subject matter as a mere representation of reality (which is quite an improvement), but which part of actual reality, and admittedly, what a tiny–fractional part of reality, is representable by the mathematics subject matter or any other single-handed discipline out there, as suggested by the concepts of multiple representations, different interpretations; or as shown by Kurt Gödel's incompleteness theorem about the incomplete nature of all mathematical proofs, arguments, or justifications, a theorem which can really shock the hell out of some young–aspiring mathematicians who may misconceive the mathematics subject matter as a complete representation of reality.

And oh, what about an inconvenient truth, insightful observation, or a whistle-blowing–teasing challenge that, while indeed the applied and algebraic approaches to the mathematics subject matter have values in terms of utilities, the analytical approach in modern pure mathematics (as in the case of the students' dreaded real analysis) is just, well, a pure mathematical fantasy; and it is actually weak in terms of mathematical consistency, rigorousness, precision, and/or accuracy required of all the mathematics concepts, ideas, or perceptions. This is because the so-called “irrational real” numbers, often considered as the backbone of pure mathematics, calculus, or real analysis, have not actually been well-defined because of the characteristics of their infinite nature and their definite existence, let alone uniqueness, which is often lost in the inconsistency associated with infinite set theory, as arbitrary infinite sets no longer have definite representative elements, is logically weak and therefore questionable; making the whole theory of “real” numbers, irrational numbers, and/or the infinite sets not so well-developed like the rest of the other number systems (e.g., rational numbers); and it has become almost impossible to do a complete, sensible, and meaningful arithmetic with the so-called “real” numbers, as arithmetic operations in this system are limited to just a few–selectable–baby examples in most real analysis textbooks.

Thus, it can easily be demonstrated with just a few insightful examples that the really hard-core pure mathematicians, often obsessed with “real” analysis, have actually been building castles in the wide open air, building fancy-superfluous theorems on a non-existent undefined basis, the indefinite-irrational numbers, often by using or preferring the vague, ambiguous, and inconsistent method of infinite choices or dubious axioms rather than a clear-cut, verifiable algorithms; and these “real” analysts end up constructing the subject matter on a shaky, wobbling, if not non-existent, foundation, as is the case with the inherited vagueness in infinite set theory. It is true that some insightful, experienced, and senior mathematicians are, however, fully aware of these foundational–developmental issues in the so-called “pure” mathematics, but they rather keep quiet because of the fear of a possible collapse of the “real numbers” theory, particularly the so-called “irrational” numbers, which do not have the largest member, or the infinite decimals such as the square roots of the prime numbers and/or their other-related transcendental infinite decimals such as the interesting-magic number e or π, whose definite-absolute existence has not yet been logically well-established, although most mathematicians often point to Dedekind’s cuts or similar constructions, which do not really work in practice, or the often invoked equivalent classes of Cauchy sequences argument, which is also an ambiguous concept because all Cauchy classes are said to be the same near a limit point and therefore have no uniqueness whatsoever. Perhaps as a direct consequence of this inherited inconsistence in the theory of “real” numbers, we may have all witnessed (as inertial observers) in one way or another how mathematical fantasies, the inappropriate misuses of mathematics, the recent venturing of fancy–over-exaggerated–mathematical equations into real-world situations of the financial systems (banking), where they might not have been suitably applied, might have arguably led to the collapse of the banking systems worldwide in an almost domino effect fashion.

Still others in the larger mathematics community perceive mathematics subject matter as just a tool, a skill set, and an acquirable toolkit, which happens to be useful for studying and approximating reality or some part of it. This is because, although mathematics is indeed regarded as such a valuable, powerful, and efficient tool, other tools are out there and are still necessary because thinking as a skill set is not limited to mere manipulations of symbols, as it appears to be the case with mathematics, physics, or any other disciplines with computational components or dimensions. Mathematics, like physics, does not know how to deal well with non-physical realities [11], objects, or concepts such as attitudes, beliefs, perceptions, and/or motivations. Hence, for a fuller understanding of the subject matter, all the tools available out there are essential, and learners should be encouraged and provided with the opportunities to explore, discuss, argue, and/or debate mathematics concepts; debates similar to the infinity-concept debate [12] or the derivative versus the integral–conceptual debate [13], where two or more skilful, knowledgeable, and passionate–enthusiastic mathematicians square off, engaging each other in civilised-professional debates or conversations; because such conversations, discussions, or conceptual debates can help generate, stimulate, and maintain the overall public interest in the subject matter.

The infinity debate [12] is about how the concept of infinite sets has become a paradise for some mathematicians, but a probable joke for others. Almost everything in mathematics is nowadays being defined vaguely, uniformly, and smoothly in terms of infinite sets notion: e.g., clearly differently appearing–distinctly looking geometric objects with different-unique sets of parametric equations like a *line*, a *curve*, or a *circle*, are all viewed at the end of the game as mathematically the same or equivalent concepts because they are, well, all made up of an infinite set of points: a probable source of a joke which claims that a really hard-core mathematician can no longer tell the real difference between a mug, a cup of coffee, or a doughnut; a joke usually attributed to topology, a study of surfaces, as its origin. Whereas the derivative versus the integral debate [13] is a humorous–actors type debate or conversations between two or more respectful–accomplished mathematicians, all for the purpose of inspiring and motivating the mathematics subject matter for diverse students and learners of various academic backgrounds and/or cultures.

5. Teaching Mathematics with Technology

Teaching mathematics with technology (for instance, smart boards, overhead projectors, or computer simulations) is not a new phenomenon, like teaching mathematics with passion, but teaching with the aid of supplementary technological learning tools or materials such as the newly introduced emerging educational robotics is quite a new breed. With these new educationally customised robots [14][15][16], along with the appropriate use of social media resources such as popular YouTube, Google, Wikipedia, and other Internet-based search engines freely accessible, available, and almost explosions of various knowledge domains in the World Wide Web [17], learning can be really revolutionised and maximised as learning time (which used to be restricted, constantly fixed, or limited) is now an unrestricted free learning variable; learning time can now become so dynamic, so flexible, and so unlimited. However, these independent learning tools, which are contextual features of the learning environment, would also need independent learners as well in order to take full advantage of these revolutionary learning tools, resources, or materials.

6. Teaching Mathematics as a Storytelling

This technique of teaching mathematics as a storytelling[18], anecdotes, or narratives [19][20] through stories-related problems is quite an impressive, innovative, and interesting method of teaching and learning; not only because it provides some really real meaning [21] to what is often perceived by many as a meaningless task (the pointlessness and dryness associated with mathematics), but also because it exposes some of the limitations, gaps, or apparently hidden weaknesses of the mathematics subject matter: the fact that professional mathematicians themselves, at the end of the day, have to resort to using their naturally endowed language skill sets such as interpersonal communication competence (a valuable skill set which can be acquired or learned), where a language (e.g., mathematics)–which in the absence of attached meanings is just a collection of meaningless symbols, noise, or sound–contextualises, facilitates understanding, and provides some really real meanings to this highly symbolic, visual, and unspoken emotionless language such as the mathematics subject matter. It goes without saying that even at what is said to be its purest theoretical form (e.g., real analysis, number theory, and/or graph theory), the meaningful mathematical arguments, justifications, or proofs that are perceived to be the most elegant are often presented through natural language skill sets (e.g., the mathematical view's points, reasoning, interpretations, clarifications, imaginations, and/or innovations), language skill sets, or the communication competence expressed in its fullest glory or potential. In a formula, for example, $\sum_{i=1}^{p} \deg v_i = 2q$, which claims and argues that the *sum of the degrees of the verticesof any graph (G) equals twice the number of edges of G*; an elegant proof consisting of just a single one-liner or a sentence is as follows: *when summing up the degrees of the vertices of a graph G, each edge of G is counted twice, once for each of the two vertices incident with the edge* [22]. A consequence (or *corollary*) of this formula is to show that *every graph contains an even number of odd vertices*, which shall be justified or proved later on (in section 12) after some precise *definitions* are provided.

Even in the case of the natural laws (e.g., gravity), natural laws, which are nothing more than generalised, descriptive temporary patterns, trends, or regularities in the world that are stated, expressed, or frozen in mathematical forms, the really-real meanings behind those formulas, assumptions, or contexts where the laws momentarily apply (because of the presence of uncertainty, unpredictability, complexity, entropy, or chaos), are always conveyed and explained through the naturally-endowed language skill sets or communication competence in the form of storytelling, the context-related stories which provide for meaningful interpretations or reinterpretations of such frozen mathematical formulas, phenomena, and/or laws.

7. Flipping the Mathematics Classroom Setting

Flipping the classroom is an emerging-interesting teaching technique where the traditional-lecture-styled mathematics classroom setting is somehow flipped [23][24][25], an innovative instructional approach in which the learning environment is transformed into an exploratory, dynamic, and interactive atmosphere, where the teacher guides students as they discuss, problem-solve, and apply subject matter concepts with a certain degree of versatility, flexibility, or creativity in the subject matter such as mathematics. Flipping the classroom setting as a teaching and learning technique or strategy is perceived and taunted by many classroom practitioners [23][24][25] as the incoming-inevitable classroom of the future influenced largely by the growing-current generation of rapidly changing digital technologies, which encourage students to be proactive rather than passive participants in their learning process. In the flipped classroom environment, students are required to watch pre-recorded lectures or related videos outside of the classroom setting and are then expected to come to the classroom fully prepared for active participation, interactions, and discussions.

Besides being a tool for facilitating students' learning experience, the flipping the classroom technique or strategy could be a solution to the age-old classroom problem, challenge, burden, or anxiety associated with being abruptly and suddenly assigned (for the first time) to teach a time-consuming, difficult course or class; as this technique allows classroom teachers to have certain degrees of freedom to be versatile, flexible, or creative. In this case, a first-time-assigned classroom teacher–who may just be ahead of students by just a few pages in the content subject matters–may delegate the task of lecturing in the classroom by assigning his/her students to just watch some preselected-content-related high-quality video lectures freely available on YouTube and then use the normal classroom time period for group discussions, interactions, and/or participations.

8. Non-mathematical Metaphors, Analogies, Memes, Embodied-actions or Gestures

The appropriate use of both the mathematical-related and non-mathematical metaphors, analogies, memes, and/or embodied actions, motions, or gestures in the mathematics classroom [26][27][28][29][30], to facilitate and enhance students' understanding of the subject matter concepts, is now a welcomed teaching and learning technique or strategy as revealed and supported by everyday research suggested, research-based, or research-informed best teaching practices. Both mathematical and non-mathematical metaphors (even in the advanced mathematics classes) such as the sayings, statements, or slogans such as: *mathematics is a story*; *mathematics is work*; *mathematics is a journey;* and/or *mathematics is discovery* [26] are useful and meaningful teaching and learning tools, techniques, or strategies because they help facilitate learning by providing the much-missing contexts for enhancement of student learning experience. There is a common saying and sometimes overused claim that mathematics is everywhere, but that is just a saying because a real passionate–enthusiastic teacher would care enough to show or demonstrate where such real contexts exist and where mathematics concepts can be derived or extracted.

9. Using Contextualised–Authentic–Real World Mathematics Examples

Using authentic and contextualised real-world mathematics examples means examples that reflect real-life experiences, as in the following examples:

> *When a friend of the family asked Ray Charles how old he and his sister Joyce were, he answered in the following fashion: If I were 3/5 as old as I am and Joyce were only ¾ as old as she is, together we would be three years older than I am alone. But if I were only 2/5 as old as I am and Joyce were half as old as she is, then together we would be three years younger than I am alone. How old is each now?* [31]

Authentic real-world life examples like the use of the names of popular personalities like Ray Charles (a famous American classic singer), and/or any other similar relevant cultural legends convey feelings of empathy; as one student was quoted as saying after the launch of the above story-related word problem, “I still don’t know exactly how old Ray and his sister are, but I'm really worried about them” [31].

Another typical authentic real-life example is figuring out the probability of finding the compatible Mr. or Mrs. right at a certain school, college, or university, given some information background like the students’ population and characteristics such as attractiveness, wealth, grades averages, etc.; and as it turns out, the probability of finding the Mr. or Mrs. right dwindles exponentially as more and more variables, characteristics, or criteria are considered [32].

10. Playfulness in a Serious Mathematics Classroom Setting or Environment

The teacher's playful attitude in a supposedly serious classroom setting, or taking a serious look at the fun in the classroom [33][34][35][36][37][38][39][40][41] means the teaching methods or approaches that utilize and make use of funny, humorous, and interesting mathematics examples; the idea of utilizing (in a relaxed fashion or mind-set) everything in the classroom context or environment, which can include popular music, songs, and even a teacher singing in the classroom setting; playing with words, numbers, mathematics formulas, and/or equations; an appropriate use of interactive, manipulative mathematics materials, tools, or resources like, for example, waving arms around to teach quantum physics in a creative fashion [42]. All of these teaching and learning tools, plus more out there, should be an integral part of the daily components of fun, playfulness, resourcefulness, or the teacher's artistic creativity in the classroom setting or environment.

11. Methodology

Because a reliable, valid, and feasible survey with formal questionnaire items cannot be carried out in the kindergartens, the authors of this article instead paid several visits to South Sudanese pre-primary schools to chat and converse informally with pre-primary school pupils or kindergarteners. The authors planned to conduct some physical and conceptual demonstrations with some basic mathematics and physics concepts, and these conceptual demonstrations are to be an integral part of these conversations or chats with pre-primary school pupils. After the physical-conceptual demonstrations, the authors solicited through conversations the opinions, perspectives, or views of the kindergarteners or pre-primary school learners; pupils' views with regards to the use, integration, or implementation of various teaching techniques, strategies, or teaching tools such as the use of technology for teaching and learning in the pre-primary school settings or contexts. The authors solicited the pupils' opinions informally with non-serious chatting questions such as: do you own a computer, or have you ever heard about computers in the classroom setting; are you familiar with smartphones, laptops, other machines, or educational robotics for teaching and learning; would you like to see or use machines for learning, educational robotics, or learning with the aid of supplementary technological tools; and would any other learning techniques or strategies be used in your classroom setting; and why or why not?

Informal non-serious questions like these allow the authors to assess the learning preferences of the kindergarteners with regard to the use of teaching tools such as educational robotics or other related–supplementary learning tools or techniques in the classroom setting. Also, future invitations of the authors back to the primary school settings for more physical–conceptual demonstrations, conversations, or visitations would indicate that the authors' initial interactions with the kindergarteners were a success. This is in addition to the observed general mood of the primary school pupils during those initial interactions and observations. During the interactions and demonstrations, the authors expected to observe a general positive classroom mood and/or positive emotional expressions such as pupils' facial expressions (e.g., playfulness, smiling, or laughing) and/or any other signs of excitement, enjoyment, and/or satisfaction with the demonstrated mathematics concepts.

11.1. Primary School Visits

We visited several primary schools where we had a chance to chat and converse with primary school pupils in a classroom setting so that we could interact and perform physical and conceptual demonstrations with the natural number system. After exploring students' perceptions of natural numbers, we planned to do some physical demonstrations with basic essential mathematics concepts during our conversations and interactions with primary school pupils in the classroom settings. The purpose of the physical–conceptual demonstrations is to stimulate, motivate, and inspire interest in the mathematics subject matter. We planned how to create the concept of natural numbers from scratch [43], by first perceiving and defining natural numbers uniformly as an identical string of ones (or zeros) on a blank blackboard which initially represents zero or nothing on it (e.g., writing 11111111111…; or 000000000…), an ongoing sequence of ones (or zeros) which can go on infinitely beyond our view, a sequence of ones which can be made more readable by separating them with a comma in the right place (e.g., 1, 11, 111, 1111, 11111, and so on): we could have used a string of dots, tiny cubes, or circles, but a string of ones (or zeros) looks much better for a definition on the board so as to make the point. The separated increasing groups of ones (N), with the (N + 1) as the next successor, can be beautifully represented by the Roman numerals (e.g., I, II, III, IV, V, and so on). However, performing arithmetic operations with Roman numerals can be quite a task because of the absence of zero, and it is therefore not an efficient process: there is a saying (according to math history) that there was a time period when a person would earn his/her PhD just by managing to do some real arithmetic operations with Roman numerals; and now, in the present day, one can earn a PhD on the back or by the way of popular mechanics [13].

Therefore, we can quickly retire Roman numerals (perhaps useful only for organising) shortly after introducing them and replace them on the board with the more efficient Indo-Arabic numerals: 0, 1, 2, 3, …; and we used bundles of toothpicks, which we can pull out of our pockets like magic wands, as our visible–visual aids for the concept of natural numbers representation. With our toothpicks as our preferred physical models, we can physically perform and concretely demonstrate all the arithmetic operations, along with their associated properties of commutativity, associativity, and distributive properties for addition and multiplication operations.

Also pulled from our pockets (like magic wands) were pieces of candy which we used for the purpose of attention getting; and also to demonstrate how some fundamental-basic concepts of physics are subconsciously wired and deeply embedded into our brains. In this demonstration, we randomly threw some pieces of candy into the air for the pupils to catch or miss; and if or when they happened to miss, however, the pupils then would not own such a candy but would give it to the next pupils sitting next to them. This activity shows that the brain is always thinking ahead of time as it automatically or subconsciously calculates, correctly guesses, and predicts the deliberately computable quantities such as the candy's path, distance, speed, time, and the angle at which a candy was thrown or projected.

After the basic physical-conceptual demonstrations, we planned to solicit the kindergarten and primary school pupils' opinions, views, perspectives, or wishes about the potential use of newly emerging educational robotics as supplementary learning and teaching tools in classroom settings. However, having identified and introduced ourselves as university lecturers and researchers, we were then not allowed voluntary permission to enter the classrooms on each of the first several visits we made (during the first term of the school year 2022, from February to May) to at least four different primary schools, namely, Hai-Negil Nursery, Kindergarten & Primary School (JCC's Centre), Atala-Bara West & East Primary Schools, and Taqwa-Junior Academy & Primary School. In some of these primary schools, there were suggestions that we could be allowed entry if we could guarantee some kind of "kick-backs," a local terminology and the latest euphemism for bribery, and/or if we could present some form of written permission from either of the two ministries of education (both the general and higher or tertiary educations), ministries led by folks who appeared to hate the education system or development as they openly neglected it through their corrupt practices, yet they still run and preside over it. A permission we were reluctant to obtain for a couple of reasons. First, it would mean giving an honor in the form of a request and having to deal with the perceived classroom haters or strangers, folks who were perceived to care not at all about either the schools, students, or the larger population; and secondly, it would appear like coercion: we wished to establish a voluntary, cooperative, and collaborative working relationship with the primary school teachers as we did earlier with some secondary schools within and around Juba city. The suggestion that we can actually pay our way into the classroom settings (though it may be with only just a few South Sudanese valueless pounds) is an option which we declined because it appeared too unethical.

11.2. Conversations with Primary School Administrators

Although we were at first denied voluntary access to the primary school classroom settings, we were able to chat with some school administrators–whose names are withheld to preserve confidentiality–locally referred to as headmasters, mistresses, or head-teachers, who narrated to us some stories of severe economic negligence by the country's top political and military leaders, the policy makers who are widely perceived by the public to have badly mismanaged the country and its abundant natural resources. Blames such as finger-pointing, wild rumours, and gossiping (e.g., privately talking about the various forms of corruptions and scandals) are very popular topics to start any kind of conversation in South Sudan, as everybody finger-points and sings the corruption songs, even the most corrupt themselves. The narrated stories were mostly about the country's self–inflicted unnecessary civil war, the resulting economic disaster, and the self–inflicted oppression, all allegedly caused by the continuing–endless corruptions by the country's politico-military leaders since the country's independence from the Sudan about twelve or thirteen years ago.

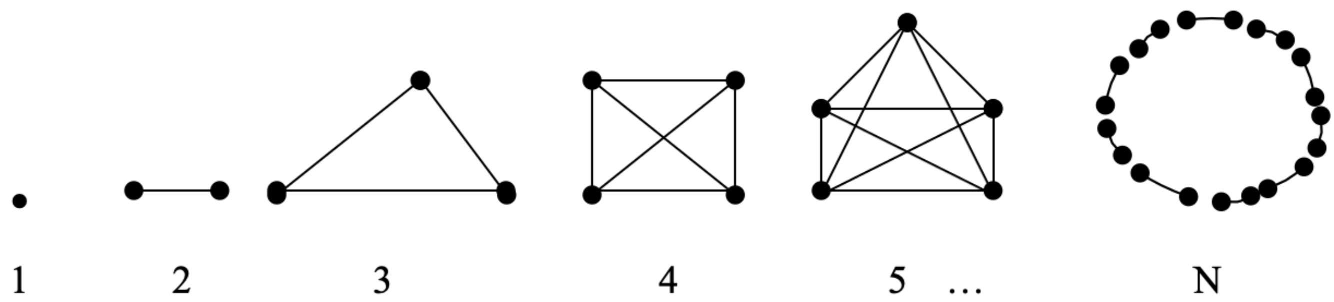
12. Resorting to Desk Work

After first coming short in securing the initial entries into the primary school classroom settings, we then resorted to desk work in order to continue exploring and reviewing the existing literature, where we sat down on several occasions to discuss, brainstorm, and then conceptualise a visible contextualising model (Picture 1 diagram), which elegantly sums up or mathematises our exploration and review of literature-based teaching and learning techniques. The research suggested teaching and learning strategies coming out (on an almost daily basis) from the research community as the research outputs. During our review of research-based teaching and learning techniques or strategies, we utilised a mixture of scholarly-based Internet search engines or tools such as ResearchGate, Google Scholar, and also some of the high-quality TED Talks and/or popular lectures available on YouTube. The rationale for using the popular TED Talks and YouTube, but high-quality lectures, is because of our overall goal of making the mathematics subject matter attractive to the larger public or population; this is in addition to our perspective or view of considering all the contextual features of the learning environment. This is because those apparently non-peer-reviewed sources available on YouTube are so popular with the general public, viewed by millions, and therefore there must be something that is quite right about them in comparison to the traditionally preferred sources, the so-called refereed or peer-reviewed resources, which almost nobody cares to read at all [1], and which also comprises the majority of our references.

In fact, it is those apparently unusual and unconventionally non-peer-reviewed sources[2][3][4][10][12][13] that initially inspired this review article in the first place, rather than the usually preferred traditionally peer-reviewed sources. We deliberately included these sources because they appeared to us as thought-provoking–food for thought popular literature. Hence, the elusive concept or delicate phenomenon such as interest generation, stimulation, and maintenance (in the classroom setting) is a multidimensional phenomenon, and achieving it successfully requires the inclusive, fair, and balanced use of all the available resources, all the contextual features of the learning environment, and not just a few selected–preferred sources or references.

12.1. Conceptualised Picture for Contextualising Mathematics Concepts

Picture 1 Diagram: A conceptualised picture, a structure, or a model for the natural numbers' representation: A model, in our perception, is a visible–visual mental aid or picture where mathematics concepts can be derived or extracted, as in the following picture, diagram, or structure.



The Picture 1 diagram (taken as a whole) is made up of a sequence of dots arranged in a unique, creative way which, when connected with an edge or a line, forms a sequence of N-sided regular polygons whose vertices (indicated by dots) can be represented by an increasing sequence of natural numbers (1, 2, 3,..., N), from which an arithmetic series can be derived, justified, or proved using mathematical induction in the context of secondary school mathematics. The first dot (labeled as Fig. 1) supposedly represents the first-ever-introduced teaching technique, strategy, or method of teaching since the beginning of teaching as a profession (perhaps, for example, the one-to-one tutoring); the next two dots or vertices (indexed as Fig. 2) represent two distinct teaching approaches which are connected with an edge or a line to indicate an existing relationship between the two approaches, a relationship supposedly introduced by a third researcher or so. The next three dots or vertices (Fig. 3), which form a connected graph, constitute a triangular method of teaching and learning with three interactive-dynamic components or dimensions, and it is a supposed improvement over the previous two methods that have only one or two components. Labeled as Fig. 4 is a teaching method that uses four interacting-dynamic components. Fig. 5 has five interactive-dynamic components or dimensions, and it can be referred to as a pentagonal approach for teaching and learning; and Fig. 6 (if drawn) would have six interactive-dynamic components or vertices …: the patterns of dots can continue as the vertices of N-sided regular polygons as the number of dots or vertices approaches infinity, as indicated by N number of dots or vertices on a circle (see Fig. N, the last figure in the Picture 1 diagram).

In conceptualizing the Picture 1 diagram, we conducted a thought experiment (a reflection in and/or on actions) where we perceived, envisioned, or visualized a presumed research inputs–outputs function box scenario; where the *outputs* (the number of dots or vertices on a regular polygon) were the published–suggested techniques or strategies, while the *inputs* were the presumed total research submissions, which are (in this case) a sum total of both the declined or rejected research work and the accepted–acceptable published final results, plus the associated preprints. Given our daily–common experiences with the nature of research work (although nobody really likes to admit failures, or maybe only a very tiny fraction of active researchers may acknowledge failures as reflected in their research profiles), the research *output* is obviously a countable tiny subset of the total research*input*, which in turn is a proper subset of the natural numbers system. Hence, the following *proposition*, *theorem*, or *claim* can be made, extracted, or derived from the Picture 1 diagram.

The number of dots allocated or placed on the vertices of the subsequence regular polygons in the Picture 1 diagram can be taken as a *countable non-empty subset of natural numbers which always has a smallest member*: this is a *well-ordering principle*, *proposition*, or *theorem* which can be justified and proved at the level of introductory real analysis or university-level set theory. Hence, a one-to-one correspondence, bijection, or cardinality with natural numbers can be established as N (the number of dots or vertices) approaches infinity: in this case, N dots or vertices represent the number of ongoing–suggested teaching methods, teaching techniques, or strategies coming out (on an almost daily basis) from various research outputs, contributions, or teaching and learning suggestions.

The number of dots (n vertices, which is a set) in each figure of the Picture 1 diagram (our contextualizing model) can mathematically be related to its corresponding number of edges E in the following mathematical fashion, rationale, or reasoning:

Definition: A graph G is called r-regular if the degree of every vertex, that is, the number of edges incident to each vertex, is r (meaning the same) for all vertices in G. A graph is complete if every two of its vertices are adjacent; and a complete graph of order n is denoted by K_n, and the number of vertices n is (n - 1)-regular. Therefore, K_n has the maximum possible size (the total number of edges) for a graph with n vertices. Observe that each component of Picture 1 diagram is a K_n complete graph and satisfies this definition.

Proposition: If G is r-regular and the number of vertices is n (as can be seen in each component of the Picture 1 diagram), then its corresponding number of edges $E = \frac{(n)(r)}{2}$.

Proof: By the degree-sum formula [22], provided earlier in section 6, the sum of the degrees of the vertices is always twice the number of edges. However, the degree of each vertex in each component of the Picture 1 diagram is always the same number (r). This implies (n)(r) = 2(E), and therefore $E = \frac{(n)(r)}{2}$, completing the proof.

Corollary: Since every two distinct vertices of K_n graph are joined by an edge, the number of pairs of vertices in K_n is "n choose 2," symbolically denoted $\binom{n}{2}$; and so the size of K_n, the total number of edges, is $\binom{n}{2} = \frac{n(n-1)}{2}$.

Up to the time we wrap up our exploration of the literature, a newly introduced or proposed teaching method, technique, or strategy, a classroom-tested practical method of teaching (separately labelled as Picture 2 diagram because of space constraints), can have at most seven (7) interactive-dynamic components or dimensions (e.g., *motivation*, *preparation*, *knowledge*, *skills set*, *context*, *outcomes*, and *reflection*), namely the revised-enhanced ICCM: Interpersonal Communication Competence Model for teaching and learning mathematics subject matter [24].

12.2. The Interpersonal Communication Competence Model (ICCM)

Picture 2 diagram: This picture, provided below, which would have been Fig. 7 in the Picture 1 diagram if it had been drawn there, is also an r-regular complete K_n graph because it satisfies all the properties given in the *definition*. Observe that this graph (Fig. 7) has seven (7) vertices, with each vertex having a degree of six (6), and its total number of edges (its existing internal dynamic–interactive relationships between these vertices or components) is 21; as can be confirmed using the formula provided by the *proposition* (or *theorem*), and also by the *corollary*, where, in particular, "n choose 2" becomes "7 choose 2" $= \frac{7(6)}{2} = 21$.

This Fig. 7, K_n graph, is a newly proposed dynamic–interactive ICCM model for teaching and learning mathematics in a classroom setting [44]; an instructional approach which claims and argues that an effective classroom teacher is always proactive, versatile, and performs in a dynamic-interactive fashion. That is to say, the teacher must always be passionate, skillful, and knowledgeable in the subject matter; always prepares, motivates, and inspires learners; visualizes, contextualizes, and assesses learning outcomes; and then reflects–on almost daily bases–on their teaching and learning practices, reflecting both during and after the lessons deliveries.

The Fig. 7 diagram, graph, or picture can be further mathematised or illustrated in the following definition.

Definition: A vertex is called even or odd according to whether its degree is even or odd. Hence, the following result is a consequence (or *corollary*) of the degree–sum formula given earlier in section 6.

Proposition (or *theorem*): *Every graph contains an even number of odd vertices*.

Proof (or *justification*): Let G be a graph. If G contains no odd vertices, as in Fig. 7 (the ICCM model), then the result follows immediately because all the vertices of Fig. 7 (which is also an r-regular graph) are even vertices, each having a degree of 6 ($r = 6$). Suppose, however, that G contains k odd vertices; we denote them by $v_1, v_2, \ldots, v_k$. If G contains even vertices as well (as in Fig. 7, an r-regular graph, a complete graph), we then denote them by $u_1, u_2, \ldots, u_n$.

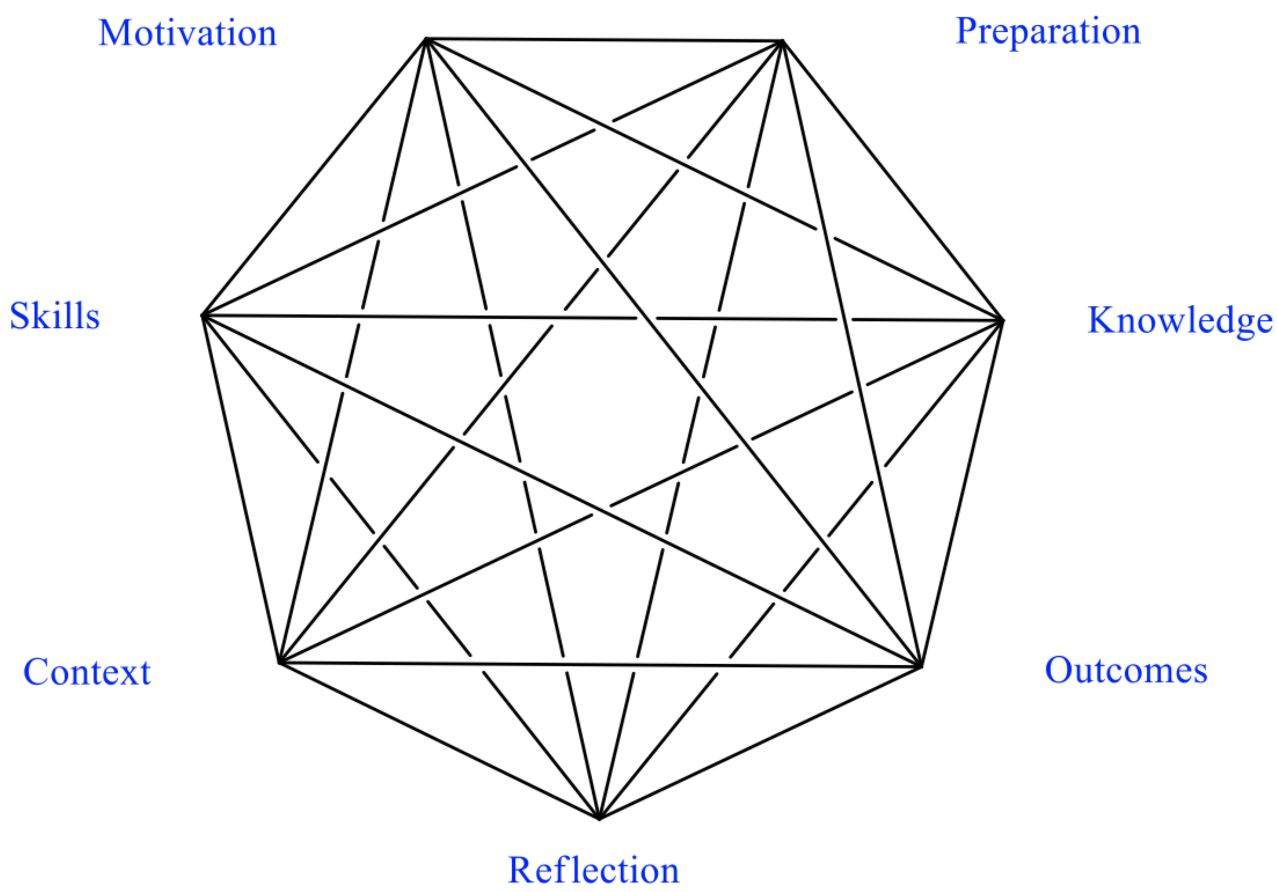
Using the degree–sum formula mentioned earlier in section 6, $(\deg v_1 + \deg v_2 + \ldots + \deg v_k) + (\deg u_1 + \deg u_2 + \ldots + \deg u_n) = 2q$,

where q is the number of edges in graph G.

Since each of the numbers $\deg u_1, \deg u_2, \ldots, \deg u_n$ is even, we then have $(\deg v_1 + \deg v_2 + \ldots + \deg v_k) = 2q - (\deg u_1 + \deg u_2 + \ldots + \deg u_n)$.

However, each of the numbers $\deg v_1, \deg v_2, \ldots + \deg v_k$ is odd. Therefore, k must be even, that is, G has an even number of odd vertices. If G has no even vertices, we then have $(\deg v_1 + \deg v_2 + \ldots + \deg v_k) = 2q$, from which we again conclude that k is even. Therefore, the claim or the *proposition* that *every graph contains an even number of odd vertices* is true.

An alternate proof of the above *proposition* or *theorem*, an elegant version which uses the natural language skills sets such as reasoning, interpretations or the interpersonal communication competence is as follows: *Suppose the sum of the degrees of the odd vertices is x and the sum of the degrees of the even vertices is y. Then the number y is obviously even, and the number x + y, being twice the number of edges (as provided by the degree-sum formula), is also even. So x is necessarily even. Now if there are p odd vertices, the even number x is the sum of p odd numbers, and therefore p must be an even number as well.*



After a deliberate–practical application of the Fig. 7 teaching and learning model, a model with 21 different–nonlinear–dynamic interactions (with the number of edges indicating the existing internal relationship between the components), either paralysis, saturation, procrastination, cognitive overload, or exhaustion may kick in or simply pop up; because applying a teaching method, technique, or strategy with more than seven (7) interactive–dynamic components is a heavy, daunting task, as the task of teaching naturally relapses back to a teaching approach with fewer and fewer interactive–dynamic components (dimensions). After a relapse, a cycle of improvement or growth may pop up from somewhere, usually after some insights emerge, especially after the reflection component or dimension is applied and/or utilized.

The Fig. 7 picture or diagram, a graph theory or a set theory concept, can easily be converted or related back to secondary school level geometric or trigonometric concepts by just asking some contextualized questions suggested by the picture itself. Related questions such as, for example, find a formula for the total area of Fig. 7; or show that the sum of the decrees of the exterior angles of the Fig. 7 picture (or any other regular polygon for that matter) is 360 decrees? A primary school pupil (in this context) can be challenged to first identify the total number of non-overlapping triangles in the picture, and then estimate the area of the picture using such distinctive triangles. This approach of asking context-suggested questions (to relate seemingly different–various concepts) is in line with the concept of teaching and learning mathematics as a conceptual and relational understanding [45][46], a horizontal bird's-eye view of the subject matter rather than the usual–traditional vertical view of the subject matter.

13. Some Reflective Observations, Conjectures, or Insights

Although our planned trips or visitations to primary school settings did not take place smoothly as initially hoped, some reflective observations, insights, or takeaways from this review article can be made. One of these takeaways is that Maslow's hierarchy of needs or the primary-essential basic needs such as food, shelter, and security appear to be the priority at the South Sudanese primary schools instead of secondary or tertiary needs such as a quality primary education system; as most of those primary schools were run by volunteer teachers rather than well-rounded, trained, and experienced primary school teachers, one of the most probable reasons for first denying us entry into some of those classroom settings, as those volunteers probably did not want us (as researchers) to witness the learning environments in those primary school settings. Another possible takeaway is that the desired teaching quality known as *creativity* in the classroom setting can really be any technique or strategy (either adapted or creatively crafted) that is perceived by the students to be amazing, amusing, and/or funny, that is, anything with the ability to grab students–learners' immediate attention, curiosity, or interest in the learning process.

We also observed that by creatively asking or posing some context-related or context-suggested conceptual questions, Picture 1 diagram can be used as a visible-visual aid, a rich context, or a physical model for extracting and deriving most of the essential, if not all, the fundamental-basic physical concepts of the mathematics subject matter: this is an observation similar to Euclid's insightful observation that all the geometry's concepts can be derived or extracted from a single axiom. This view of mathematics, perspective, or approach is in line with the concept of teaching and learning mathematics as a conceptual, relational, and contextual understanding [47]. For example, Fig. 2 (a separation or distance between any two points or vertices) in the diagram can be used as a visual-physical model for deriving the familiar distance formula, a half-way formula, and/or the mathematics most popular theorem–the Pythagorean Theorem, and also for solving most distance or average speed-related problems [44]; Fig. 3 can be used as a context for deriving a general formula for the area of a triangle whose sides are all known (namely the Heron's formula); and Fig. 4, when perceived as a piece of paper folded in half infinitely many times, and/or even better, a piece of cake divided equally among friends, can be used as a physical model for deriving geometric series and other related–infinite series; and Fig. 4 can also be used for deriving most of the single-variable functions, elementary or transcendental functions such as the exponential functions, as well as their inverses: the logarithmic functions and their associated various properties. The last figure in the diagram (Picture 1 diagram), Fig. N, which can be regarded as an inflated or a ballooned-up picture of Fig. 1 (the base case scenario), can be used to introduce the basic circular or trigonometric functions and also the fundamental–basic concepts of limits or calculus.

Furthermore, useful mathematical concepts such as the concept of a vector quantity and its various properties can also be introduced using the Picture 1 diagram as a visible–visual context or a model for teaching and learning; and even some fundamental concepts of graph theory (often a college or university level topic) are also suggested by the picture (Picture 1 diagram). As discussed in detail earlier in the related sections, each figure in the Picture 1 diagram satisfies all the properties of a complete K_n graph, that is, each vertex in each component of the Picture 1 diagram is adjacent to the next vertex or every other vertex; and also each vertex in each figure has the same degree (r) and is said to be an r-regular in graph theory language, where the degree (r) is the number of edges incident with a vertex. Therefore, each vertex (v) in each figure of the Picture 1 diagram (or Picture 2 diagram, for that matter) has (v-1) edges incident with such a vertex. Hence, the Picture 1 diagram (which also contains the Picture 2 diagram as its proper subset) suggests and provides a visible–visual mental aid; or a semi–concrete physical model, a rich context for deriving and extracting some of the most basic concepts of the mathematics subject matter.

Another food for thought is that mathematical concepts or mathematics itself as a subject matter, although it is indeed a valuable tool (e.g., its utilities as it appears powerful, useful, and timeless), is in fact a very narrowed, limited, and one-dimensional perspective as it appears or tends to be unaware of the existence of other useful tools or knowledge domains out there in the metric space; mathematics is a strict-linear perception imposed (or focused) on certain kinds of generic reality; as can be easily demonstrated on a blackboard or whiteboard by a simple definition of natural numbers as a continuous–uniformly–smooth string of identical objects or sequences, an ongoing–continuous–sequence whose relative size approaches infinity (or goes beyond our view); and yet somehow leaves most of the finite board blank, untouched, empty, or unused. Mathematics, like physics (its related cousin), deals well only with inanimate objects (e.g., points or particles in space) such as in the case of classical physics where things are dramatically and emotionlessly thrown off a cliff (or a very tall building), but it would never explain why, for example, a person may choose to end his/her dear life by jumping off a cliff or a building.

The last but not the least takeaway or observation is that contextualizing mathematics concepts for facilitating students' understanding is hardly less important than the abstraction or generalization (the preferred end-game of mathematics) as such visible–visual mathematics contexts have useful values as they provide the often perceived dry mathematics with some real-meaningful contexts. Such visible–visual contexts, in turn, suggest the posing of more appropriately related questions (extensions) because in the absence of more related questions or extensions, one ends up only with just a huge collection of answers or solutions without any actual problems being solved; similar to scratching a body's surface when or where it does not itch.

14. Wrap-up Comments, Remarks, or Conclusion

After more than a year, we were finally allowed some access during our second round of visits to some of the primary school classroom settings (during the second term of the 2023 school year, May to August). We were at last able to interact with primary school pupils and were also able to implement our long-planned physical and conceptual demonstrations in some of the primary schools mentioned in section 11. We were able to observe that the primary school pupils were so relaxed during our physical–conceptual demonstrations with the basic mathematics and/or physics concepts; and also the pupils' cooperating mathematics teachers told us that all their pupils would like us to visit them again in the future and even on a regular basis (something which we were eager and excited to hear). Thus, our overall primary purpose of facilitating, motivating, and inspiring students' learning appeared to have worked out just fine; also, as well, our secondary-aimed objective or goal of making mathematics attractive, that is, selling out the mathematics subject matter usually by generating, stimulating, and/or maintaining students' interest in mathematics through physical-conceptual demonstrations, appeared to have worked out as intended, as students were visibly excited during our classroom interactions. Besides, the primary school pupils appeared to be welcoming new ways of teaching and learning in their classroom settings; as none of them resisted our interventions (e.g., there were no signs of observable tensions, interruptions, or disruptions) and none of the pupils suggested any kinds of hindering of their learning process as a result of our interactions with them in their primary school classroom settings.

Because we appear to be just exploring and reviewing the existing literature, we may be perceived as not contributing any new knowledge, discovery, or invention. However, bringing to the attention or forefront a knowledge domain that may be otherwise overlooked, blurred, or overshadowed by the research community's daily surges or waves of research outputs (often referred to as new teaching techniques and/or strategies; the daily surges, sequential-like outputs, or series of explosions of various and diverse knowledge domains out there in the World Wide Web [17][48], daily waves usually driven mostly by individuals' personal validations, an almost minute-by-minute instant self-gratifications, a burning desire or pressure of having to publish something or anything in a given time) is hardly less than an important, valuable contribution; especially when viewed from the point of view of the classroom's daily teaching and learning practices rather than from the usual theories-driven observations or perspectives, given the existing-widening gap between daily classroom practices and the theories about these practices. This article can therefore be seen as a bridge, an edge, or a relating link between the classroom practices and the related theories.

We skimmed and scanned through vast–existing literature resources, paid careful attention and read those sources that grab our immediate attention and/or curiosity, those sources which we have access to as practicing classroom teachers. Other similar or different reviewers may consider on similar or different topics other alternative sources of literature for their reviews; and they may claim and argue either similarly or differently because the relative size of the existing literature is so dense and congested with a lot of useful and everyday literature suggested research–based teaching and learning techniques; and/or various–effective teaching strategies [49][50][51][52][53][54]. However, the explored, reviewed and recommended techniques or strategies we highlighted here can also be useful to the STEM education programs; for the overall purpose of making attractive (e.g., selling out) the mathematics subject matter to a larger students' population, as these recommended approaches or methods of teaching and learning would help increase the prospective number of students who may pick up the mathematics and its related subjects as their preferred majors to pursue further. Therefore, the highlighted and recommended approaches of teaching and learning can actually increase the relative size of students who enroll in the STEM related programs, disciplines and/or education.

As we claimed and argued earlier in the preceding sections, the classroom environment can be so dynamic, so chaotic, and so complex with so many interacting–moving parts or components; and it is still at the end of the day a classroom teacher's role to find a way to fit together, manage, and harmonize all those interactive–moving parts or dynamic components, often by making use of all the available contextual–features of the present-day classroom setting (e.g., the rapidly changing technological tools, resources, or techniques), as there is no single technique or strategy that always works; and by always embracing the concept of an open mind-set, continuous improvement, development, and/or growth; and also by utilizing contextually–relevant and culturally–responsive methods of teaching and learning in the classroom setting [49][50].

In our next–upcoming article or a sequel, we shall further justify the rest of our *claims* that might have been otherwise not yet well-argued, supported, or substantiated with convincing argument. For example, as a future direction, we can further substantiate most of the assertions, hypotheses, conjectures, or insights we observed in section 13 of this article; and we can do that by further mathematising or illustrating Picture 1, our claimed–contextualizing model for teaching and learning mathematics subject matter, from a suggestive visible–visual context. We can, for the benefits of daily mathematics classroom practices, demonstrate several more examples of how and why various fundamental–essential concepts of mathematics can be extracted, derived, or justified from a given–related picture as a visible–visual context: a daily practical approach we referred to as *contextualized mathematics* or *experimenting with mathematics concepts*, for a lack of a better description.

We would like to re-emphasize that contextualizing mathematics concepts has a learning value as it aids in students' understanding and comprehension; that is to say, visibly visualizing abstract mathematics concepts (similar to zooming into or near zero) is not less a valuable or important contribution or task than the usually preferred generalization of the mathematics concepts (which could mean zooming out to infinity unnecessarily fast). In fact, if anything, the two approaches (either zooming into or way out of view) are equivalent mathematics concepts; because given a general–abstract mathematics concept, one (say a mathematician) may easily provide the situations where it would immediately apply. However, it may not be an easy task to recognize and provide a related real–concrete and visible–visual context where such a general-abstract concept can be extracted, derived, or justified. If contextualization (or experimentation) of mathematics concepts can be so easily achieved, then there may be no need for a burning desire for too much heavy reliance on an elusive, ambiguous, and shaky set theory of vague indefinite-infinite domains and/or ranges [12]. The current messy–vague set theory as the foundation of mathematics, a messy–chaotic task which (apart from its obvious inexactness, inaccuracies, or imprecisions) only compounds the already existing–apparent conservation of the mathematical difficulties for all various and diverse learners of the mathematics subject matter and its related subjects, disciplines, or programs.

Just so that we hit the point home with this type of argument or point of view, it does not really matter whether one is pushing or taking a limit to zero or infinity as long as the algebraic expression (argument) in question is a legitimate–an appropriate limit problem when a limit point is plugged in, which results in the form such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$, an infinitesimal or indeterminate form, a kind of irrational behavior but which indicates the existence of a limit point or a bound. The same result is always obtained, and also the same amount of work is done or required as in the following limit problem, for example,

$\lim_{d\to 0}\left(\frac{2d}{\frac{d}{30}+\frac{d}{60}}\right) = \lim_{d\to\infty}\left(\frac{2d}{\frac{d}{30}+\frac{d}{60}}\right) = 40$, where d stands for the distance or separation between any two points or vertices as in Fig. 7 diagram: this kind of limit is a suitable solution to the distance or average speed–related problem mentioned earlier in the discussion of section 13. Therefore, visibly contextualizing mathematical concepts with a picture or diagram is an equally valuable concept as the generalization of mathematical concepts.

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