Review of: "New adaptative numerical algorithm for solving partial integro-differential equations"

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Potential competing interests: No potential competing interests to declare.

In this manuscript, the author attempted to propose a numerical method based on orthogonal Bernoulli polynomials (OBP) for solving parabolic partial integro-differential equations. The matrices of OBP were derived and utilized to obtain the approximate solution. With the proposed method, the given problem was transformed into a nonlinear algebraic system. On the one hand, he did some experimental tests to validate the good accuracy of the numerical algorithm. On the other hand, by comparing the proposed technique with some other methods, the advantage of this approach was highlighted.
However, overall, there are significant flaws with this article. Therefore, major revision has to be done.

Major comments:

$\bullet$ The article lacks innovative points. In fact, there should already be a lot of related work in place.

$\bullet$ There are too many typos in the entire text, which need to be carefully checked. Meanwhile, regarding the use of grammar, it is hoped that the author can carefully consider it repeatedly.

$\bullet$ In terms of scientific rigor, the description of space in the Section 4 is not clear and the expression is confusing. Meanwhile, the characterization of mathematical models is not well presented.

$\bullet$ When writing a paper, please pay attention to different language environments, as there are many places in the article where italicized and nonitalicized fonts are used interchangeably for certain symbols and equations. Even in some parts of this manuscript, () is used when referencing equations,

but there are others that are not. Please maintain consistency.

$\bullet$ Please cite some relatively laster articles and pay more attention to authoritative journals for references, such as:
https://doi.org/10.1007/s13540-023-00198-5,
https://doi.org/10.1016/j.cam.2023.115287,
https://doi.org/10.1007/s10092-023-00533-5,
https://doi.org/10.1007/s10444-023-10050-2.

$\bullet$ The effect of the figures and tables is not sufficiently highlighted. Please provide a reasonable, accurate, and detailed description in each example, combined with corresponding graphs and tables, so that readers can better understand the meaning paper want to express.

Minor comments:

$\bullet$ In page 2, the line above equation (3), the word ``oppeared" should be ``appeared". The same problem appears in the line above equation (22). Furthermore, it would be better to replace the word ``integral" with the ``integral term".

$\bullet$ In Page 3, the line above equation (5), what is the meaning of ``OBPs"? Does this word refer to ``orthogonal Bernoulli polynomial"? If that's the case, before abbreviations appear here, a prompt should be provided in the previous text.

$\bullet$ In page 3, equation (5), ``$i=0,1,\dots$" should be replaced by ``$i=0,1,\dots,N$". This type error in this manuscript needs to be thoroughly checked.

$\bullet$ In page 3, equation (6), the definition of $\Phi_{N,b}$ is given. However, all the symbols used in the following text are $\Phi_{b,N}$. Please maintain strict consistency.

$\bullet$ In page 3, Section 3, first line, remove the charecter ``t" between ``technique" and ``to".

$\bullet$ In page 4, the equation between (15) and (16), check and correct errors in punctuation usage. The same problem appears in the last equation in page 6 and the equation below (32).

$\bullet$ In page 6, top of the page, ``$k=0,1,2,\dots,r_{1}.$ are Gauss ...", what does using a period here mean?

$\bullet$ In page 6, the sentence above Section 4, ``Gaus'' should be replaced by ``Gauss''.

$\bullet$ In page 7, top of the page, ``$k=(k_{1},k_{2};\dots,k_{d})$'' should be replaced by ``$k=(k_{1},k_{2},\dots,k_{d})$''.

$\bullet$ In page 7, bottom of the page, ``The following theorems gives'' should be replaced by ``The following theorems give''. Furthermore, why didn't you give a period at the end of the sentence?

$\bullet$ In page 8, Proof 1, ``32 reduces to 29'', do 32 and 29 here refer to equations? During the process of writing the manuscript, please pay attention to accurate expression.

$\bullet$ In page 8, the line above equation (34), review and correct grammar error. The same problem appears at the bottom of page 9 and the fourth line in page 15.

$\bullet$ In page 11, the equations below (40), pay attention to maintaining consistent spacing.

$\bullet$ In page 11, the sentence above Example 4, remove the period before ``better accuracy than ...''.

$\bullet$ In page 13, Table 3, please come up with a suitable name for Table 3 to highlight its intended expression. Similar problems arise in other graphs and tables.

$\bullet$ In page 15, top of the page, ``Le given'' should be replaced by ``Consider''.

$\bullet$ In page 18, reference [5], ``MAathematics of Computation'' should be replaced by ``Mathematics of Computation''.