# Review of: "Non-Hermiticities even in quantum systems that are closed"

Alexander Lopez[1]

1 Escuela Superior Politécnica del Litoral

Potential competing interests: No potential competing interests to declare.

Dear editors of Qeios,

Thanks a lot for allowing me to review the manuscript entitled: “non-Hermiticities even in quantum systems that are closed”. I have carefully read the work and considered that this work could provide interesting physical insight to the community interested in topological systems. In particular, the author proposes to analyze both polarization and magnetization in systems which can be closed in multiply connected spaces, a case in non-Hermitian physics that is not usually discussed in the literature. Its discussion, contrasting continuous and discrete models, could indeed shed some light on the better understanding of quantum behavior at the most fundamental level. Yet, I would like the author to consider the following points:

1. The abstract is rather long and full of details that could be provided in the introduction, where there is enough room for setting up the motivation and goals of the paper. In my opinion, the potential readers would very much benefit for a succinct statement of the main results of the paper, in such a way to motivate these readers to further go through the manuscript in the main body of the paper.
2. In the first paragraph after the abstract there is not reference whatsoever to former works. Although the author mentions briefly that some previous works have dealt with paradoxes related to the Erhenfest and Hellmann-Feymann theorems, the authors states that: “Apart from this novelty, we remind the reader that the above-mentioned paradoxes had been noted in applications of the Ehrenfest theorem and Hellmann-Feynman theorems, with some related but separate discussions on the quantum mechanical uncertainty relations. These few works were totally disconnected to each other and the whole issue has been largely ignored, until recently – when a new analysis of the matter seems to lead to interesting possibilities.” I consider the proper citations of the related works should be included for the benefit of the potential authors. This lack of references should also be addressed in the next paragraph, where the author states that: “and a particularly interesting follow up that includes an additional electric field) for a pedagogical analysis”.
3. I might have not completely gotten the authors point in the connection between the Ehrenfest theorem and non-hermiticity of a quantum mechanical operator. From Sturm-Liouville theory, for a proper definition of any operator, its domain and boundary conditions are essential. For example, and without loss of generality, when considering a one dimensional system, by stating that the velocity operator for this system is just v=i[x,H]/\hbar, would be not enough from a formal point of view, as the boundary conditions determine the proper domain of validity and the actual self-adjoint domain of the operator considered. Indeed, the proper self-adjoint condition for unbounded operators, such as

kinetic energy, relies on the proper definition of its domain, see for instance references Methods of Modern Mathematical Physics-Functional Analysis Vol. 1, Michael Reed and Barry Simon, 1980 as well as Fundamental Properties of Hamiltonian Operators of Schrodinger Type, Tosio Kato, Transactions of the American Mathematical Society, 1951, Pg. 195-211. I would kindly ask the author to discuss a bit further on this point, if possible.

4. There seem to be typos in the last terms of equations (4) and (5) as the author write “\Omega*H-H+\Omega”, and I guess it should read “\Omega*H-H*\Omega”. The dimensionality of the different terms of the equation do not seem to match, provided \Omega and H are non dimensionless. Could the author please check this?
5. When discussing the “Bohr-type” nature of the Aharanov-Bohm effect, I am not that sure if is fair to assume that this phenomenon is not topological. Could the author please further discuss why he/she considers this Aharonov-Bomh effect to be non-topological?
6. I am not sure of whether the sentence “Note that this sharpness of $\gamma_0$ is absolutely necessary, in order for the sharpness (or precise quantization) of the Hall conductance (as shown by the Laughlin argument)” is clear. It seems to be lacking a complement. Could the author please check this issue?
7. I would kindly ask the author to review the phrasing of the sentence “Hence it seems that the emergent non-Hermitian term (although its initial origin, the boundary, has disappeared due to the folding) is an absolute must to be considered, in order for the IQHE to be explained. [And it is plausible to wonder whether this need (of consideration of the emergent non-Hermiticity) might be generalizable to all topological effects.]” as the wording turns out to be not so clear.
8. I would suggest the author to consider some standard literature on physical realizations/proposals for topological effects such as:

M. V. Berry (1984). "Quantal Phase Factors Accompanying Adiabatic Changes". *Proceedings of the Royal Society A*. **392** (1802): 45–57

S. Pancharatnam (1956). "Generalized Theory of Interference, and Its Applications. Part I. Coherent Pencils". *Proc. Indian Acad. Sci. A*. **44** (5): 247–262

Y. Aharonov and D. Bohm (1959). "Significance of electromagnetic potentials in quantum theory". *Physical Review*. **115** (3): 485–491

M. Z. Hasan and C. L. Kane, (2010-11-08). "Colloquium: Topological insulators". *Reviews of Modern Physics*. **82** (4): 3045–3067,

C. L. Kane and E. J. Mele, "Quantum Spin Hall Effect in Graphene". *Physical Review Letters*. **95** (22): 226801 (2005).

C. L. Kane and E. J. Mele, "$Z_2$ Topological Order and the Quantum Spin Hall Effect". *Physical Review Letters*. **95** (14): 146802, 2005-11-23.

B. A. Bernevig, T. L. Hughes, and S. C. Zhang, “Quantum spin Hall effect and topological phase transition in HgTe quantum wells”, Science 314 (5806), 1757-1761, 2006

B. A. Bernevig and S. C. Zhang, “Quantum spin Hall effect”, Physical Review Letters 96 (10), 106802, 2006