Commentary

Einstein's 1911 Approach to Gravitational Redshift: From Nothing to Too Much

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In 1911, Einstein revisited his earlier application of the equivalence principle to gravitational phenomena, deriving simple expressions for gravitational redshift, time dilation, and light deflection. In this paper, I critically examine the logical structure of Einstein's 1911 derivation of the gravitational redshift (and time dilation) and argue that it contains an internal inconsistency that gives rise to an infinite regress paradox. Specifically, the derivation implies that the presence of a finite gravitational redshift necessitates its doubling–and so on *ad infinitum*–when the result is reintroduced into the same reasoning. I reconstruct Einstein's argument step by step, highlight the source of the paradox, and discuss its implications for the logical coherence of the heuristic method underlying his early gravitational reasoning. Although general relativity later provided a rigorous mathematical formulation for gravitational redshift and time dilation, I contend that this paradox should not be dismissed as historically irrelevant. Instead, it exposes conceptual tensions at the foundations of Einstein's heuristic approach, raises questions about the unrestricted application of the equivalence principle to electromagnetic phenomena, and offers insight into the philosophical underpinnings of general relativity.

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1. Introduction

In 1907, Einstein introduced the equivalence principle [1], proposing that a system at rest in a homogeneous gravitational field is locally indistinguishable from a uniformly accelerated system. Using this principle, he sought to extend the framework of special relativity to include gravitational phenomena. In that early work, Einstein was able to derive, for the first time, several key gravitational

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effects: the gravitational redshift, gravitational time dilation, and the influence of gravity on light propagation, such as the variation in the light velocity and the deflection of light rays.

Einstein himself later expressed dissatisfaction with this preliminary attempt to unify gravity and relativity[1]. He therefore revisited the problem in 1911 [2], presenting a much simpler derivation of gravitational redshift, time dilation, and light deflection, again based on the same equivalence principle.

In this paper, I argue that Einstein's 1911 derivation contains a subtle logical inconsistency that leads to a paradox: the very existence of a certain amount of gravitational redshift appears to imply the necessity of twice that amount, and so on *ad infinitum*.

Section 2 reconstructs Einstein's 1911 derivation of gravitational redshift and the associated time dilation. Section 3 exposes the paradoxical structure underlying the argument, which results in an infinite regress. Finally, the concluding section considers the significance of this paradox within the broader context of Einstein's fully developed theory of general relativity.

2. Einstein's 1911 gravitational redshift derivation

Consider two material systems, S_1 and S_2, at rest in a local, uniform gravitational field $\mathbf{a}$ (Fig. 1). S_1 and S_2 are separated by a distance *d*. Consider further a reference frame K_0. System K_0 is a free-falling (gravitation-free) system located near S_2 with an initial instantaneous velocity relative to S_2 equal to zero.

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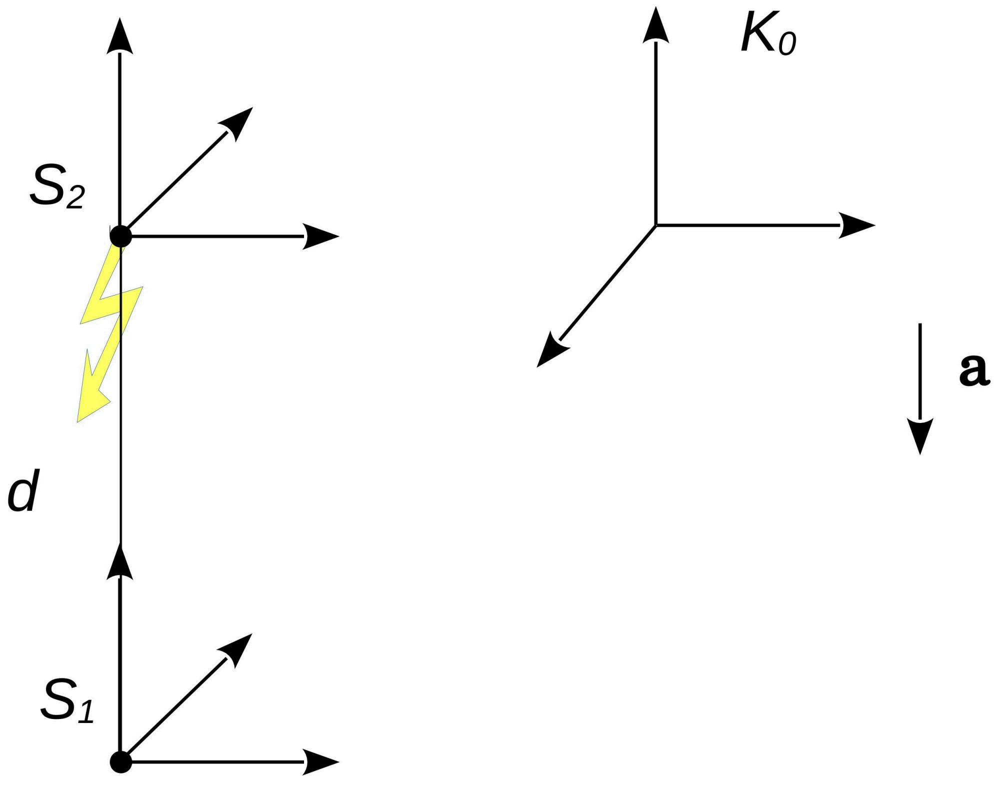

Figure 1. Material systems S_1 and S_2 are at rest in a local, uniform gravitational field $\mathbf{a}$. The reference frame K_0 is a free-falling (gravitation-free) system located near S_2 with zero initial velocity relative to S_2. According to the equivalence principle, this is equivalent to systems S_1 and S_2 accelerating upward with acceleration $-\mathbf{a}$, while the frame K_0 remains inertial and at rest.

Suppose further that a ray of light of frequency ν_2 is emitted by S_2 towards S_1 when the relative velocity of the free-falling frame K_0 with respect to S_2 and S_1 is still equal to zero. The ray of light reaches S_1 after a time nearly equal to d/c. According to the principle of equivalence, this situation is physically equivalent to one in which K_0 is at rest, and S_2 and S_1 accelerate with acceleration $-\mathbf{a}$ and initial velocity equal to zero. When the ray of light arrives at S_1, the velocity of S_1 relative to the stationary frame K_0 is equal to $v = ad/c$. Therefore, *from the point of view of any observer in frame* K_0, the ray of light received at S_1 has a frequency ν_1 as follows

$$\nu_1 = \nu_2 \left(1 + \frac{v}{c}\right) = \nu_2 \left(1 + \frac{ad}{c^2}\right), \tag{1}$$

where the second term is the first-order Doppler formula valid for $v \ll c$.

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For ad, Einstein substituted the gravitational potential Φ of S_2, taking that of S_1 as zero, and assumed that the relation (1), deduced for a homogeneous gravitational field, would also hold for other forms of field. Then, Einstein arrived at the well-known (approximate) formula for the gravitational redshift (in this example, it is actually a blueshift)

$$\nu_1 = \nu_2 \left(1 + \frac{\Phi}{c^2}\right). \tag{2}$$

From this result, Einstein also derived the formula for gravitational time dilation. In Section 3 of [2], he raises the following question:

> "On superficial consideration equation (2) or (2a) [*our equations (1) and (2)*], respectively, seems to assert an absurdity. If there is constant transmission of light from S_2 to S_1 [*when both systems are stationary in a gravitational field*], how can any other number of periods per second arrive at S_1 than is emitted from S_2?"

Einstein's reply is, in his own words, "simple". Suppose that, during the time interval Δt_2 (as measured by a clock at rest at S_2), S_2 emits n waves. Then, from the definition of frequency, we have $n = \nu_2 \Delta t_2$. Let S_1 receive these same n waves during the time interval Δt_1 (as measured by a clock at rest at S_1). Then, again, according to the definition of frequency, we have $n = \nu_1 \Delta t_1 = \nu_2 \Delta t_2$. Hence, equation (2) leads to the gravitational time dilation formula

$$\Delta t_2 = \Delta t_1 \left(1 + \frac{\Phi}{c^2}\right). \tag{3}$$

Therefore, from gravitational redshift, Einstein derives the gravitational time dilation.

3. The infinite regress paradox

At first glance, Einstein's 1911 derivation appears flawless–direct, elegant, and seemingly inevitable. Yet, upon closer examination, it conceals what may be called a "doubling-to-infinity" paradox.

In Section 3 of [2], considering the purely kinematical accelerated version of the S_1-S_2 system, the stationary observer K_0 does not directly observe any frequency shift but infers that the accelerated observer S_1 should perceive one due to the Doppler effect associated with S_1's motion relative to the radiation emitted instantaneously by S_2. In the corresponding case where S_2 and S_1 are stationary within a gravitational field, the freely falling observer K_0 (and Einstein himself) asks how a frequency

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shift could occur in radiation exchanged between two relatively stationary observers. The resolution of this question leads to the discovery of gravitational time dilation (equation 3).

A point deserves emphasis here: in the purely kinematical accelerated setup, the frequency shift inferred by K_0 is nothing more than the ordinary first-order Doppler effect associated with the motion of S_1 during the light propagation time. The essential point is that the first-order Doppler shift for light in a moving system–an effect of order $O(v/c)$–is not, by itself, caused by any change in the rate of time in the observer's frame. Any special relativistic time dilation associated with the acquired velocity $v = ad/c$ would contribute only at order $O(1/c^4)$, whereas the Doppler shift discussed here is of order $O(1/c^2)$. In fact, the observability of the frequency shift exactly according to the Doppler formula in the kinematical setup requires that the observer's (S_1's) time rate be the same as that of the system K_0, according to which S_1 must observe that precise frequency shift.

At this stage, nothing prevents us from repeating Einstein's derivation, now incorporating the newly obtained result of gravitational time dilation alongside special relativity and the equivalence principle. If the equivalence principle is taken symmetrically, the conclusion reached in the gravitational case must also apply to the equivalent accelerated case from which the original Doppler shift was inferred.

In this modified reasoning, the stationary observer K_0 again infers that S_1 experiences a frequency shift due to the Doppler effect associated with the motion of S_1 relative to the radiation emitted by S_2. However, K_0 now also recognizes that time in S_1 runs more slowly than in S_2 by the amount given in equation (3). Consequently, S_1 would appear to experience *two* distinct, but equal-in-magnitude, contributions to the frequency shift: one due to the Doppler effect inferred by K_0 (the very same Doppler shift that enabled the original derivation of the gravitational redshift and time dilation), and another arising from the fact that the time in S_1 runs more slowly by the same factor given in equation (3). The total result would therefore amount to a *doubling* of the original redshift obtained by Einstein.

It is worth noting that most contemporary authors (see, for instance, [3], [4], [5], [6]) hold that the only acceptable explanation of gravitational redshift is in the modification of the rate of a clock exposed to a gravitational potential, and that the energy–and therefore the frequency–of a photon is in fact conserved as it propagates in a static gravitational field[2].

The doubling argument described above is already puzzling, but in principle it can be iterated indefinitely. Returning to Einstein's reasoning, one could again ask how a different number of oscillations per second–now *twice* that of Einstein's original derivation–might be observed at S_1 compared with S_2, in the corresponding case where S_2 and S_1 are both are stationary within a gravitational field. By the same logic, one would answer that time at S_1 runs slower (now *twice* as slow) than at S_2, and thus the process could be repeated without bound. The result is an infinite regress–a paradoxical amplification of the redshift *ad infinitum*.

4. Discussion and conclusions

The paradox outlined in the previous section undermines the logical validity of Einstein's 1911 derivation of gravitational redshift and gravitational time dilation. It cannot be regarded as a consistent or self-contained argument. Nevertheless, the existence of the complete theory of general relativity–within which both effects can be rigorously and elegantly derived–has led most scholars to retrospectively downplay the significance of Einstein's early heuristic derivations (dating from 1907 and 1911).

Today, most relativists seem to embrace Wittgenstein's famous motto: "He must, so to speak, throw away the ladder after he has climbed up it." The ladder represents Einstein's preliminary heuristic constructions, while the heights attained symbolize the discovery of the already existing "precious stone" of the fully developed general theory of relativity.

However, we regard this widespread attitude as more dangerous than simply sweeping the dust under the carpet. The mature theory of general relativity did not emerge *ex nihilo*, particularly with respect to its physical consequences. It is, rather, a mathematical framework that incorporates and formalizes results that Einstein had previously reached through heuristic reasoning and thought experiments[3]. The 'physical content' of general relativity comes not from the sophisticated mathematics that make the theory so intimidating, but from the original heuristic thought experiments that inspired it.

In our view, a more fitting analogy replaces Wittgenstein's ladder with an architectural one: the completed theory of relativity is a magnificent high-rise built upon tentative and uncertain foundations–Einstein's heuristic derivations. Such foundations cannot simply be discarded, for no high-rise, however elegant or structurally refined, can stand without them.

Therefore, the paradox presented in this paper deserves to be taken seriously when analyzing and reassessing the philosophical foundations of general relativity–just as another paradox, discussed by the present author, has shown that the gravitational redshift is incompatible with the (locally applied) principle of energy conservation [7].

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Can we still live comfortably at the top of our luxurious high-rise, knowing the precarious nature of its foundations?

The above-mentioned “doubling-to-infinity” paradox may have a broader implication for fundamental physics. Einstein’s 1911 derivation relies on an unrestricted application of the equivalence principle to all physical phenomena, including electromagnetic ones. By applying the Doppler formula to the accelerated version of the relatively stationary S_1-S_2 system, Einstein infers a blueshift. He then transfers this result, via the equivalence principle, to the case in which S_1 and S_2 are stationary in a gravitational field.

As discussed in Section 3, this reasoning does not ultimately resolve the apparent “absurdity” identified by Einstein (see the end of Section 2). Instead, it generates the “doubling-to-infinity” paradox, suggesting that some step in the inferential chain may be illegitimate.

Since the Doppler analysis in the accelerated frame appears straightforward, one possible conclusion is that Einstein’s unrestricted extension of the equivalence principle to electromagnetic phenomena deserves closer scrutiny. It may be that the equivalence principle, while robust for purely mechanical processes, cannot be applied without qualification to electromagnetic ones.

In this regard, it is worth noting that even the renowned relativist Wolfgang Rindler, while discussing the puzzle of an electric charge at rest on the surface of the Earth—which, according to the equivalence principle, should radiate but, according to energy conservation, should not—writes:

> “Much has been written on these paradoxes, but the proper solution seems to have been first recognized by Ehlers: *it is necessary to restrict the class of experiments covered by the equivalence principle to those that are isolated from bodies or fields outside the cabin*. In the case of the charges discussed above, their electric field extends beyond the cabin and is, in fact, ‘anchored’ outside; since radiation is a property of that whole field, it follows that these ‘experiments’ lie outside the scope of the equivalence principle.” [8]

Further doubts about the general validity of the equivalence principle are also raised in [9] in connection with the compatibility between gravitational redshift and energy conservation.

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Data Availability

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Footnotes

[1] About Einstein's 1907 derivation, Earman and Glymour remarked that "the treatment itself, however, was uncharacteristically cumbersome, principally because of the circuitous explanations of the concepts involved. Despite the explanations, some of the meanings remained obscure" [3]. Their assessment is consistent with Einstein's own later dissatisfaction with this early attempt to extend special relativity to gravitation.

[2] In that modern interpretation, however, a further problem arises. If the energy of a photon truly remains constant as it propagates through a static gravitational field, then the very notion of gravitational redshift becomes problematic. According to the Planck–Einstein relation, $E = h\nu$, the photon's energy E is directly proportional to its frequency ν, with Planck's constant h being invariant everywhere. If both E and h retain the same values at emission and reception, then, by definition, the *measured* frequency must also remain unchanged. The receiver would therefore *detect* a frequency $\nu = \frac{E}{h}$, identical to that measured by the emitter. Gravitational time dilation at the receiver's location cannot alter the universality of physical laws such as the Planck–Einstein relation ($\nu = \frac{E}{h}$) when it is combined with the assumption of constant photon energy throughout the field. Under these conditions, no gravitational redshift would be observable.

[3] Earman and Glymour [3] summarize this continuity particularly well: "In 1916 Einstein presented the first derivation of the red shift from his newly completed general theory. Though couched in terms of the

formalism of the new theory, the derivation actually relied on the same ideas as the 1907 and 1911 heuristic derivations, especially the idea that the rate of clocks is affected by the gravitational field."

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