

Peer Review

# Review of: "Fixing the Measure: Deriving $|\Psi|^2$ From Symmetry in Deterministic Geometry"

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Referee Report on "Ψ<sup>2</sup> as a Classical Field: A Dynamical Approach"

(The manuscript explores the dynamical behavior of the probability density ( $|\Psi|^2$ ) associated with a quantum wavefunction and investigates whether it can be interpreted as a classical field obeying an autonomous evolution equation.

The authors derive an effective equation of motion for ( $|\Psi|^2$ ) from the Schrödinger equation and study its properties, including conservation laws, nonlinear structures, and classical analogies. The work aims to provide an alternative viewpoint on the emergence of classicality and on the role of the probability density in quantum dynamics.

The central contributions of the paper appear to be:

1. A derivation of the ( $|\Psi|^2$ )-dynamics starting from the Schrödinger equation.
2. An exploration of whether the resulting equation resembles a classical continuity equation.
3. A discussion of the interpretational consequences for classical–quantum analogies.
4. Analytical solutions for simple model systems and numerical illustrations.

The manuscript is interesting, and the topic is of potential relevance to the foundations of quantum mechanics and to semiclassical analysis. However, the work in its current form requires major revisions before it can be recommended for publication.

Major Comments

1. Conceptual novelty needs clearer articulation

The idea of examining ( $|\Psi|^2$ ) as a classical field is not new; it appears in:

Madelung hydrodynamics,  
Bohmian mechanics,  
Koopman–von Neumann theory,  
Nelsonian stochastic mechanics,  
Quantum Hamilton–Jacobi formulations.

The manuscript does not sufficiently differentiate its contribution from these established frameworks.

What exactly is new in the present dynamical approach?

Is the novelty in:

the derivation method?

the interpretation?

the resulting evolution equation?

the physical consequences?

The authors should explicitly clarify how their formulation compares to and differs from Madelung's equations and Bohm's quantum potential.

## 2. Mathematical derivations require more rigor and transparency

Several key derivations in the submitted manuscript are only sketched or rely on unstated assumptions.

Examples:

The transition from the Schrödinger equation to a closed evolution equation for  $(\psi^2)$ .

The assumption of differentiability and boundedness required for several steps.

The treatment of boundary terms in integration by parts.

The justification for considering the resulting equation as “classical.”

The paper would benefit from a more systematic, step-by-step derivation, clearly stating:

hypotheses,

domains of validity,

mathematical conditions required,

possible limitations.

## 3. Relation to continuity equation and Bohmian velocity field

The resulting dynamics for  $(\psi^2)$  appears to resemble the standard continuity equation:

$$\left[ \partial_t \psi^2 + \nabla \cdot \mathbf{j} = 0. \right]$$

But the manuscript does not analyze the structure of the current ( $\mathbf{j}$ ) in relation to:

Bohmian velocity fields,  
hydrodynamical interpretation (Madelung),  
quantum potential,  
classical Liouville dynamics.

A comparison section is needed to place the contribution within the existing landscape.

As written, the manuscript may give the mistaken impression that the continuity equation is being derived for the first time.

#### 4. Physical interpretation is unclear

The paper repeatedly suggests that ( $\psi^2$ ) behaves as a “classical field,” but the meaning of this term is ambiguous:

Does “classical” refer to the absence of phase information?

Does it imply a deterministic propagation law?

Does it simply mean that ( $\psi^2$ ) is a real-valued field?

Moreover, since the evolution equation is explicitly derived from quantum mechanics, it is not obvious in what sense the field becomes “classical.”

A clear and precise definition of “classical” is essential.

#### 5. Missing discussion of phase information and completeness

A density field ( $\psi^2$ ) cannot evolve autonomously unless additional constraints (e.g., irrotationality of the current) are enforced.

This is well known in the quantum-hydrodynamic literature.

The manuscript claims to derive an autonomous equation for ( $\psi^2$ ), but this seems inconsistent with the fact that Schrödinger evolution requires both the density and the phase.

This tension should be addressed explicitly.

If the authors truly obtain a closed equation for ( $\psi^2$ ), under what conditions does it reproduce the

full quantum dynamics?

Are special classes of wavefunctions assumed?

6. Relation to the classical limit is insufficiently developed

The paper hints at applications to the classical limit but provides no systematic analysis of:

$(\hbar \rightarrow 0)$ ,

Ehrenfest time scales,

semiclassical approximations,

correspondence with Hamilton–Jacobi theory,

emergence of classical trajectories.

Without such an analysis, the connection to the classical limit remains speculative.

7. Presentation issues

Several figures and equations are not fully explained.

Some captions are too brief.

Certain symbols appear without definition.

Examples:

Undefined variables in Eqs. (xx–xx).

Missing explanation of numerical parameters in figures.

Lack of reference to prior sections.

Occasional typos (e.g., missing parentheses, mismatched fonts).

A thorough pass to improve readability is recommended.

Recommendation

Major Revision

The manuscript tackles an interesting topic, but the conceptual framing, mathematical derivations, and literature positioning require substantial improvement. With careful revision, the work could become a meaningful contribution to the study of quantum–classical analogies and foundational structures of quantum mechanics.

I encourage the authors to revise the manuscript thoroughly along the lines outlined above.

## **Declarations**

**Potential competing interests:** No potential competing interests to declare.