

Peer Review

Review of: "Inverse Evolution Data Augmentation for Neural PDE Solvers"

Jerome Pousin¹

1. Camille Jordan Institute UMR CNRS 5220, Institut National des Sciences Appliquées de Lyon, France

Dear Editor,

Please find hereafter my comments concerning the review of the paper "Inverse Evolution Data Augmentation for Neural PDE Solvers" authored by Chaoyu Liu¹, Chris Budd², Carola-Bibiane Schönlieb.

The subject of the article concerns the approximation of solutions of evolutionary equations with neural networks by interpolating the evolutive operator at particular time points. More specifically, the possibilities of generating data in an appropriate way are investigated. The subject is of interest particularly for evolution PDEs modeling physical phenomena.

First of all, I remark that the sentence 'By substituting Eq (5) into (4) we derive Eq(6)' on page 6 is not correct. The relation between the direct evolution equation (ascending time evolution) and the inverse evolution (descending time evolution) on pages 5-6 is misunderstood.

The relation between the direct and inverse solutions can be described with the following change of coordinates:

$$(t, x) \rightarrow (\tau, x)$$

with $t = T - \tau$. By setting $u(\tau, x) = v(T - \tau)$ we have:

d

$$dtv(t, x) = F(v(t, x)); (1)$$

$$v(0,x) = g(x). \quad (2)$$

And then for u

d

$$d_\tau u(\tau,x) = -F(u(\tau,x)); \quad (3)$$

$$u(T,x) = g(x). \quad (4)$$

By using Euler's schemes for the previous equations does not lead to relation (5) on page 6.

I would like to end my review with some remarks concerning the differential operator F . If it is a 2nd order self-adjoint operator (as the Laplacian, for example), then it is known that its spectrum is constituted of eigenvalues going to $1+\infty$ and that the eigenfunctions are a Hilbert basis of appropriate functional spaces ($L^2(\Omega)$ and $H^1(\Omega)$). Thus, solutions have decreasing toward zero properties when $t \rightarrow \infty$. This could be used for the next paragraph 'Initialization for Inverse Evolution'

The present version of the article cannot be considered for publication. I suggest that the authors prepare a corrected version of the paper that I could continue to review.

Sincerely Yours,

Jerome Pousin

Attachments: available at <https://doi.org/10.32388/4SW02Z>

Declarations

Potential competing interests: No potential competing interests to declare.