Qeios

Review on the paper 'Circuits, Currents, Kirchhoff, and Maxwell' by Dr. Eisenberg.

As one can understand from reading this work, the main purpose of the author is to establish new term in classical electrodynamics, the total current.

The author states that a role of the total current in the modern version of electrodynamics is downgraded. For example, the author writes
Despite this history, almost everyone today thinks of current as the flow **J** of charge (with mass), just as the currents in our plumbing and in the ocean are the flow of mass.

But the author makes some statements which are at least incorrect.
1. The mathematical way of saying SOMETHING is conserved is

$$\mathrm{div}(\mathbf{SOMETHING}) = 0$$

It is not so. The law of the total charge conservations states that

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$$

and $\mathrm{div}\boldsymbol{j} \neq 0$ does not mean that the charge does not conserve.

The condition of the absence of the longitudinal EM waves is $\boldsymbol{\nabla} \cdot \mathbf{E} = 0$ but the radiated EM waves leave the system so they are not conserved.

2. Physics textbooks (e.g., [18, 28]) show how the displacement current arises from special relativity as a property of space.

This is also not the case. The displacement current is a consequence of the total charge conservation as discovered by Maxwell.

As Feynman writes
https://www.feynmanlectures.caltech.edu/II_18.html
Maxwell began by considering these known laws and expressing them as differential equations, as we have done here. (Although the ∇ notation was not yet invented, it is mainly due to Maxwell that the importance of the combinations of derivatives, which we today call the curl and the divergence, first became apparent.) He then noticed that there was something strange about Eq. (18.1).

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\epsilon_0 c^2}$$

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If one takes the divergence of this equation, the left-hand side will be zero, because the divergence of a curl is always zero. So this equation requires that the divergence of $\boldsymbol{j}$ also be zero. But if the divergence of $\boldsymbol{j}$ is zero, then the total flux of current out of any closed surface is also zero.

Maxwell appreciated this difficulty and proposed that it could be avoided by adding the term $\partial \boldsymbol{E}/\partial t$ to the right-hand side of Eq. (18.1)

$$c^2 \boldsymbol{\nabla} \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\epsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}.$$

Thus, one can conclude that derivation of Eq. (5) by the author

$$\boldsymbol{\nabla} \cdot \boldsymbol{J}_{total} = \boldsymbol{\nabla} \cdot [\boldsymbol{\nabla} \times \mathbf{B}] = 0$$

is known procedure originally used by Maxwell. So it is unclear why does the author re-write the procedure of Maxwell but treats it as an introducing new physical quantitiy.

3. Despite the author introduces new term, the total current, he does not give any example of application of this term to some electrodynamical system. I mean it is not clear why the use of this term in one of the Maxwell equations is better than convenient use of $\frac{\boldsymbol{j}}{\epsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}$.

Therefore, I cannot estimate a content of this work as valuable.