Review of: "An alternative foundation of quantum theory"

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The mathematical foundations of Quantum Mechanics is still one of the most important topics in Mathematical Physics. Topos models of reconstructions of physical theories developed our knowledge about the logical foundations of Quantum Mechanics. The focus of this research is to formulate a new alternative axiomatic setting for Quantum Mechanics on the basis of group theory, group representation theory and category theory where the structure of operators attached to accessible physical variables are considered. The interconnections between physical variables, mathematical variables and theoretical variables are considered from the perspective of group theory where the author discusses how the ordinary quantum formalism can be derived from his new collection of postulates. Having a one-to-one correspondence between a suitable group G and a space Ω_{θ} of variables generated by some accessible functions is the core of this research work. The group G is acting transitively on Ω_{θ} such that the group action is proper and therefore a left-invariant measure does exist on Ω_{θ}.

This is an interesting research effort, however, it seems to me there are rigorous mathematical gaps in this paper which should be considered.

1- According to the text, variables take values only at interactions such that these values are only related to the other system (such as an observer) affected by the interaction. A variable is called accessible, if it takes a definite value relative to an observer such that other theoretical variables generated by the observer's mind are called inaccessible. Inaccessible variables are purely mathematical variables which do not take any values and in this work they are addressed as notions.

Entering some philosophical point of view in several parts of this research made some difficulty in the forward of the construction of a consistent mathematical formalism. For example, using the phrase "the mind of the observer" is not helpful for a mathematical purpose. It causes several questions such as (i) does any observer should have mind? (ii) What is the mind of a non-human detector? (iii) Is it a consciousness mind or a subconsciousness mind? Is there any difference between the (in)accessibility of variables in consciousness mind and subconsciousness mind? This kind of questions could raise naturally from the perspective of observers of this paper as "accessible variables". Therefore my first advise is to revise the text in a mathematical way to present explicitly the original purpose.

2- The paper does not address the fundamental role of the theory of computation in recognizing (in)accessible variables. Real numbers are classified into two separate types namely, computable numbers and non-computable numbers. A real number a is called computable if there exists a finite terminating algorithm which could approximate a in any arbitrary small neighborhood. Computable functions are applied to determine computable numbers. Rational numbers and algebraic numbers are examples of computable numbers. However, it is shown that almost every real number is non-

computable. Now if we follow the language of this paper, then serious questions are raised. For example, what is the relation between accessible variables and computable numbers? Is there any accessible variable which is not computable? It seems to me an understandable way is to interpret inaccessibility of variables as non-computable variables. The other challenge is about algebraic numbers. For example, is $\sqrt{2}$, as a computable number, accessible? Because its decimal presentation has infinite non-repeating terms which is not clearly a “definite value”. In other words, only integer and rational numbers could be applied for labeling accessible variables which is not enough to build Quantum Mechanics. So it is observed that the concept of "accessible variable" is not well-defined in this text.

3- Regarding Comments 1,2, all five postulates should be rewritten to clarify the relation between accessible variables and their computability situation? In other words, in which system of numbers these postulates are given? In particular, Postulate 3 is problematic. Because it could be only valid for computable numbers. There is NO non-computable real value such that all computable real values can be seen as functions of it.

4- The paper does not address the logical foundation of this new axiomatic setting. The main question is, in which topos or Heyting algebra we can evaluate propositions derived from these postulates?