Research Article

Quantum Coherence Between Mass Eigenstates of a Neutrino Can Be Destroyed by Its Mass-Momentum Entanglement

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If a neutrino or antineutrino produced in the decay of an unstable particle is not entangled to its accompanying particles, its mass is necessarily correlated with its momentum. In this manuscript, I illustrate that this entanglement would destroy the quantum coherence between the neutrino's mass eigenstates in both the momentum and position representations, which was overlooked by other authors in previous investigations of entanglement and coherence associated with neutrino oscillations. I further point out that the states of a neutrino and an electron become nonseparable after their charged-current interaction. This nonseparability leads to decoherence for neutrinos propagating in matter, but was not taken into consideration in previous investigations of the matter effect.

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To interpret flavor transformations of neutrinos, each of the three flavor eigenstates is assumed to be formed by a coherent superposition of three mass eigenstates[1][2][3]. Due to the time-evolving phase differences among the probability amplitudes associated with these eigenstates, an initial flavor eigenstate will evolve into a superposition of all the three flavor eigenstates, whose populations oscillate with time. In a recent manuscript[4], I proved that the electron antineutrino produced from the β decay of a neutron cannot be in a coherent superposition of different mass eigenstates, as a consequence of their correlations with different joint momentum states of the antineutrino and the accompanying particles, i.e., the electron and proton. This result somehow contradicts with the conclusions drawn in other papers[5][6][7], where the decoherence caused by the entanglement between the neutrino and the accompanying particles was also correctly recognized, but oscillations were still predicted. As detailed below, this inconsistency is due to the fact that the entanglement between the mass and momentum of the neutrino can also destroy the coherence among its mass eigenstates, which was not taken into consideration in these investigations.

The state of the entire proton-electron-antineutrino system, produced by the β decay, can be written as

$$|\psi\rangle = \sum_j \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} d^3\mathbf{P}_{p,j} d^3\mathbf{P}_{e,j} F(\mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j}) \left|\bar{\nu}_j\right\rangle \left|\mathbf{P}_{\nu,j}, \mathbf{P}_{n,j}, \mathbf{P}_{e,j}\right\rangle, \quad (1)$$

where $|\mathbf{P}_{\nu,j}\rangle$, $|\mathbf{P}_{p,j}\rangle$, and $|\mathbf{P}_{e,j}\rangle$ respectively denote the momentum eigenstates of the antineutrino, proton, and electron, and σ_j denotes the distribution region of the joint antineutrino-proton-electron momentum associated with the antineutrino mass eigenstate $|\bar{\nu}_j\rangle$. As proved in Ref. [4], there is no overlapping between the momentum distribution regions associated with different antineutrino mass eigenstates, that is, $\sigma_j \cap \sigma_k = \emptyset$ for $j \neq k$. With a suitable momentum distribution of the neutron undergoing the β decay, the antineutrino can be disentangled with the accompanying particles. In this case, the antineutrino's mass is necessarily correlated with its momentum, described by the entangled state

$$|\varphi_\nu\rangle = \sum_j \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} G(\mathbf{P}_{\nu,j}) \, |\bar{\nu}_j\rangle \, |\mathbf{P}_{\nu,j}\rangle \,, \tag{2}$$

where $\mathbf{P}_{\nu,j} \neq \mathbf{P}_{\nu,k}$ for $j \neq k$. When the antineutrino's momentum degree of freedom is traced out, the mass degree of freedom is in a classical mixture, described by the density operator

$$\rho_\nu = \sum_j D_j |\bar{\nu}_j\rangle\langle\bar{\nu}_j|, \tag{3}$$

where $D_j = \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} |G(\mathbf{P}_{\nu,j})|^2$. This implies that the entanglement with the momentum destroys the coherence among the mass eigenstates, prohibiting occurrence of flavor oscillations, which can be interpreted in terms of complementarity[8][9][10][11][12][13][14][15]. The information about the mass eigenstate of the antineutrino is encoded in its momentum. The fact that which eigenstate the antineutrino is in can be determined by measuring its momentum in principle is sufficient to destroy the coherence among the mass eigenstates. This decoherence has been overlooked in previous investigations of entanglement and coherence in neutrino oscillations by other authors[5][6][7].

This decoherence can also be illustrated in the position representation, in which the evolution of the wave function is given by

$$|\varphi_\nu(t)\rangle = \sum_j \int d^3\mathbf{r} f_j(\mathbf{r},t) |\bar{\nu}_j\rangle |\mathbf{r}\rangle, \tag{4}$$

where

$$f_j(\mathbf{r},t) = (2\pi)^{-3/2} \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} G(\mathbf{P}_{\nu,j}) e^{i(\mathbf{P}_{\nu,j}\cdot\mathbf{r} - E_j t)}, \tag{5}$$

$E_j = \sqrt{p_j^2 + m_j^2}$ with m_j being the mass of the jth mass eigenstate, and $|\mathbf{r}\rangle$ denotes the position eigenstate. The coherence between $|\bar{\nu}_j\rangle$ and $|\bar{\nu}_k\rangle$ manifested on the detection of the antineutrino is given by

$$\begin{aligned} C_{j,k} &= \int_D d^3\mathbf{r} f_j^*(\mathbf{r},t) f_k(\mathbf{r},t) \\ &= (2\pi)^{-3} \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} \int_{\sigma_k} d^3\mathbf{P}_{\nu,k} G^*(\mathbf{P}_{\nu,j}) G(\mathbf{P}_{\nu,k}) \int_D d^3\mathbf{r} e^{i[(\mathbf{P}_{\nu,k} - \mathbf{P}_{\nu,j})\cdot\mathbf{r} - (E_k - E_j)t]}. \end{aligned} \tag{6}$$

where D is the detection region of the antineutrino. When the size of the detector is much larger than that of the antineutrino's wavepacket, $\int_D d^3\mathbf{r} e^{i(\mathbf{P}_{\nu,j} - \mathbf{P}_{\nu,k})\cdot\mathbf{r}}$ can be well approximated by taking the integral over the whole space. As $\mathbf{P}_{\nu,j} \neq \mathbf{P}_{\nu,k}$, such an integral is zero, which implies that the coherence $C_{j,k}$ vanishes. This result can also be understood in terms of the position-dependent phase difference between $\left|\bar{\nu}_j\right\rangle$ and $\left|\bar{\nu}_k\right\rangle$ owing to the associated momentum difference. We note that such phase differences were also included in previous investigations, exemplified by the statement *"Since $p_{x,i} \neq p_{x,j}$, phase differences exist between the components at the point of detection"* in Ref. [6]. However, the statement "*As a result, the interference effects of neutrino oscillations arise solely from the different momenta in the components in the final state"*

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is incorrect. Actually, the interference effects of mass eigenstates (internal degree of freedom) would be averaged out when integrating the position (external degree of freedom) over the large volume of the detector, as the position-averaged value of the phase factor caused by the corresponding momentum difference is zero.

Therefore, if the state of the electron antineutrino produced from the β decay of a neutron consists of three mass eigenstates, the mass degree of freedom of the antineutrino is necessarily entangled with the momentum degrees of freedom of the accompanying particles or/and that of the antineutrino itself. This conclusion holds for the neutrino or antineutrino produced in the weak charged-current decay of other unstable particles, including mesons and muons. It is also applicable to solar ^{8}B neutrinos, which are produced by the reaction [3]

$$^8\mathrm{B} \rightarrow {}^8\ \mathrm{Be} + \mathrm{e}^+ + \nu_e. \tag{7}$$

I further note that even if solar ^{8}B neutrinos can be initially in a superposition of mass eigenstates, they cannot adiabatically evolve into a pure mass eigenstate, where the population of the electron flavor eigenstate was assumed to be about 1/3 [3], as will be detailedly interpreted below.

Previously, the flavor transformation of solar ^{8}B neutrinos was attributed to the matter effect proposed by Mikheyev and Smirnov by extending the idea of Wolfenstein, referred to as the MSW effect [16][17][18]. This effect originates from the charged-current (CC) interaction between the electron neutrino and the background electrons in matter, which can be described by the effective Hamiltonian,

$$H_{cc} = \frac{G_F}{\sqrt{2}} \nu_e^+ \gamma_4 \gamma_\lambda \left(1+\gamma_5\right) \mathbf{e}\mathbf{e}^\dagger \gamma_4 \gamma_\lambda \left(1+\gamma_5\right) \nu_e, \tag{8}$$

where ν_e and $\mathbf{e}$ denote the fields associated with the electron neutrino and electron, respectively. Using the Fierz transformation, the Hamiltonian was rewritten in the form of

$$H'_{cc} = \frac{G_F}{\sqrt{2}} \nu_e^+ \gamma_4 \gamma_\lambda \left(1+\gamma_5\right) \nu_e \mathbf{e}^\dagger \gamma_4 \gamma_\lambda \left(1+\gamma_5\right) \mathbf{e}. \tag{9}$$

Then the electron field was considered as a static background, whose state is not affected by the CC interaction, so that $\mathbf{e}^\dagger \gamma_4 \gamma_\lambda \left(1+\gamma_5\right) \mathbf{e}$ can be replaced with $\delta_{\lambda,4} N_e$, where N_e is the number density of electrons. With this treatment, the Hamiltonian H'_{cc} is effectively equivalent to an external potential for the neutrino, given by $V = \sqrt{2} N_e G_F$.

The matter effect was interpreted in terms of the postulation that each neutrino flavor eigenstate is formed by a linear superposition of three mass eigenstates

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U_{\alpha j} |\nu_j\rangle\,, \tag{10}$$

where j labels the mass eigenstate, and $\alpha = e, \mu, \tau$ denotes the flavor of the neutrino. $|\nu_e\rangle$ was supposed to be approximated by a superposition of $|\nu_1\rangle$ and $|\nu_2\rangle$, i.e., $U_{e3} \simeq 0$ [3]. Under this assumption, neither H'_{cc} nor the free Hamiltonian can couple $|\nu_e\rangle$ to $|\nu_3\rangle$, and thus the population of $|\nu_3\rangle$ can be neglected for the initial state $|\nu_e\rangle$. Then the dynamics can be described in a two-dimensional subspace $\{|\nu_e\rangle\,, |\nu_\beta\rangle\}$, where

$$|\nu_\beta\rangle = \mathcal{N}_\beta (U_{\tau 3} |\nu_\mu\rangle - U_{\mu 3} |\nu_\tau\rangle), \tag{11}$$

with $\mathcal{N}_\beta = \left(|U_{\tau 3}|^2 + |U_{\mu 3}|^2\right)^{-1/2}$. Within this subspace, the Hamiltonian can be approximately expressed as

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$$H \simeq V\left|\nu_e\right\rangle\left\langle\nu_e\right| + \sum_{\eta,\xi=e,\beta} M_{\eta,\xi}(p)\left|\nu_\eta\right\rangle\left\langle\nu_\xi\right|, \tag{12}$$

where

$$M_{\eta,\xi}(p) \simeq \sum_{j=1} \frac{m_j^2}{2p} U_{\eta j}^* U_{\xi j}, \tag{13}$$

with $U_{\beta j} = \mathcal{N}_\beta(U_{\tau 3}U_{\mu j} - U_{\mu 3}U_{\tau j})$. Here p denotes the neutrino momentum, which is much larger than the mass (m_j) associated with each mass eigenstate. The trivial common energy, described by $p\sum_j |\nu_j\rangle\langle\nu_j|$, has been discarded.

When $V \gg m_j^2/2p$, the electron flavor approximately coincides with the eigenstate of the Hamiltonian with the larger eigenenergy. If the electron number density is changed sufficiently slowly, the neutrino adiabatically follows the corresponding Hamiltonian eigenstate during its propagation. On the solar surface, V can be neglected as compared to $m_j^2/2p$ so that the eigenstates of the Hamiltonian coincide with the mass eigenstates. This implies that the initial electron flavor eigenstate evolves to the mass eigenstate with the larger mass ($|\nu_2\rangle$) when the neutrino reaches the solar surface. This mass eigenstate remains invariant until being detected on the Earth. The resulting probability $P_{|\nu_e\rangle\to|\nu_e\rangle}$ is approximately equal to $\left|U_{2e}^\dagger\right|^2$, which was assumed to be about $1/3$ [3].

This treatment has overlooked the crucial fact that the CC reaction leads to neutrino-electron entanglement when the neutrino is in a superposition of the electron flavor eigenstate and the other two flavor eigenstates before the reaction[19]. As the neutrino and the electron can be transformed into each other by their CC reaction, it helps to make the presentation more clear to refer the original neutrino and the original electron to as particle 1 and particle 2, respectively. The CC reaction transforms the state $|\nu_e, \mathbf{p}_1\rangle_1 |e, \mathbf{p}_2\rangle_2$ into $|e, \mathbf{p}_3\rangle_1 |\nu_e, \mathbf{p}_4\rangle_2$, where the subscripts "1" and "2" outside the kets label the two particles, and $\mathbf{p}_1$ ($\mathbf{p}_3$) and $\mathbf{p}_2$ ($\mathbf{p}_4$) are their momenta before (after) the reaction. If particle 1 is initially in the flavor eigenstate $|\nu_e\rangle$ and $\mathbf{p}_1 = \mathbf{p}_4$, the Fierz rearranging is equivalent to relabelling the two particles, which does not cause any problem. However, when it is initially in a superposition of $|\nu_e\rangle$ and $|\nu_\beta\rangle$, it will be entangled with particle 2 by the CC reaction. To illustrate this point, we suppose that the two-particle system is initially in the state

$$|\psi_0\rangle = \left(C_e|\nu_e, \mathbf{p}_1\rangle_1 + C_\beta|\nu_\beta, \mathbf{p}_1\rangle_1\right)|e, \mathbf{p}_2\rangle_2. \tag{14}$$

In this case, the CC reaction actually corresponds to a conditional dynamics, by which particle 1 exchanges its state with particle 2 when it is initially in the electron flavor eigenstate, but nothing occurs if it is initially in the other two flavor eigenstates. This conditional state swapping evolves the system to the entangled state

$$|\psi\rangle = C_e|e, \mathbf{p}_3\rangle_1|\nu_e, \mathbf{p}_4\rangle_2 + C_\beta|\nu_\beta, \mathbf{p}_1\rangle_1|e, \mathbf{p}_2\rangle_2. \tag{15}$$

It should be noted that the electron transformed from the neutrino does not have the same momentum as the original electron, i.e., $\mathbf{p}_3 \neq \mathbf{p}_2$. Such momentum differences have been used to identify neutrino-electron scattering events in SNO experiments[3]. This quantum entanglement is masked by the Fierz rearranging and the subsequent replacement of the electron part in the Hamiltonian with a number. We further note that the Fierz rearranging is valid for calculation of the e-ν_e scattering amplitude, which is irrelevant to the quantum coherence between $|\nu_e\rangle$ and $|\nu_\beta\rangle$. However, it overlooks the fact that $|\nu_e\rangle$ and $|\nu_\beta\rangle$ are carried by different particles after the CC reaction, which is essential for correct description of the neutrino state evolution in matter. In other words, the states of the two particles are no longer separable after their CC interaction, so that the electrons participating in such interactions cannot be treated as a static background for the neutrino, and their effects cannot be modeled as a potential, which cannot reflect the effect of quantum entanglement produced by the conditional dynamics.

Due to the quantum entanglement, each of the two particles is essentially in a mixture of the neutrino and electron states. This critical point can be illustrated more clearly by the reduced density operators for these particles, each obtained by tracing out the degree of freedom of the other particle, given by

$$\begin{aligned}\rho_1 &= \mathrm{Tr}_2\left(|\psi\rangle\langle\psi|\right)\\ &= |C_e|^2 |e,\mathbf{p}_3\rangle_1\langle e,\mathbf{p}_3| + |C_\beta|^2 |\nu_\beta,\mathbf{p}_1\rangle_1\langle\nu_\beta|,\\ \rho_2 &= \mathrm{Tr}_1\left(|\psi\rangle\langle\psi|\right)\\ &= |C_e|^2 |\nu_e,\mathbf{p}_4\rangle_2\langle\nu_e| + |C_\beta|^2 |e,\mathbf{p}_2\rangle_2\langle e,\mathbf{p}_2|.\end{aligned} \tag{16}$$

Under the subsequent free Hamiltonian dynamics, ρ_1 and ρ_2 evolve as

$$\begin{aligned}\rho_1' &= |C_e|^2 |e,\mathbf{p}_3\rangle_1\langle e,\mathbf{p}_3| + |C_\beta|^2 |\varphi_1,,\mathbf{p}_1\rangle_1\langle\varphi_1,,\mathbf{p}_1|,\\ \rho_2' &= |C_e|^2 |\varphi_2,\mathbf{p}_4\rangle_2\langle\varphi_2,\mathbf{p}_4| + |C_\beta|^2 |e,\mathbf{p}_2\rangle_2\langle e,\mathbf{p}_2|,\end{aligned} \tag{17}$$

where

$$\begin{aligned}|\varphi_1\rangle_1 &= u_{p_1}|\nu_\beta\rangle_1 + v_{p_1}|\nu_e\rangle_1,\\ |\varphi_2\rangle_2 &= u_{p_4}^*|\nu_e\rangle - v_{p_4}^*|\nu_\beta\rangle.\end{aligned} \tag{18}$$

u and v depend on time as

$$\begin{aligned}u_p &= \cos(\lambda_p t) - i\frac{\Delta_p}{\sqrt{\lambda_p^2+\Delta_p^2}}\sin(\lambda_p t),\\ v_p &= \frac{-i\lambda_p}{\sqrt{\lambda_p^2+\Delta_p^2}}e^{i\theta_p}\sin(\lambda_p t),\end{aligned} \tag{19}$$

where $\Delta_p = [M_{e,e}(p) - M_{\beta,\beta}(p)]/2$, $\lambda_p = |M_{e,\beta}(p)|$, and $\theta_p = \arg[M_{e,\beta}(p)]$. After this free evolution, the total $|\nu_e\rangle$-state population is

$$\begin{aligned}P_{|\nu_e\rangle} &= \int d^3\mathbf{p}\left({}_1\langle\nu_e,\mathbf{p}|\rho_1'|\nu_e,\mathbf{p}\rangle_1 +_2 \langle\nu_e,\mathbf{p}|\rho_2'|\nu_e,\mathbf{p}\rangle_2\right)\\ &= |C_e u_{p_4}|^2 + |C_\beta v_{p_1}|^2.\end{aligned} \tag{20}$$

Such a probability does not present the cross terms proportional to $C_e^* C_\beta$ and $C_e C_\beta^*$. This is due to the fact that the state components $|\varphi_1\rangle_1$ and $|\varphi_2\rangle_2$ have different momenta and are carried by different particles, so that quantum interference cannot occur.

If one only concerns about the neutrino part in the two-particle system, its behavior just after the CC reaction can be effectively described by the classically mixed state

$$\rho_\nu = |C_e|^2 |\nu_e,\mathbf{p}_4\rangle\langle\nu_e,\mathbf{p}_4| + |C_\beta|^2 |\nu_\beta,\mathbf{p}_1\rangle\langle\nu_\beta,\mathbf{p}_1|. \tag{21}$$

However, it should be born in mind that the two mixed state components are essentially carried by two different particles. The validity of this description can be illustrated by examining the subsequent free Hamiltonian dynamics, which evolves ρ_ν to

$$\rho_\nu' = |C_e|^2 |\varphi_2,\mathbf{p}_4\rangle\langle\varphi_2,\mathbf{p}_4| + |C_\beta|^2 |\varphi_1,\mathbf{p}_1\rangle\langle\varphi_1,\mathbf{p}_1|. \tag{22}$$

The resulting neutrino's $|\nu_e\rangle$-state probability is the same as Eq. (13). This equivalence further confirms that the CC reaction indeed destroys the quantum coherence between $|\nu_e\rangle$ and $|\nu_\beta\rangle$. When a second CC reaction occurs, these two particles will be further entangled with a third particle. Under the competition between the coherent coupling and CC-reaction-induced decoherence, the population of $|\nu_e\rangle$ is progressively decreased while that of $|\nu_\beta\rangle$ is increased until reaching the steady state

$$(|\nu_e\rangle\langle\nu_e| + |\nu_\beta\rangle\langle\nu_\beta|)/2. \tag{23}$$

For simplicity, we here have discarded the momentum degrees of freedom. For this mixed state, the gain of the $|\nu_e\rangle$-state population originating from the $|\nu_\beta\rangle \rightarrow |\nu_e\rangle$ transition cancels out the loss due to the $|\nu_e\rangle \rightleftarrows |\nu_\beta\rangle$ transition. Therefore, the probability $P_{|\nu_e\rangle\rightarrow|\nu_e\rangle}$ should not be smaller than 1/2, which is inconsistent with the solar ^{8}B neutrino experiments[3][20].

The observed deficit of solar ^{8}B electron neutrinos can be well explained in terms of a new mechanism, where the neutrino can oscillate among different flavors by interacting with the Z bosonic field[19]. If the Z bosonic field is allowed to connect different neutrino flavors, its virtual excitation induces the coherent couplings among these flavors, given by the Hamiltonian

$$\mathcal{H} = \sum_{\alpha,\beta} \xi_{\alpha\beta}|\nu_\alpha\rangle\langle\nu_\beta|, \tag{24}$$

where $\alpha, \beta = e, \mu, \tau$. On the other hand, the decoherence induced by the CC reaction can be modeled as the incoherent transformation

$$\rho \rightarrow S_e \rho S_e + S_{\mu+\nu}\rho S_{\mu+\nu}, \tag{25}$$

where $S_e = |\nu_e\rangle\langle\nu_e|$, $S_{\mu+\nu} = |\nu_\mu\rangle\langle\nu_\mu| + |\nu_\tau\rangle\langle\nu_\tau|$, and ρ denotes the density operator of the neutrino before the CC reaction. Under the competition between the coherent coupling and decoherence effect, the neutrino has a unique steady state, given by

$$\rho_s = \frac{1}{3}\sum_\alpha |\nu_\alpha\rangle\langle\nu_\alpha|. \tag{26}$$

This steady state corresponds to the equal classical mixture of the three flavors, which is not affected both by the coherent coupling and by the CC-reaction-induced decoherence. This steady state is in well agreement with the solar ^{8}B neutrino experiments[3][20].

In summary, there necessarily exist mass-momentum entanglement for the neutrino produced by the decay of an unstable particle, if the neutrino's flavor eigenstates are inconsistent with mass eigenstates and the neutrino is not entangled with the accompanying particles. Due to this entanglement, the neutrino is essentially in a classical mixture of these mass eigenstates when all the momentum freedom degrees are traced out. Consequently, the neutrino cannot has a definite flavor at the production, and the population of each flavor eigenstate cannot be changed during the propagation. For solar ^{8}B neutrinos, even if they can be initially in a superposition of different mass eigenstates, they cannot adiabatically evolve into a pure mass eigenstate, whose electron flavor population was assumed to be about 1/3. These results imply that none of the observed flavor transformations of neutrinos or antineutrinos can be consistently interpreted in terms of the inconsistency between the flavor and mass eigenstates. The experimental results can be well explained within the framework where the coherent flavor coupling is mediated by virtual excitation of the Z bosonic field, and the CC interaction is modeled as a dephasing effect[19].

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Declarations

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