Review of: "On the Mass of (Gravitational) Potential Energy"

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- This paper introduces an intriguing question that may be asked by many students when introduced to special relativity. If energy is proportional to mass, E=mc^2, what and where is the mass associated with the potential energy? I think this question is frequently ignored, and a paper trying to clarify it is welcome, though there are some classical attempts to address the problem. Nevertheless, I do not recommend this paper as it has several problematic issues. Here, attention is centered on gravitational potential energy. Einstein knew that Newton's theory of gravitation, an instantaneous interaction at a distance, was incompatible with the theory of relativity and spent a decade working on a relativistic theory of gravitation, thus producing the general theory of relativity.
- One difficulty with the classical gravitational potential energy of a particle within a gravitational field is that it depends on a reference, and only potential energy differences can have meaning. That problem is somewhat hidden in this paper when it is stated on p.2 that potential energy actually refers to the change of potential energy, that is, to work; nevertheless, this change depends on an arbitrary choice of reference level, of a starting height for the case of a uniform field discussed in detail. That the inertia of a particle depends on an arbitrarily defined reference level is unacceptable. This issue is not considered or solved in the paper.
- In part 2 of the paper, it is argued that the gravitational mass cannot be distributed in space, as a very small change in the height of a particle could take place in a very small time and only correspond to changes within a very small sphere centered on the particle. While true, the argument ignores that a very small change in height corresponds to a very small change in potential energy; not all of the energy mgh has to come back to the particle when its height decreases by a small amount. More importantly, that a change in position produces a dynamical, not a static, change in the field is ignored. Fields propagate; accelerated masses would radiate gravitational fields just as accelerated charges radiate electromagnetic fields, and dynamical fields have dynamical properties such as energy and momentum that may propagate. This is the problem that arises when an action-at-a-distance theory is tried to be analyzed within the context of special relativity. As these aspects were ignored, the conclusion of this part of the paper cannot follow from the argument.
- The paper then uses the hydrogen molecule H_2 as an example. The molecular mass is less than the mass of two hydrogen atoms, and thus the work performed when breaking the molecule becomes part of the atomic mass, i.e., it is not distributed in space but is localized within the atoms. Unfortunately, this example is of a very different nature than the previous one, as in this case, the fields (the molecular and atomic wavefunctions and the electric field) do not extend appreciably beyond the molecule and the atoms, while the gravitational field considered in the previous and following examples is a non-screenable, long-range field that occupies all space.
- In part 3 of the paper, a thought experiment is proposed. In it, a particle is annihilated, producing a photon that moves

upwards and converts into another particle. It is argued that the total mass of the created particle should include its rest energy and its potential energy; otherwise, linear momentum conservation along the horizontal direction could be violated. The argument is nice, but it could have been simplified. One objection is that during particle annihilation, momentum must be conserved in, for example, the vertical direction. Thus, the reader would be distracted, thinking that not one but two oppositely moving photons should be created. The author argues in a footnote that the second photon could be deflected upwards, so that the conclusions would be unaffected. Nevertheless, that would give the wagon a downwards momentum, decreasing its height, its potential energy, and therefore, its mass, and complicating the analysis. To avoid that complication, in a footnote, the reader finds that the wagon rolls over a very massive system that absorbs the vertical momentum with negligible motion. The need for the footnote shows that the simplicity of the system is only apparent. Furthermore, the reader might be confused with Eq. (6). It seems that the quantized energy of the photon is converted into the upper particle and to the mass of its potential energy. However, no explicit mention is made of the gravitational Doppler shift of the frequency of the photon as it rises against the gravitational field. Thus, the quantum of energy becomes $h\nu_A$, which is smaller than $h\nu_B$ and is precisely the energy needed to create the mass m_A. Extra energy is required to create the mass $m_A g h/c^2$ associated with the potential energy. As the paper pretends that this energy is contained within the particle, a discussion is required about how it reaches it. The massless photon has no 'potential energy'. A further discussion of the meaning of a momentum that depends on the reference height would also be required.

- Also in part 3, a refinement is proposed, where it is assumed that the gravitational force is proportional to the total mass, including the mass associated with the potential energy. Thus, when work is done lifting a particle, work is also done lifting its potential energy, and thus the total mass in a uniform field is not proportional to height but to an exponential of height. The problem I see here is that if the momentum of a particle contains a contribution due to its potential energy, the equation of motion would have to differ from Newton's second law, just as when the potential momentum $q A/c$ due to a vector potential A modifies the equation of motion of an electric charge q, introducing magnetic forces. This would be required as the mass depends on an arbitrary potential energy reference level, but the equation of motion must not. Without a discussion of the correct equation of motion, the exponential increase of mass with height is not well justified.
- In the conclusion, the author dismisses a priori any criticism based on not using general relativity.
- While part 3 shows that the total mass should include the gravitational potential energy, within the stated approximations, the argument of part 2 about the localization of the mass within the particle is not convincing, as I argued above. Thus, the conclusion that it must be localized within the wagon on p. 12 is not well supported. The example of the hydrogen molecule is retaken, but as remarked above, it is not a good example, as in that case, the fields themselves are localized. The rest of the conclusions are a tentative proposal, not based on the main text, about how the mass distributes among the particles in a multiparticle interacting system.
- In summary, I found that the theme of the paper is interesting as it stimulates thinking about a fundamental problem that is usually ignored. However, I found the approach rather uncritical and incomplete. Thus, my low rating.