Zero-point Energy Conundrum for Spin ½ Particles

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Abstract

The zero-point energy concept results in an infinite energy that has not yet been fully resolved. A previous paper provided a resolution for spin 0 particles within the scope of the Klein-Gordon equation. This paper reviews the zero-point energy issue for spin ½ particles utilizing the Dirac equation. The need for the zero-point energy is confirmed in Santilli's synthesis of the neutron from the proton and the electron in the core of a star.

A possible resolution lies in establishing a cutoff energy (e.g., at the Planck scale) that negates the singularity. This singularity is avoided by establishing a restricted energy region that precludes exceeding the cutoff energy scale except under extreme circumstances (e.g., during a big bang/big crunch event). Potential energy functions, analogous to the Higgs potential or the short range hard core nuclear interaction, are introduced as possible mechanisms to establish the cutoff energy.

1.0 Introduction

The zero-point energy (ZPE) concept results in an infinite energy that has not yet been fully resolved [(1)-(2)]. The need for the zero-point energy is confirmed in Santilli's mathematical, theoretical, and experimental studies on synthesis of the neutron from the proton and the electron in the core of a star [3]. Santilli [3] refers to the zero-point energy as "universal substratum" with an extremely large energy density for the characterization and propagation of electromagnetic waves at 300,000 km/s.

A possible resolution lies in establishing a cutoff energy (e.g., at the Planck scale) that negates the singularity. A previous paper reviewed the ZPE issue for spin 0 particles within the scope of the Klein-Gordon equation [4]. In this paper, the cutoff energy restriction is addressed utilizing a spin 1/2 free field Hamiltonian within the scope of the Dirac equation [5]. Since the free field restriction precludes interactions with other particles and antiparticles, particles cannot be created or destroyed in pairs. Therefore, the free field Hamiltonian does not measure the energy of these pairs, and the 1/2 quantum energy terms can be interpreted as free fields that are not emerging from or entering into the vacuum in pairs.

Associated with the ZPE issue and the postulated physical effects are fluctuations that are typically quantified in terms of pair production and annihilation in a bubbling quantum foam, the zero-point energy of fermion fields, virtual particle vacuum bubbles and loops, and interacting virtual particles [(6)-(9)]. Although these theoretical arguments have been and continue to be made, there is no experimental basis to select if any of these proposed mechanisms occur.

A number of experiments have been proposed to validate the existence of the ZPE. These include the Casimir effect [10], Lamb shift in hydrogen ([11]-[12]), anomalous magnetic moment of the electron [13], and Fulling-Davies-Unruh effect ([14]-[17]). These theories and associated experiments provide useful information, but are not definitive. There are other possible explanations for the proposed effects including the incorporation of higher order corrections beyond the usually assumed quantum field theory (QFT) interaction terms.

This paper proposes that the ZPE singularity could be avoided by establishing a restricted energy region that precludes exceeding a cutoff energy scale except under extreme circumstances (e.g., during a big bang/big crunch event). The cutoff energy scale is established by introducing a potential energy function that restricts the accessible energy scale.

2.0 Theoretical Approach

The spin ½ methodology is provided in (1) – (6). (7) – (10) outline an approach to eliminate the ZPE singularity. Specific discussion is summarized in subsequent commentary.

$$H_0^{1/2} = \sum_{p=0}^{\infty} \sum_{r} E_p \left(N_r(p) - 1/2 + N'_r(p) - 1/2 \right) \tag{1}$$

$$\begin{aligned} H_0^{1/2} \left| \psi_{r'p'}, \psi_{r''p''}, \psi'_{r'''p'''} \right\rangle &= \sum_{p=0}^{\infty} \sum_{r} E_p \left(N_r(p) - 1/2 + N'_r(p) - 1/2 \right) \\ \times \left| \psi_{r'p'}, \psi_{r''p''}, \psi'_{r'''p'''} \right\rangle &= E_{p'} + E_{p''} + E_{p'''} \\ &+ \sum_{p=0}^{\infty} \sum_{r} E_p \left(-1/2 - 1/2 \right) \left| \psi_{r'p'}, \psi_{r''p''}, \psi'_{r'''p'''} \right\rangle \end{aligned} \tag{2}$$

$$\begin{aligned} H_0^{1/2} |0\rangle &= \sum_{p=0}^{\infty} \sum_{r} E_p \left(N_r(p) - 1/2 + N'_r(p) - 1/2 \right) |0\rangle \\ &= \sum_{p=0}^{\infty} \sum_{r} E_p \left(-1/2 - 1/2 \right) |0\rangle \end{aligned} \tag{3}$$

$$\psi(x) = \sum_{p=0}^{\infty} \sum_{r} \sqrt{\frac{m}{VE_p}} \left[x_r(p) u_r(p) e^{-ipx} + y'_r(p) v_r(p) e^{ipx} \right] \tag{4}$$

$$\Psi'(x) = \sum_{p=0}^{\infty} \sum_{r} \sqrt{\frac{m}{VE_p}} \left[y_r(p) v'_r(p) e^{-ipx} + x'_r(p) u'_r(p) e^{ipx} \right] \tag{5}$$

$$\left[x_r(p), x'_s(p') \right]_+ = \left[y_r(p), y'_s(p') \right]_+ = \delta_{rs} \delta_{pp'} \tag{6}$$

$$V(\Psi, \omega) = V_1(\Psi, \omega_1) + V_2(\Psi, \omega_2) \tag{7}$$

$$H_0^{1/2} |0\rangle = \sum_{p=0}^{p_2} \sum_{r} E_p \left(-1/2 - 1/2 \right) |0\rangle \tag{8}$$

$$V(\Psi) = \mu^2 \Psi' \Psi + \lambda \left(\Psi' \Psi \right)^2 \tag{9}$$

$$\mu^2 = 2\lambda \upsilon^2 \tag{10}$$

The free field Hamiltonian for a spin 1/2 field [18] is provided in (1). In (1), N(p) is the number operator for field particles (x) with three-momentum p and spin r, and N′(p) is the number operator for field antiparticles (y) with three-momentum p and spin r. In (1), $E^2 – p^2 = \mu^2$ where E is the field quanta energy and μ is the mass of the field particle. The momentum p is an eigenstate of the field. For example, the free field, spin 1/2 Hamiltonian operating on the particle ket | $\psi_{r'p'}, \psi_{r''p''}, \psi'_{r'''p'''}$> that has one x type Dirac particle in an eigenstate with three-momentum p′, a second x type particle with three-momentum p″, and a y type antiparticle with three-momentum p‴.

In (2), the first term is the positive particle energy and the second the infinite negative energy of the vacuum. The vacuum has -1/2 quantum of energy for each p, r for x type particles and y type antiparticles. These -1/2 quanta are the zero-point energy (ZPE), that is the source of the infinite energy singularity and associated conundrum. (1) and (2) illustrate the fact that every spinor state is superimposed on the vacuum.

As noted in (3), the infinite energy issue becomes even more obvious by considering the vacuum state |0>. Since every state is superimposed on the vacuum, every state has infinite negative energy. The unrestricted sum in (3) represents the perspective that the vacuum is endowed with infinite energy. Given the infinite energy singularity, the remainder of this paper addresses a possible mechanism, to impose an energy cutoff to mitigate this singularity.

3.0 Normal Ordering

The infinite sum of 1/2 quanta energy noted in (2) and (3) presents a significant challenge in QFT. This infinite energy associated with the vacuum would lead to an extreme curvature within the scope of general relativity. However, this physical effect has not been observed. From a theoretical perspective, the infinite energy singularity is eliminated if the sums of (2) and (3) are terminated at a large but finite energy (e.g., at the Planck scale).

One possibility to mitigate the effects of the infinite sum is to impose normal ordering [18]. Within the scope of normal ordering, all destruction operators in any term are relocated to the right hand side of that term. This operation is justified for operators that commute. For example, the discrete plane wave solutions of the Dirac equation [5] can be written as (4) and (5) [18].

In (4) and (5), $x_r(p)$ is a destruction operator. When the operator $x_r(p)$ acts on a single x type particle of spin r and three-momentum p, it destroys the state $|\psi_{rp}>$. In a similar manner, $x_r'(p)$ creates a single Dirac particle out of the vacuum. The y type operators have a similar role. $y_r'(p)$ and $y_r(p)$ operators create type y antiparticles from the vacuum and destroy type y antiparticle states, respectively. Application of $y_r(p)$ to a state containing a single y type antiparticle $|\psi'_{rp}>$ yields the vacuum state.

Normal ordering is jeopardized because these operators have the discrete anti-commutation relationships summarized in (6). All other anti-commutation relationships between the x and y coefficients are zero. Normal ordering would introduce a minus sign for these zero anticommutation relationships. In addition, (6) introduces additional terms that are not expected in the normal ordering approach. These additional terms and sign changes impact the assumed outcome of the normal ordering operation.

Although normal ordering is commonly utilized in QFT, it violates the anti-commutivity property of operators that is a fundamental postulate of the Dirac equation in quantum field theory. Therefore, it is reasonable to question the appropriateness of normal ordering to resolve the infinite energy issue for spin ½ particles.

4.0 Cutoff Potential

There is a lack of theoretical justification in addressing the vacuum infinite energy issue through normal ordering. Efforts to impose an arbitrary energy limit or explaining the ZPE in terms of the Casimir effect, Lamb shift, anomalous magnetic moment of the electron, and Fuller-Davies-Unruh effect were also not universally accepted as a complete explanation of the infinite energy issue ([1]-[2]).

In view of these approaches and their limitations, this paper postulates the existence of an energy cutoff characterized by a potential energy function that mitigates the ZPE singularity. This energy restriction eliminates the singularity and the ZPE issue. Two possible potential forms are presented. These include an analogue to the short range hard core interaction in nuclei [19] and a vacuum potential analogue of the Higgs field ([20]-[21]).

4.1 Hard Core Nuclear Potential Analogue

An energy restriction can be formally defined in terms of a potential function summarized in (7). In (7), V_1 defines an accessible energy region from $0 \leq \omega \leq \omega_1$ and V_2 represents a potential term that is sufficiently large (e.g., at the Planck scale) to preclude additional particle production from the vacuum. The use of a V_2 term to restrict the accessible parameter space is analogous to the use of a hard core nuclear potential to represent the short range repulsion between nucleons [19]. A hard core precludes the spatial interaction region of nucleons to values typically taken to be about 0.4 fm. The V_2 term effectively terminates the sum of (2) and (3) as illustrated in (8). In (8), $\omega_2^2 = p_2^2 + \mu^2$, ω_2 is the cutoff energy, and p_2 is the associated three-momentum. Terminating the sum in (8) eliminates the ZPE issue.

4.2 Vacuum Potential Analogue of the Higgs Field

An additional approach facilitates the energy cutoff, and utilizes a potential similar to the Higgs field potential. The Higgs field incorporates an energy scale that characterizes the electroweak interaction. In a similar manner, the analogue vacuum field potential is also characterized by an energy scale (e.g., at the Planck scale) that effectively terminates the (2) and (3) sums, and eliminates the energy singularity. This potential approach is similar to the application of a potential function to characterize the Higgs field.

For specificity, the Higgs analogue potential energy function is assumed to have a form of (9). In (9), μ and λ are not specifically defined, but can be selected to yield the desired energy cutoff. The Higgs field potential can be characterized by the values $\mu \approx 126$ GeV, $\upsilon \approx 246$ Gev, and $\lambda \approx 0.13$ that are derived from the parameterization of (10).

For the Higgs field υ represents the energy scale of the electroweak interaction. In a similar manner for the ZPE interaction, υ represents the energy scale that truncates the sum of (8). In (10), μ represents the field particle mass (e.g., at the Planck scale) which has yet to be determined.

5.0 Conclusions

The use of a potential describing the particles generated from the spin 1/2 free field Dirac Hamiltonian offers a possible resolution to the infinite energy singularity encountered with the vacuum. This singularity known as the zero point energy has not yet been fully resolved.

A possible resolution lies in establishing a cutoff energy (e.g., at the Planck scale) that negates the singularity. This singularity is avoided by establishing a restricted energy region that precludes exceeding the cutoff energy scale except under extreme circumstances (e.g., during a big bang/big crunch event). Potential energy functions, analogous to the Higgs potential or the short range hard core nuclear interaction, provide possible mechanisms to establish the cutoff energy.

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