Review of: "On a New Two Point Taylor Expansion With Applications"

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Potential competing interests: No potential competing interests to declare.

To avoid possible confusion in comparing the two forms (1) and (24), I suggest changing the notation from (a_m, b_m) to (b_m, c_m) in the form of the classical formula (24).

A natural question arises: are possibly the two forms (suggested and classical) equivalent, e.g., by transforming one into the other for all m?

Now, for $m \geq 0$, the terms in (1) are $[a_{2m} + a_{2m+1}(x - x_0)][(x - x_0)(x - x_1)]^m$

and in (24) are $[a_m(x - x_0) + b_m(x - x_1)][(x - x_0)(x - x_1)]^m$; then, for all x and all m, they coincide if and only if (compare the two terms of first degree in x) it is $a_{2m} = a_m + b_m$ and $a_{2m+1}x_0 - a_{2m} = a_m x_0 + b_m x_1$.

I have a preference for the suggested form (1) because it is immediate to obtain from it the 1-point Taylor expansion based at point $x_0$by considering series terms $[a_m][(x - x_0)]^m$

as a “truncation” of the terms in the 2-points formula.

It should be interesting to obtain a (r+1)-points expansion based at $x_0, x_1, \ldots, x_r$ with series terms (2-points if $r = 1$), similar to (1),

$[a_{m,0} + a_{m,1}(x - x_0) + \ldots + a_{m,r-1}(x - x_{r-1})][(x - x_0)(x - x_1)\ldots(x - x_r)]^m$

and study its properties (and what is gained by increasing r).

Working similarly to (24), one can consider the terms (obtaining equivalent expansions)

$[a_{m,0}(x - x_0) + \ldots + a_{m,r}(x - x_r)][(x - x_0)(x - x_1)\ldots(x - x_r)]^m$.