

Research Article

Modelling and quasistatic simulation of the stiffness degradation of concrete based on physical measurements

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AIM:

The description of the mechanical properties of concrete is usually done with two pairs of values "stress-strain" within the "elastic area" which are defining the slope (Young's modulus) and the peak value (strength). The post-peak-behaviour (softening) is usually defined only by a theory giving the decreasing curve e.g. exponential softening. Therefore the aim of this project is the metrological recording of the complete stress-strain-curve for the integration into the material model, reproducing the "linear" and the nonlinear area (softening) within the nonlinear simulation exactly.

The quality of simulation results should improve when using physical measured values as input data of the material model.

PROCEDURE:

According to the statistical evaluation of the experimentally determined material parameters (compressive-, bending -, tensile-, splitting tensile strength), the bending strength was identified as the material parameter with the least deviation from the mean value (relative scattering coefficient, the coefficient of variation). This leads to the objective of implementing the electronically recorded measurement data of the carried out deformation controlled 4-point-bending-tests (according to test guideline German committee for reinforced concrete DAfStB) into the material model. The experimentally recorded forces and displacements were linearly converted into normalized stresses and strains according to the rules of statics. The input for the elastic-degrading material model of the steel-fibre-concrete are 20 pairs of values, taken from the measured data of the 4-point bending test. The material model (smeared cracking method) of the unreinforced fibre concrete and its softening is thus based exclusively on experimentally measured data, taken from the 4-point-bending-tests. The theoretical material models available in "Ansys Mechanical", e.g. "Mohr-Coulomb" or "Drucker-

Prager”, have not been applied here. The usual input data of the uniaxial tensile- and compressive strength were also not included in the simulation.

When evaluating the 4-point tests, the lowest force-displacement curve is decisive (minimum work performed); this corresponds to the main crack in midspan. In the area of the damage between the load applications, the constant (bending) normal-stress corresponds to the equivalent stress because of the lack of (bending) shear-stress. The (bending) normal-stress together with the plastic strain are the experimental input values of the material model.

CONCLUSION:

Element size and deformation rate have to be minimal for a good quality simulation result. There are deviations between simulation and experiment in the „elastic zone“ and also concerning the peak value (bending strength) because within the simulation all cracks are smeared homogeneously.

Sufficient local and temporal discretization / sufficient small mesh size and deformation speed bring the nonlinear simulation close to the physical reality. The optimum quality of results is achieved with a specific mesh density and a specific deformation speed - too small values, on the other hand, will worsen the quality of results (optimization process). The elastic-degrading/multilinear-elastic simulation using a physically based material model showed no convergence problems.

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1. Material Model

Based on 4-point bending experiments (beam 15x15x70cm)

The statistic analysis of the material parameters, determined by experiments with the small specimen, delivered the following „coefficient of variation“ results for steel-fibre-concrete C30/37 containing 25kg/m³ steel fibres and 0,5kg/m³ plastic fibres.

	compression strength cylinder	splitting tensile strength	bending tensile strength deformation controlled	tensile strength CC-geometry	tensile strength bone	young's modulus cylinder	compression strength cube
	EN12390	EN12390	DAFStB	-	-	DIN1048-5	EN12390
	[MPa]	[MPa]	[MPa]	[MPa]	[Mpa]	[MPa]	[MPa]
COV coefficient of variation [%]	7,84	5,77	4,05	12,99	10,92	5,53	5,53

Figure 1: Mean scatter of the material parameters for the preferred concrete mix (COV coefficient of variation [%])

The bending tensile strength can be identified as the material parameter with the lowest coefficient of variation. Therefore, according to the project's aim, the electronic measured data of the 4-point-bending-tests (test guideline "Steel fibre concrete" of the "DAFStB german committee for reinforced concrete") is implemented in the material model.



Figure 2: 4-point bending experiment, deformation-controlled, beam 15x15x70cm, span length 60cm, test duration about 20min, test guideline "Steel fibre concrete" of the "German committee for reinforced concrete DAFStB"

The recorded experimental forces and displacements have been converted to normalized stress and strain data acc. the rules of statics. Twenty experimental pairs of values have been recalculated to stress-strain data and implemented into a multi-linear-elastic material model.

Therefore the material model of the steel-fibre-concrete is only based on the measurements of the 4-point-bending-tests. Theoretic material models provided by „Ansys Mechanical“ have not been used. No uniaxial tensile strength resp. compressive strength has been included in the model.

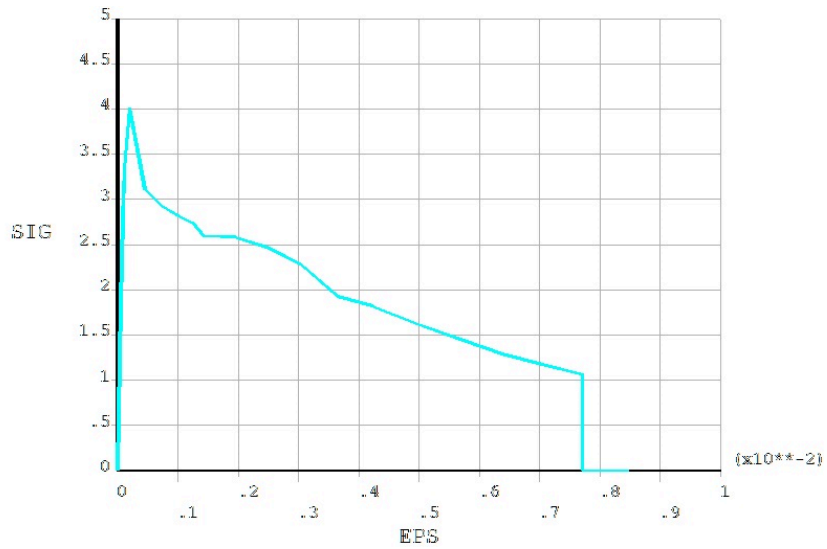


Figure 3: Input data in Ansys „Mela“ for the elastic-degrading material model of the steel-fibre-concrete. 20 pair of values force-displacement (4-point-bending experiment) have been recalculated to stress-strain data.

Please find here a short video file (1min) of the 4-point-bending experiment:

https://www.researchgate.net/profile/Juergen_Ries

Subsequent some figures concerning the quasistatic simulation of the 4-point bending test.

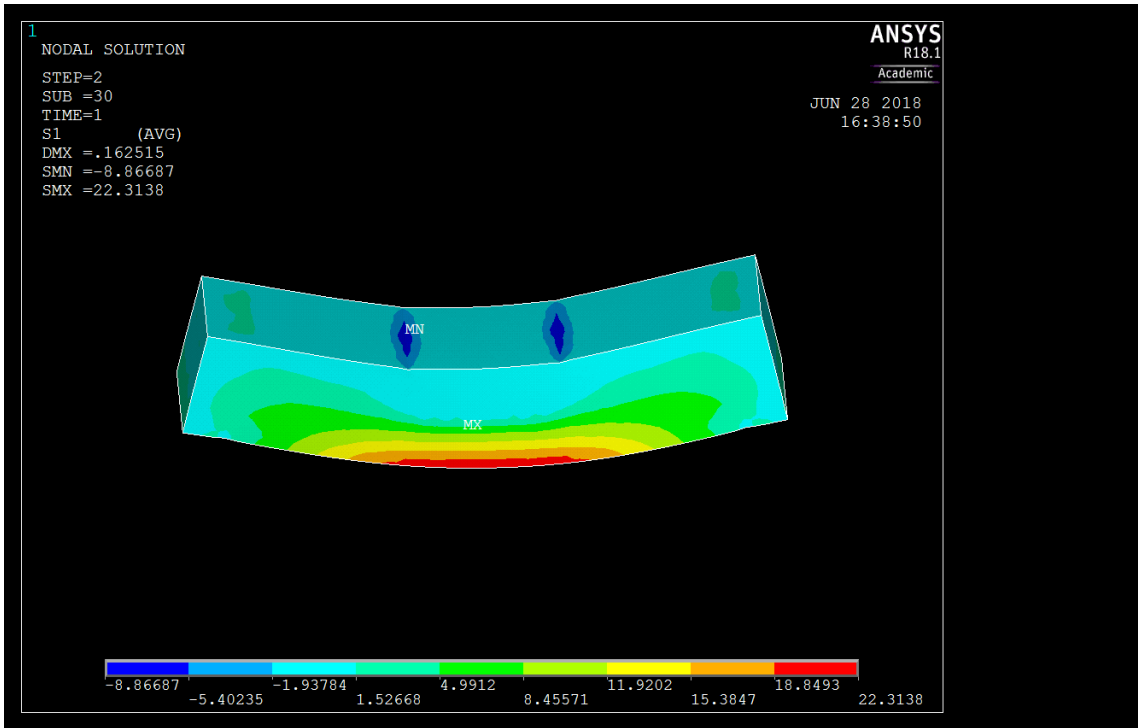


Figure 4: Side view, first principal stress

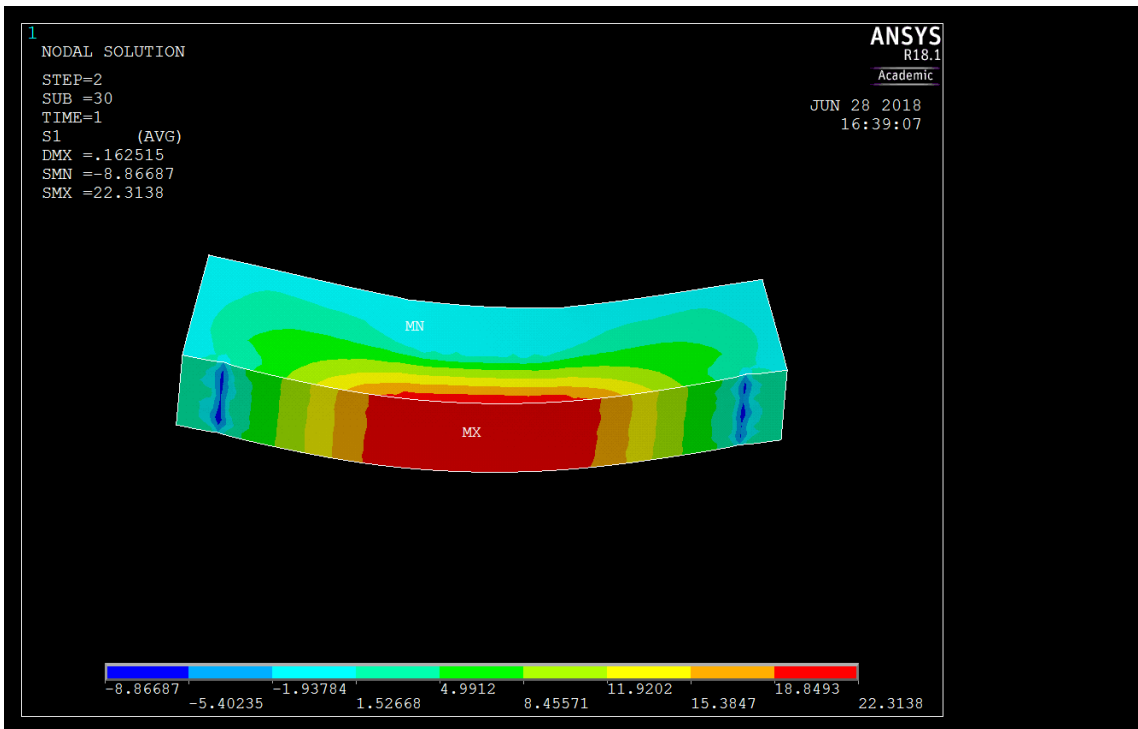


Figure 5: Bottom view, first principal stress

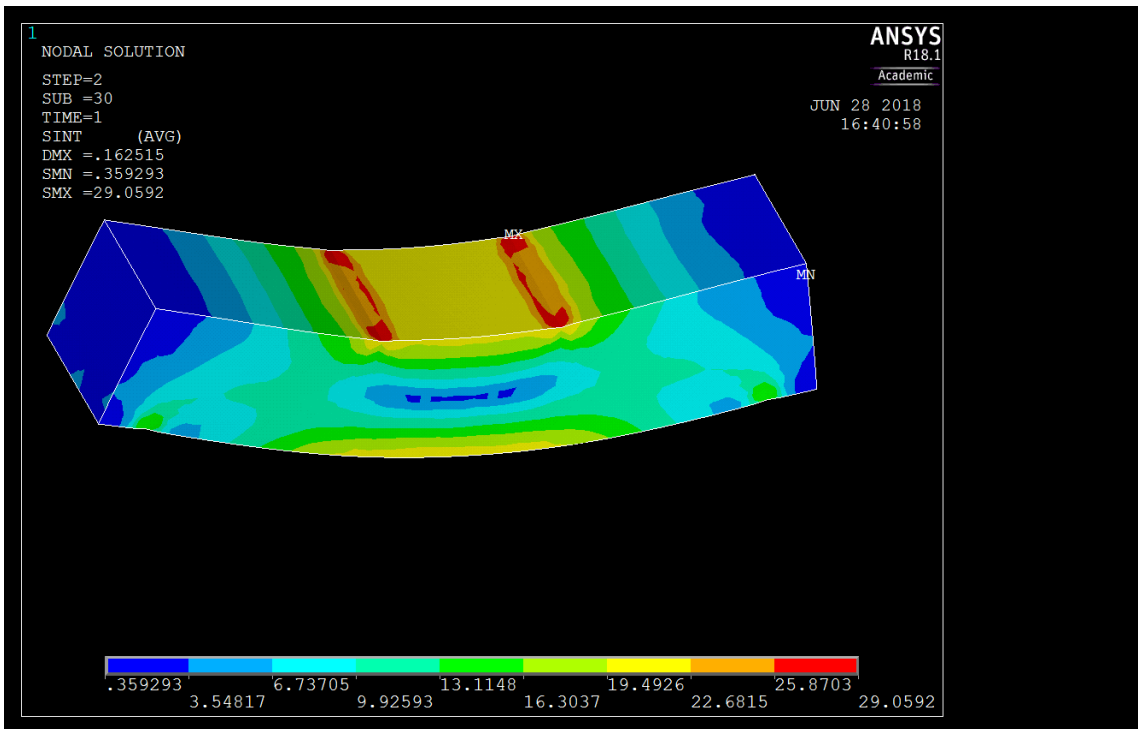


Figure 6: Side view, equivalent stress

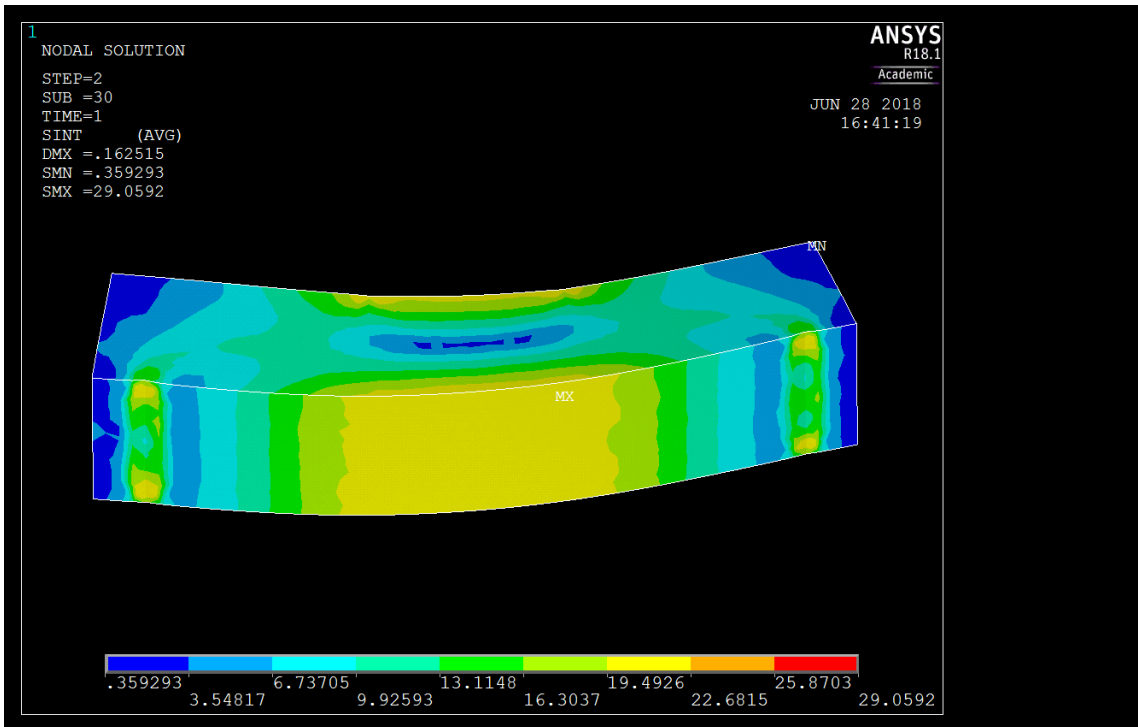


Figure 7: Bottom view, equivalent stress

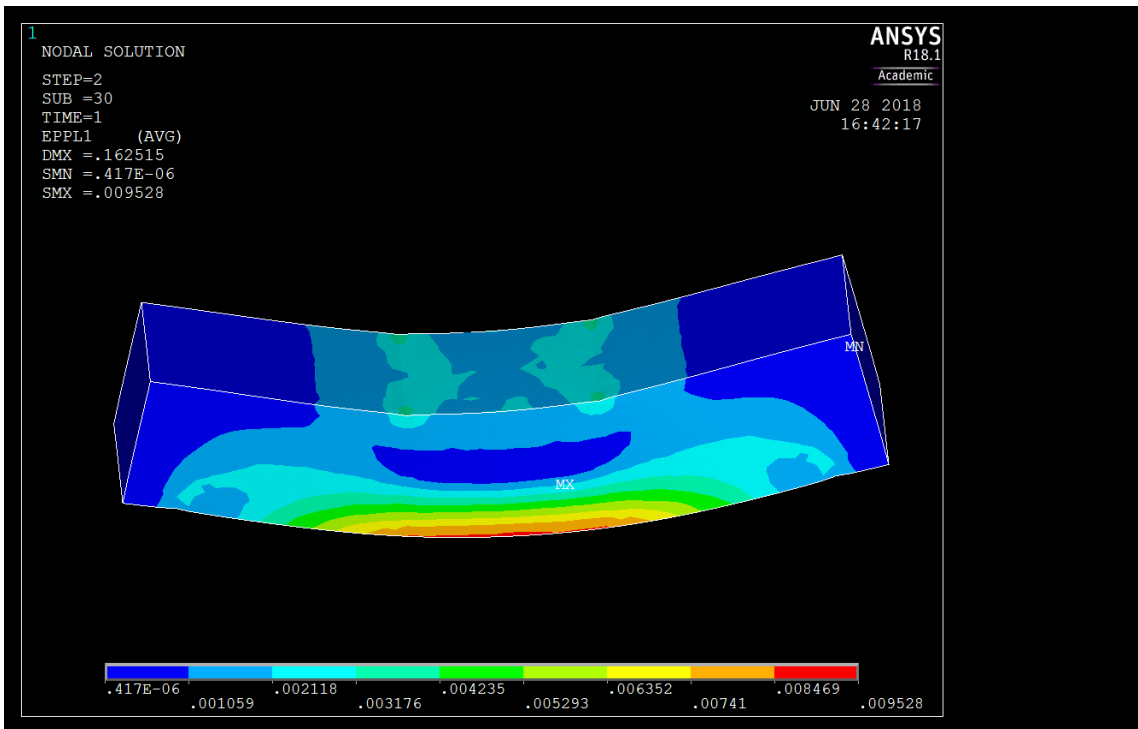


Figure 8: Side view, first plastic strain

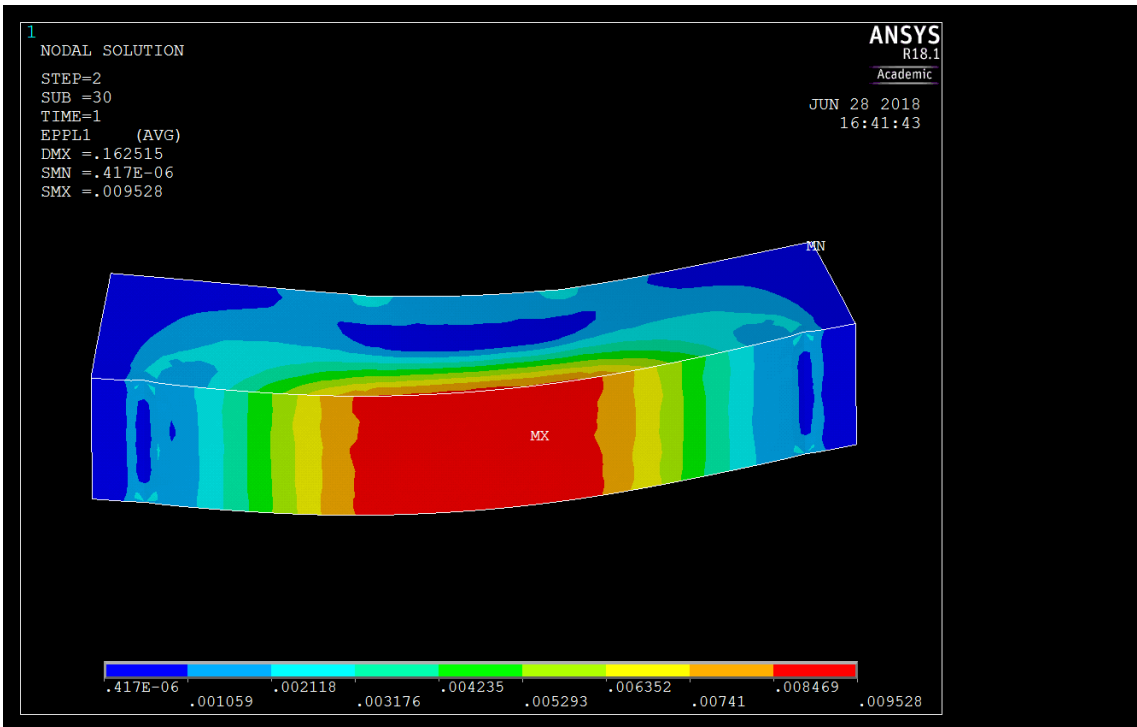


Figure 9: Bottom view, first plastic strain

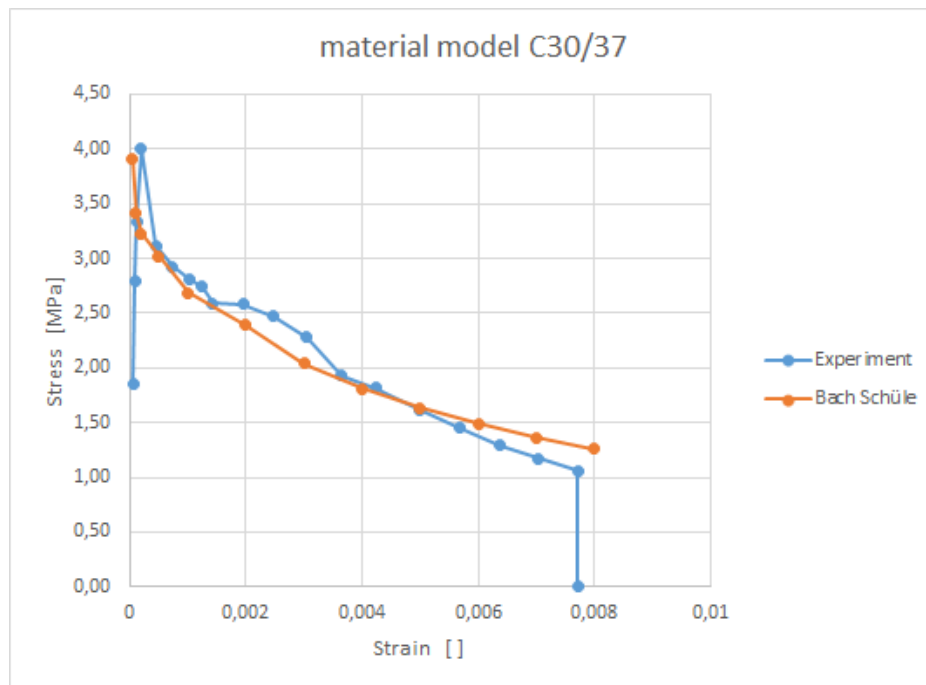


Figure 10: Comparison between experimental data and the theory of „Bach-Schüle“

The power law from Bach and Schüle $\sigma^n = E * \epsilon$ is very close to the experimental curve and describes the softening of the concrete very well.

Within the evaluation of the 4-point-bending tests, the lowest force-displacement-curve is decisive (minimal serviced work); this refers to the main crack in midspan.

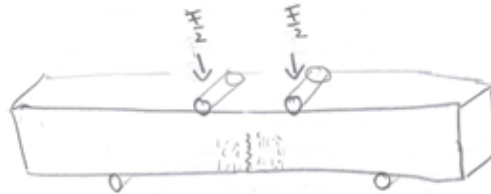


Figure 11: Main crack located in the mid-span and symmetric area of degradation

Acc. to the rules of static there is a constant bending moment between the applied loads, but there is no shear force. The area of degradation spreads out uniformly to the left and right side of the main crack located in midspan. In the area of damage between the applied loads, the normal stress from bending equals the equivalent stress - because of the missing shear stress. Normal stress and plastic strain represent the input values of the material model.

The next graph shows the simulation/verification of the 4-point-bending-test. All implicit simulations within this article have been done in Ansys Mechanical.

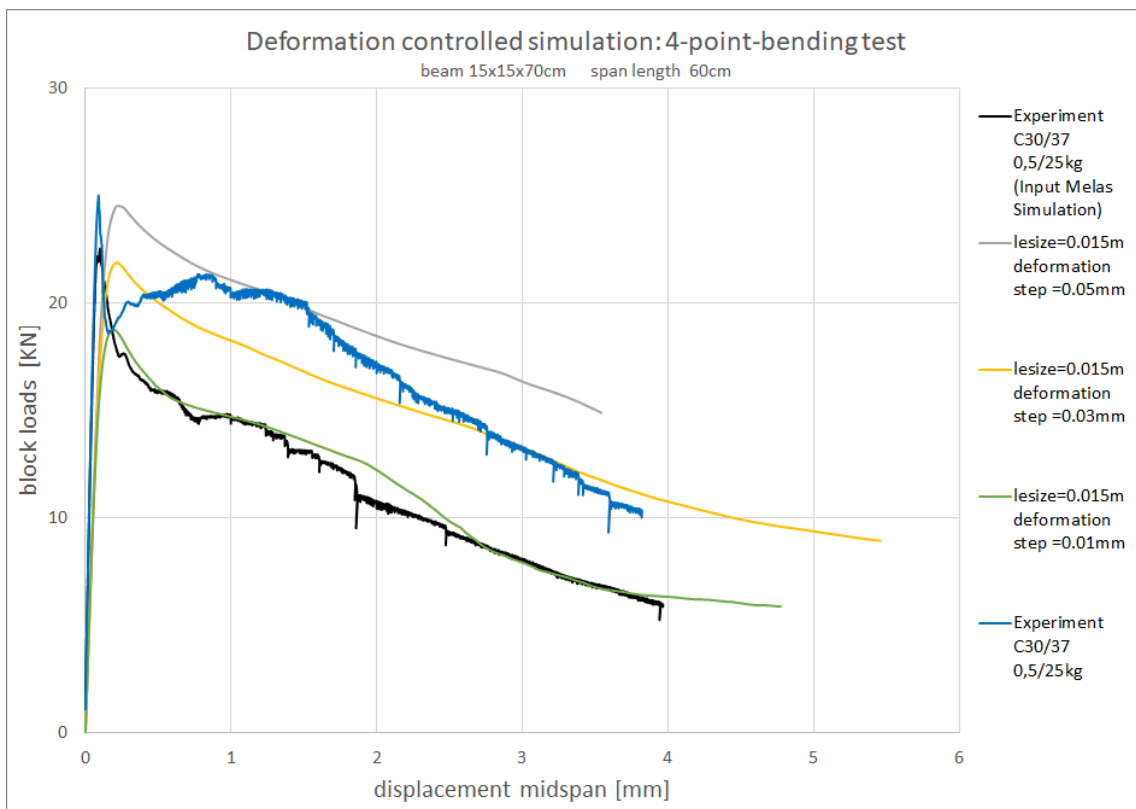


Figure 12: Experiment and quasistatic simulation of the 4-point-bending-test done in „Ansys Mechanical“; the calculation time increases with decreasing deformation step / with increasing time discretization.

Element size and deformation rate have to be minimal for a good quality simulation result. There are deviations between simulation and experiment in the „elastic zone“ and also concerning the peak value (bending strength) because within the simulation all cracks are smeared homogeneously. The crack energies below the curves are nearly identical; the softening behaviour/stiffness degradation of the simulation is very close to the experimental values. The simulation is very close to the lowest experimental curve; this curve has been used as input values for the material model (Mela).

2. Sensitivity of the simulation results

(ceiling 4,80x4,80x0,16m)

The FEM model represents a ceiling which is poured on the construction site, with load carrying action in both directions. The reinforcement (BSt 500) has been implemented with 0,5 Vol% (smeared distribution) in both directions of the ceiling with bilinear stress-strain behaviour. The results of the

simulations are illustrated in the following force–displacement–curves. Only the displacement-controlled simulation shows the results of the softening- / post-peak-behaviour. The forces at the vertical axis are the sum of applied mechanical loads. For a hydraulic load application on 4 areas with only one pump, the forces of the simulation would only be $\frac{1}{4}$.

The span length of the ceiling depends on the grid dimension (here: 1,50m) of the vertical fixing points; e.g. 1,50 – 3,0 – 4,50 – 6,00m ... etc. The decision led to 4,50m of span length because of the slab thickness 16cm (slenderness ratio). All calculations have been proceeded with the „arc-length-method“.

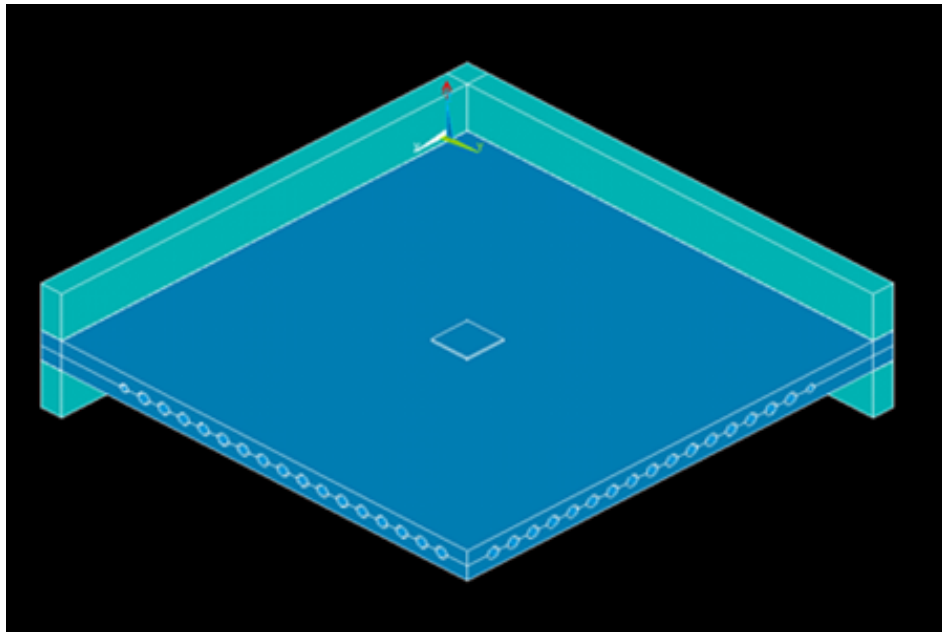


Figure 13: quarter model 2,40x2,40m ; block load area / deformation area 20x20cm

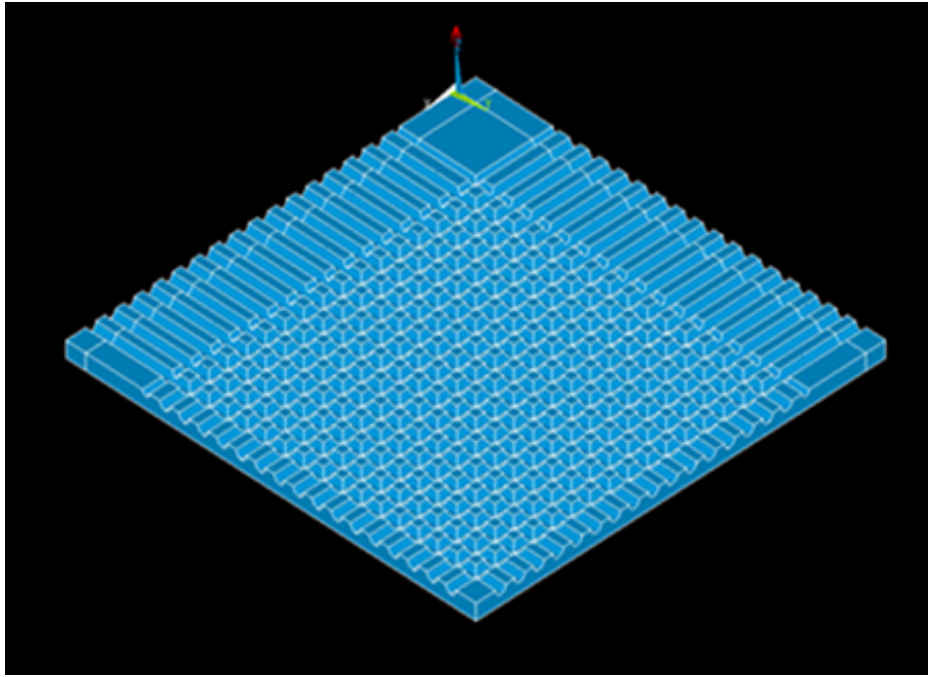


Figure 14: quarter model 2,40x2,40m ; view inside the CC-structures of the ceiling

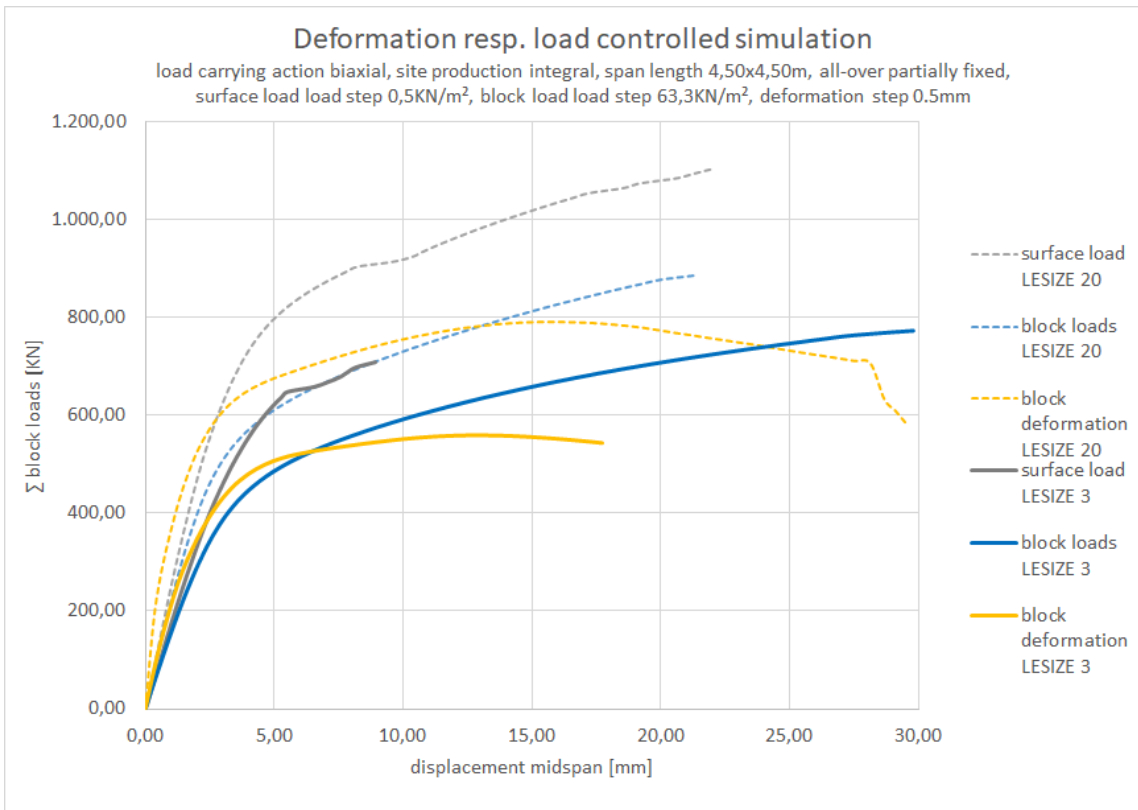


Figure 15: Influence of mesh size (LESIZE) and load case – the displacement controlled simulations show the softening after the peak of the load; the calculation time increases with decreasing element size / with increasing geometric discretization (LESIZE).

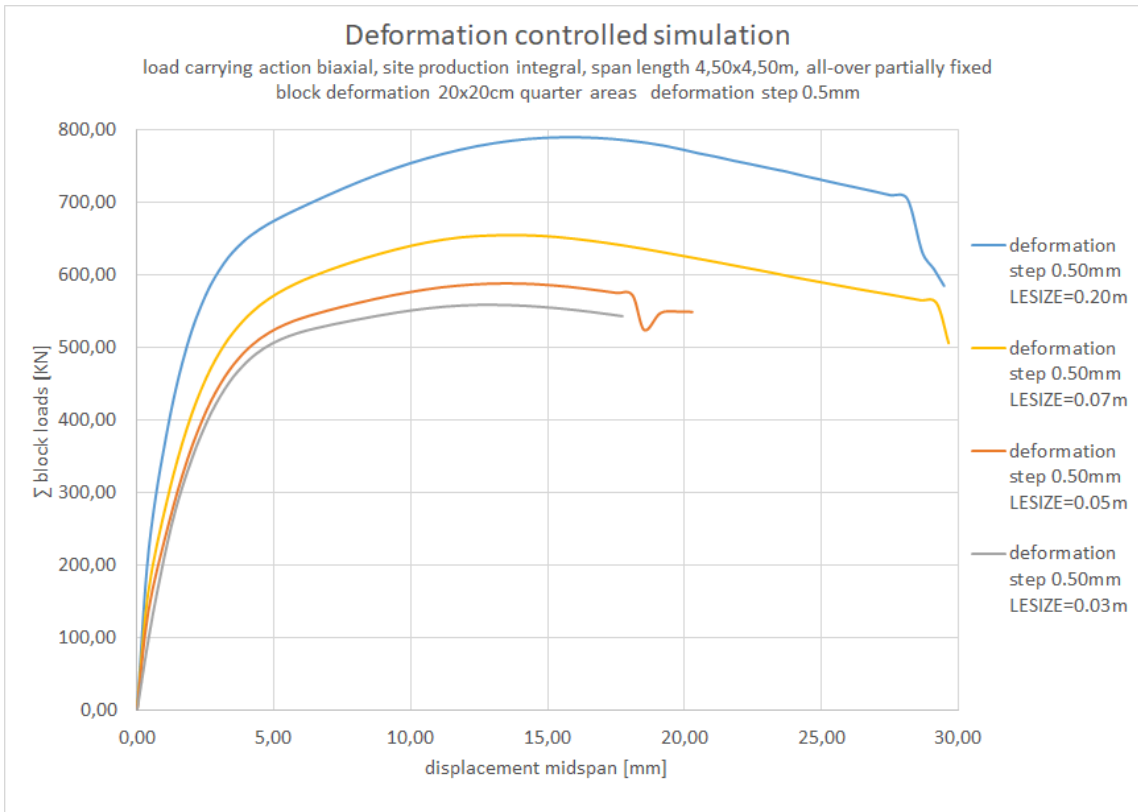


Figure 16: Influence of the element size/mesh size: with smaller mesh size the simulation converges towards the physical reality; the calculation time increases with decreasing element size / with increasing geometric discretization (LESIZE).

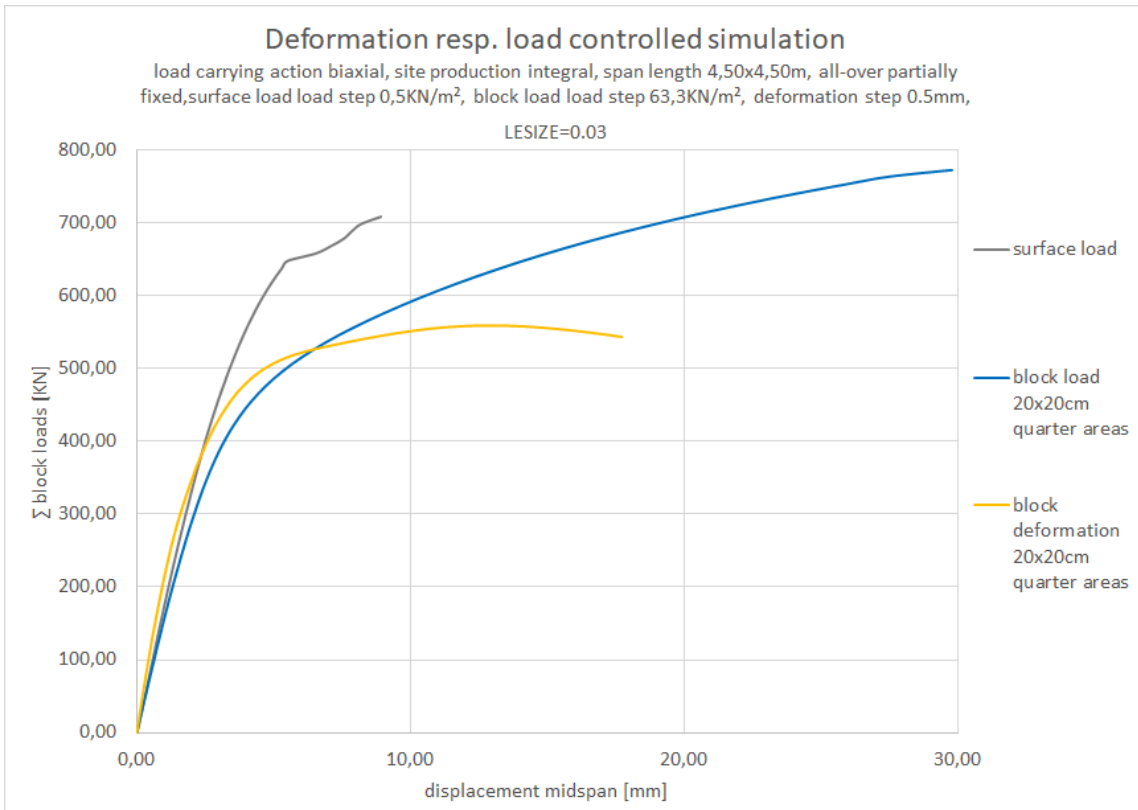


Figure 17: three different load scenarios: only with the displacement-controlled simulation the softening can be shown, the force-controlled simulations converge only pre-peak.

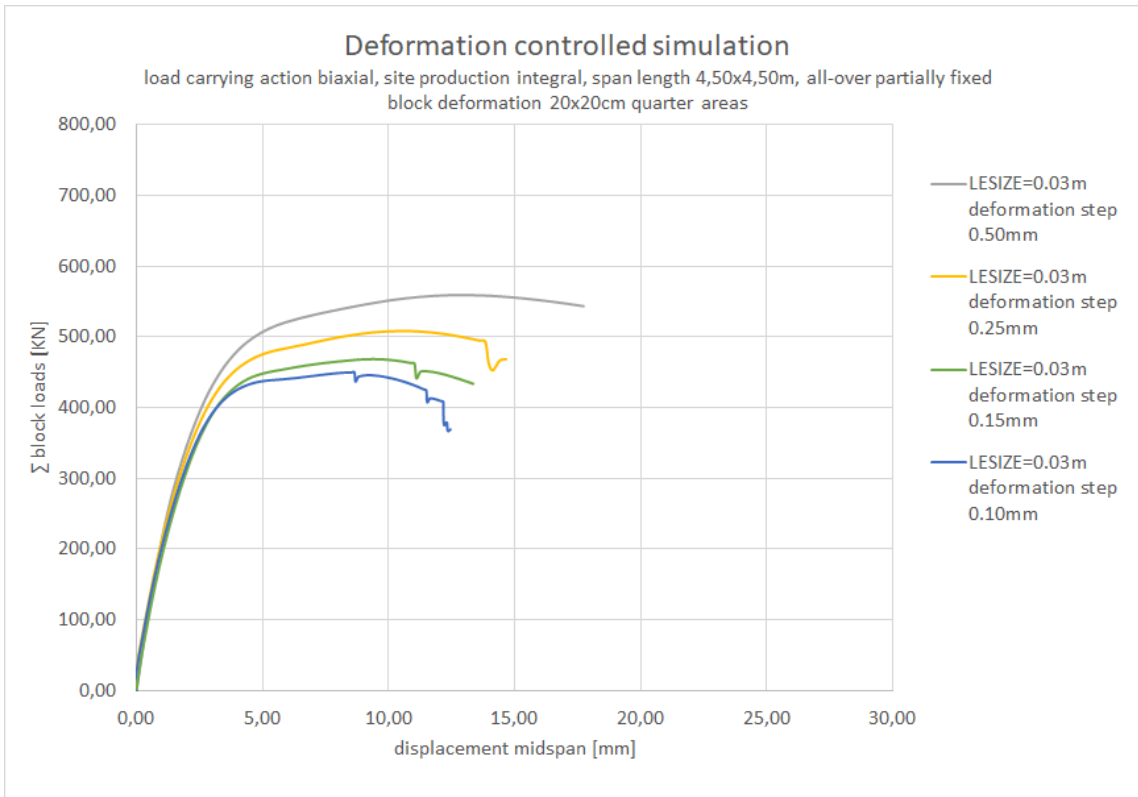


Figure 18: Influence of deformation rate: the deformation rate has to be small enough as well, to receive a simulation result near to physical reality; the calculation time increases with decreasing deformation step / with increasing time discretization.

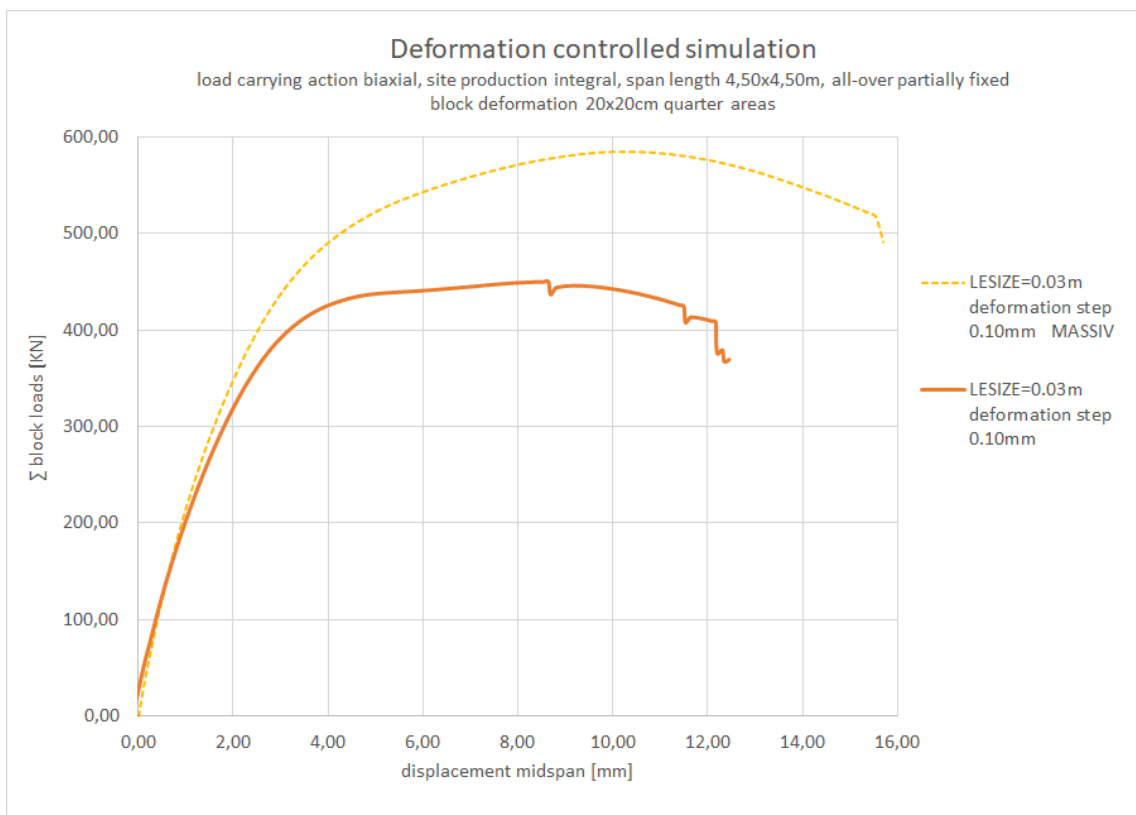


Figure 19: Comparison of the massive and hollow ceiling, the prediction for the experiment

Till about 300kN (15kN/m²) the massive and the hollow construction can be seen as equivalent, the curves are parallel to each other. The massive construction will hold a higher load level for >400kN. This load level won't be reached in reality, for normal use the level is scheduled with 5kN/m² (100kN). The degree of restraint of the simulated supports has to be calibrated with the experimental strain measurements along the supports.

With smaller mesh size and smaller deformation rate, the simulation will get closer to physical reality. The boundary conditions of the simulation have to be calibrated with the experimental strain measurements along the supports. The elastic degrading/multilinear elastic simulation using a physically based material model showed no convergence problems. The physically measured strains at the upper side of the ceiling (photogrammetry) will be compared with the simulated values; the differences will show good and bad areas. Finally, optimization potentials can be identified by specifying the origin of the differences.

The element size converged at LESIZE=3cm, the deformation step size converged at 0,1mm. The increase of local discretization or time discretization / the reduction of mesh size or deformation step size (time step size) will minimize the strain energy – the physical process will be reproduced.

Up to the target value (element size 3cm or deformation rate 0,1mm), the simulation results showed the behaviour of a monotonic convergence. When the target value was exceeded, the behaviour of the results changed to an oscillating convergence, which in turn severely degraded the quality of the results.

Subsequent some pictures of the last simulation / prediction for the experiment (LESIZE=0.03m deformation step=0.1mm). The print of displacement is 10x inflated.

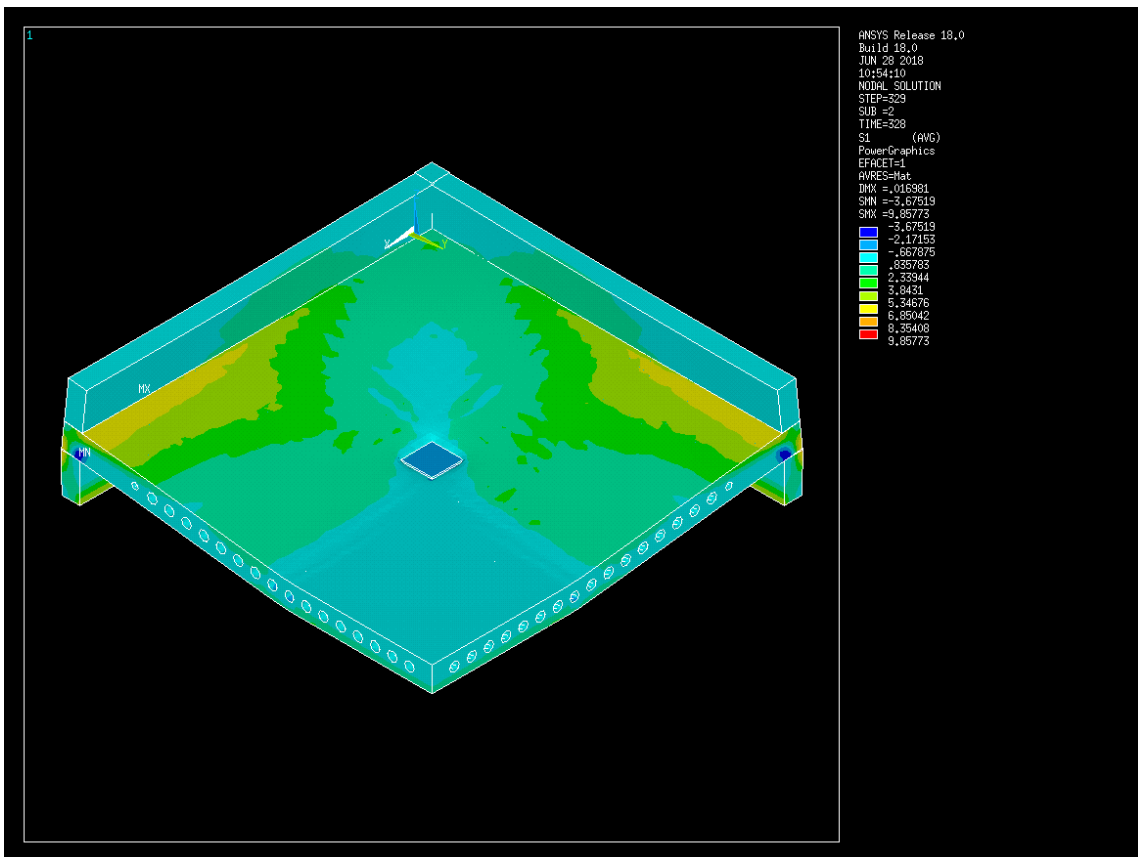


Figure 20: Top side, first principal stress

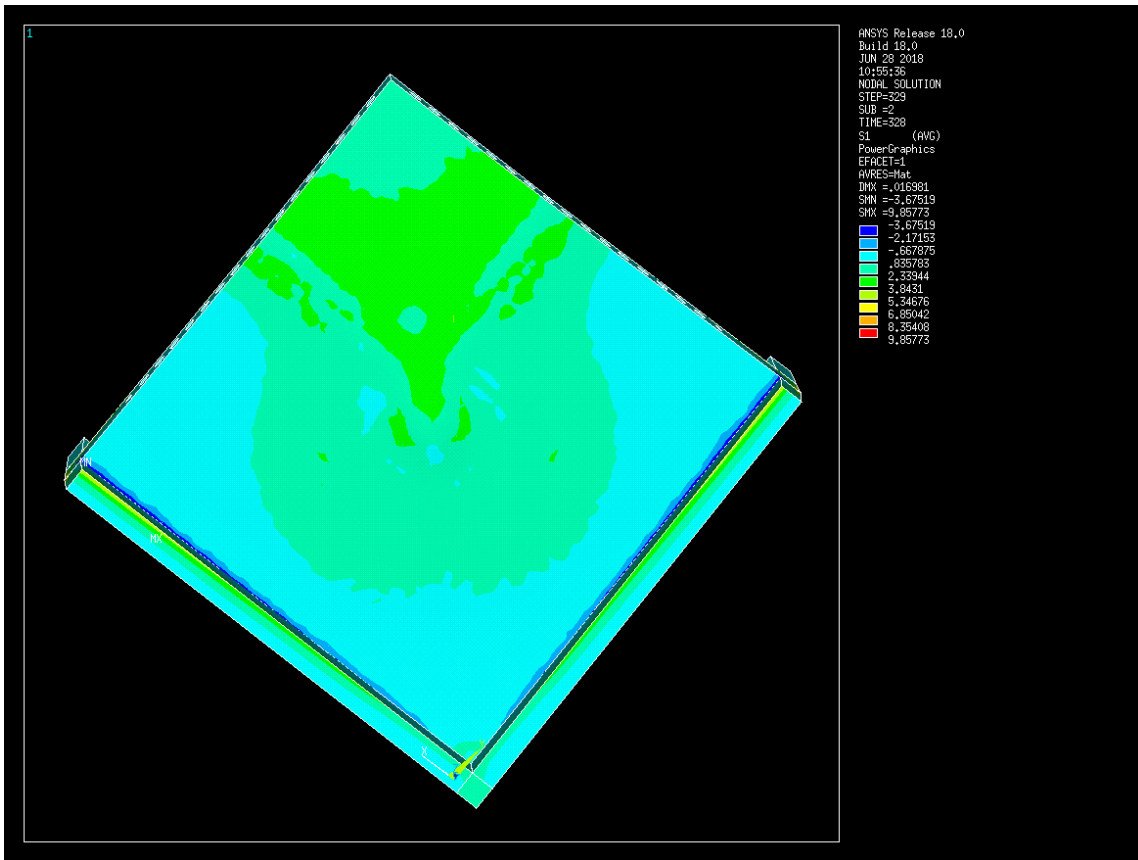


Figure 21: Lower side, first principal stress

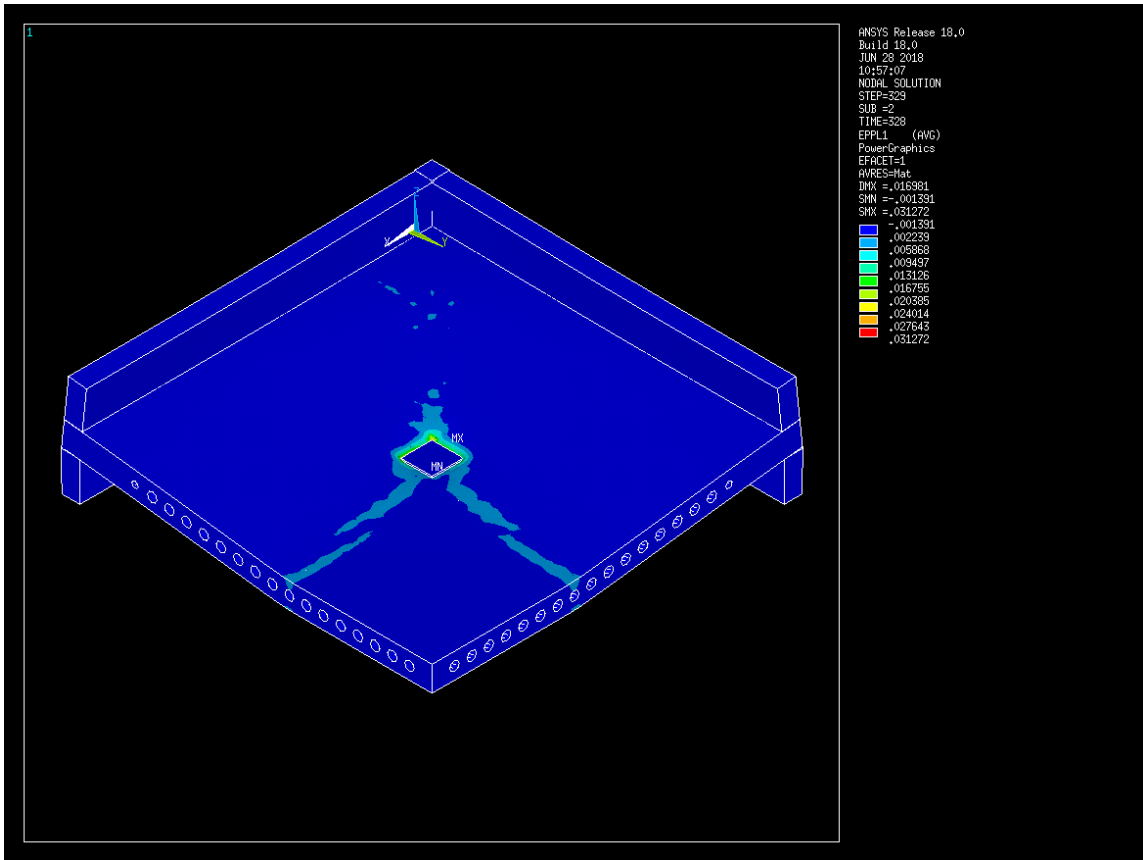


Figure 22: Top side, first plastic strain

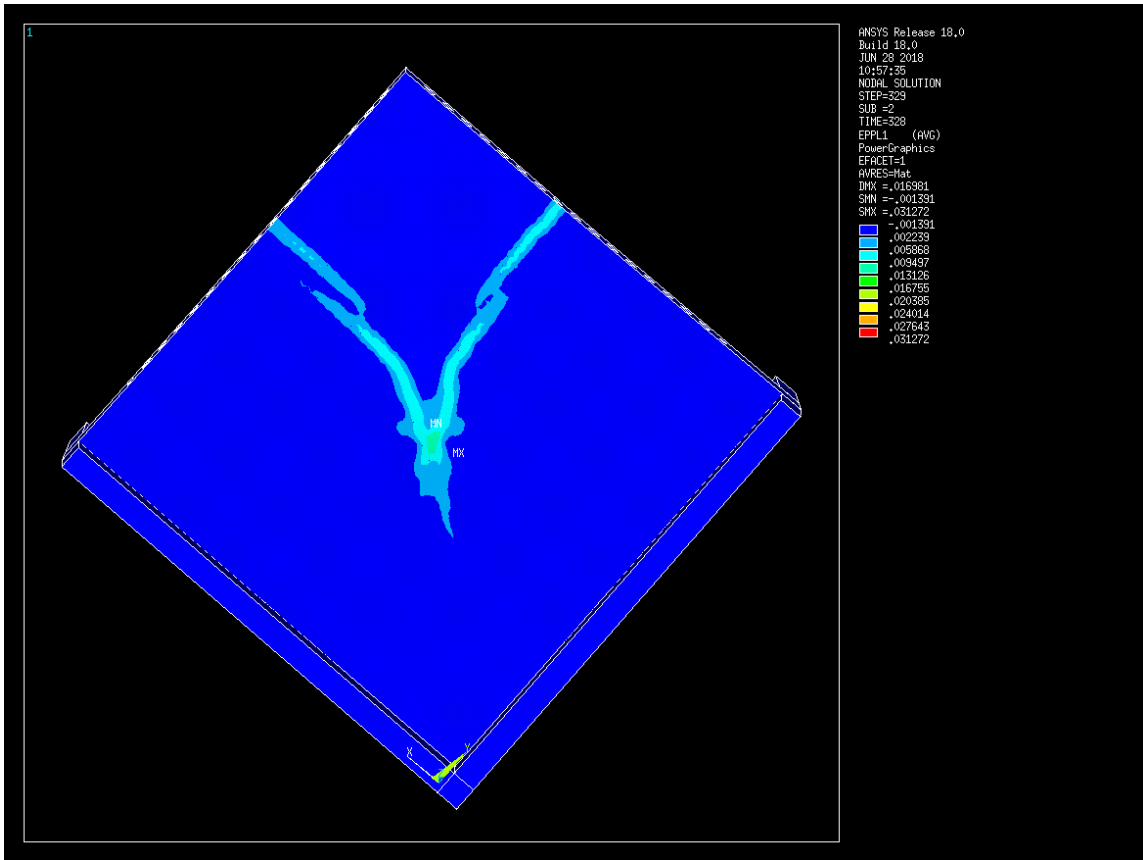


Figure 23: Lower side, first plastic strain

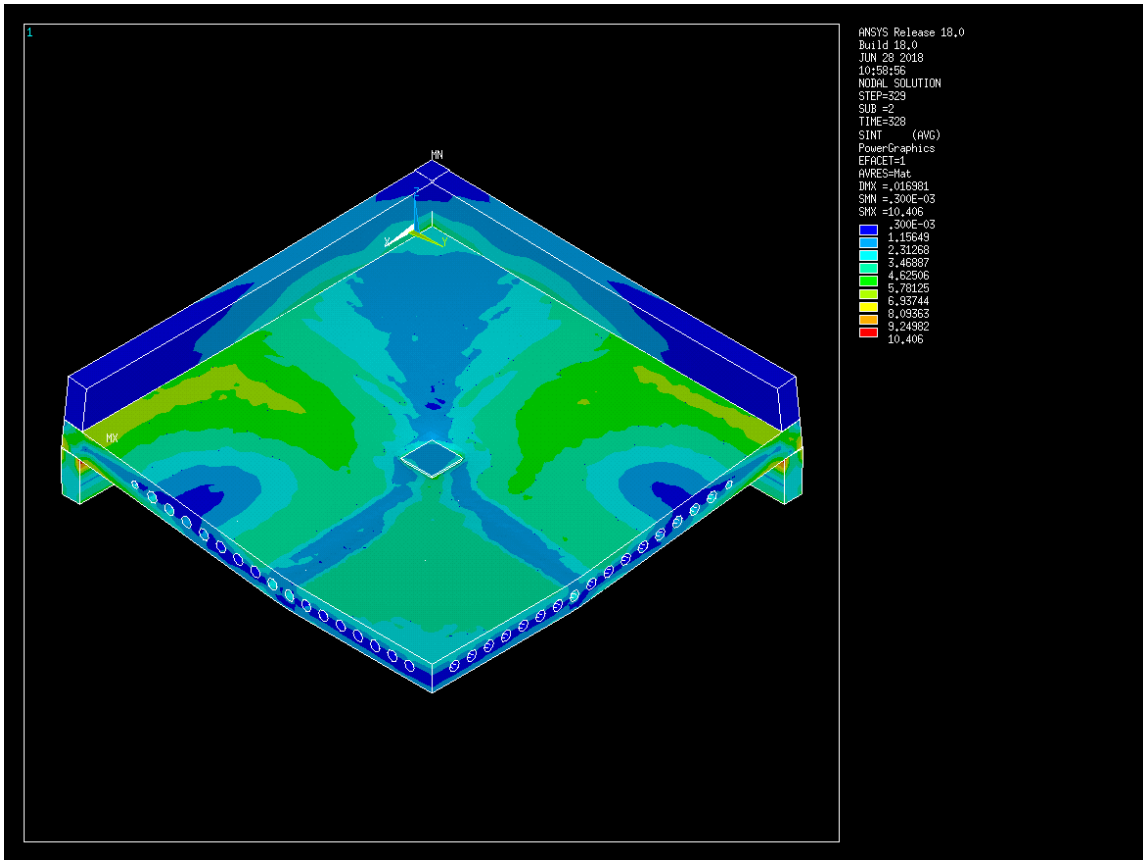


Figure 24: Top side, equivalent stress

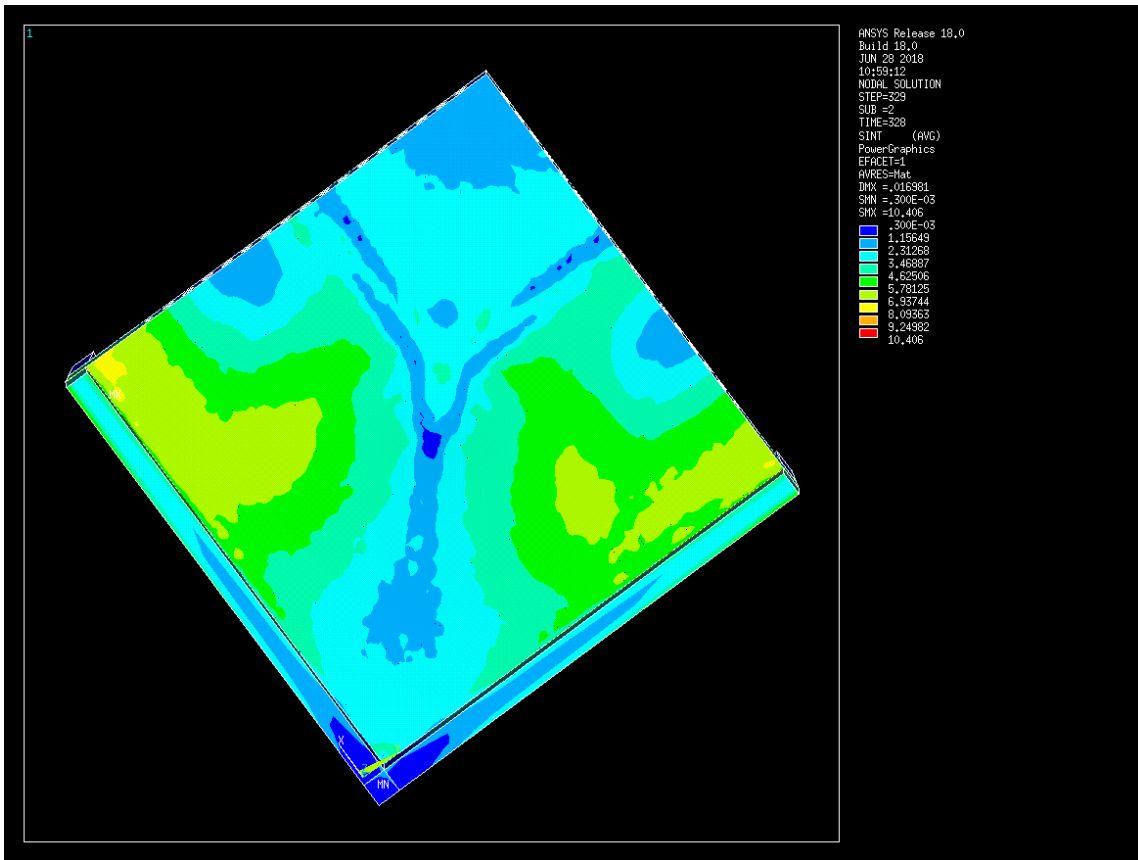


Figure 25: Lower side, equivalent stress

3. Quality of results

Thoughts and remarks

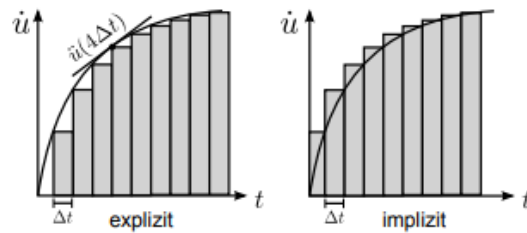


Figure 26: Explicit methods calculate the lower sum,
implicit methods deliver the upper sum ^[1]

Above figure shows the reduction of the global error in connection with reduced time step Δt ; smaller stripes will reproduce the curve more exactly. But with a higher amount of stripes, the numerical mistake will also increase – see next figure.

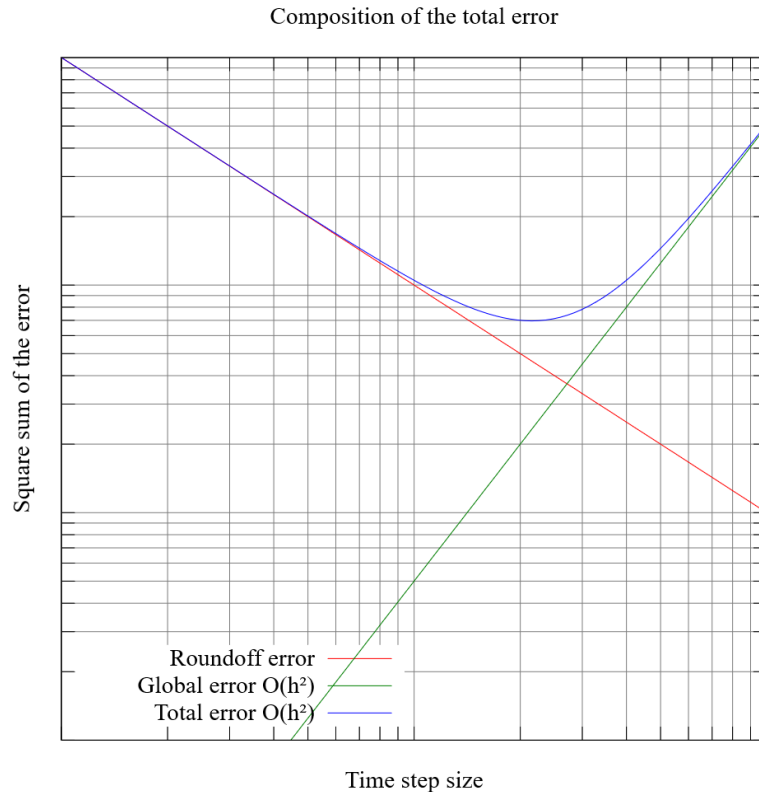


Figure 27: Total error as the sum of rounding error and global error / method error [2]

With the reduction of the global error for smaller time steps the rounding error will increase. Adding both errors will show an optimal time step size, where the total error will be minimized. From this follows that with a time step size chosen too small the result quality will get worse because of the increasing rounding error. [2]

This relationship should also be valid for the element size – with smaller element size the rounding error will increase while the global error/method error will decrease. Therefore too small element sizes will worsen the result quality because of the increasing total error.

4. Summary and outlook

With decreasing element size or deformation speed, the simulation gets closer to physical reality. Reducing these parameters over a certain level the force-displacement-curve did not get lower anymore and took place above the last curve – this behaviour has to be reviewed more exactly in future research projects. The optimal result quality with a minimized total error can be reached with certain values for

element size and deformation speed; these parameters have to be found. The exceeding of these optimal parameters has to be avoided; choosing an element size or deformation speed too small will cause a significant worsening of the total result quality. The origin of the abrupt worsening result quality should have its source within the increasing numerical error of the calculation; with decreasing element size or time step (deformation step/load step) the numerical error increases.

Choosing the discretization of geometry or time will depend on the system stiffness. For a short beam, the parameters have to be much smaller compared to a ceiling with larger span length; e.g. applicable evaluation criteria could be the slenderness ratio. Even within the implicit quasistatic analysis, the discretization of time by deformation step or load step has a big influence on the simulation result. Therefore the quasistatic simulation has to be seen as a time-dependent simulation.

Hypothesis:

The author formulates the hypothesis, that the convergence concerning the deformation speed (here: 0,1mm) results from the deformation speed of the 4-point-bending experiment. The experimental curve of the 4-point-bending test is time-dependent; the usage as the input of the material model transfers the damping behaviour of the concrete to the model. The equation of motion includes the velocity as the first derivative of movement. This means the only unknown variable in the model is the discretization of geometry / the element size.

More detailed information concerning the project „CC-technology“ can be found here:

https://www.researchgate.net/profile/Juergen_Ries

The references have been translated by the author.

References

1. ^a Simon Adler. (2014). *Entwicklung von Verfahren zur interaktiven Simulation minimal-invasiver Operationsmethoden. Fakultät für Informatik, Dissertation Uni Magdeburg*, <http://www.vismd.de/lib/exe/fetch.php?media=files:misc:adler2014.pdf> (Stand 17.04.2019)
2. ^{a, b} Ingo Berg. (2019). *Über die Genauigkeit numerischer Integrationsverfahren*. http://beltoforion.de/article.php?a=runge-kutta_vs_euler&hl=de (Stand 17.04.2019)