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Peer Review

Review of: "Inverse Evolution Data Augmentation for Neural PDE Solvers"

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This paper presents a novel data augmentation method based on inverse evolution for training neural operators on evolution PDEs. The proposed approach leverages insights from inverse processes to generate augmented data while maintaining consistency with implicit numerical schemes. The authors validate their method using Fourier Neural Operator (FNO) and UNet on various PDEs, including Burgers' equation, the Allen-Cahn equation, and the Navier-Stokes equation. The results show improved performance, robustness, and computational efficiency compared to traditional data generation methods.

While the paper is well-written and contributes to an important problem, there are several limitations and areas for improvement that should be addressed before resubmission. Below are my specific comments and suggestions:

Major Comments & Recommendations

1. Stability Issues in Inverse Evolution

- The method is noted to be **unstable** for sharp-interface problems and chaotic PDEs.

- While normalization and rescaling techniques are introduced, they modify the solution space. A more detailed discussion on how this affects generalization is needed.

- Can an **adaptive scheme** be designed to mitigate instability without altering the original solution space?

2. Clarify Initialization Strategy for Inverse Evolution

- The paper suggests using **randomly combined previous solutions** for initialization, but this lacks systematic justification.

- Would physics-based initialization strategies improve data quality?

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- More details on how initialization affects data diversity and training stability should be provided.

3. Scalability to High-Dimensional PDEs

- The method is validated on 1D and 2D PDEs, but its applicability to 3D problems is unclear.

- Can the approach scale efficiently for high-dimensional systems such as 3D Navier-Stokes turbulence?

- If testing in higher dimensions is computationally infeasible, a discussion of expected challenges and solutions would be valuable.

4. **Computational Overhead and Trade-offs**

- The paper claims inverse evolution reduces computational cost, but higher-order schemes require additional derivatives.

- A comparison of accuracy vs. computational cost should be included.

- Would semi-implicit methods further improve efficiency?

5. Comparison with Other Data Augmentation Methods

- The paper briefly mentions existing augmentation techniques (e.g., coordinate-based transformations) but does not compare them.

- A quantitative comparison with baseline augmentation methods would strengthen claims of superiority.

6. Potential Bias in Augmented Data

- The augmented data is noted to contain higher-frequency components than the original.

- Does this introduce bias into training, making models favor certain solution behaviors over others?

- A discussion on the effect of augmentation on solution diversity should be included.

7. **Impact on Other Neural Architectures**

- The method is tested on FNO and UNet, but its applicability to Physics-Informed Neural Networks (PINNs) or Transformers is unclear.

- Would inverse evolution augmentation benefit other architectures commonly used in PDE solving?

8. Lack of Formal Error Analysis for Augmented Data

- The empirical accuracy of generated data is validated, but a theoretical error analysis is missing.

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- How does inverse evolution data compare with true implicit solver solutions in terms of numerical error?

- Providing formal error bounds would enhance the rigor of the study.

9. Handling Chaotic PDEs and Long-Term Predictions

- Many evolution PDEs, such as Navier-Stokes turbulence, exhibit chaotic behavior.

- How does inverse evolution augmentation perform in long-term rollouts where small errors grow over time?

- A discussion on applicability to chaotic PDEs would improve the paper's relevance.

10. Improve Clarity of Figures and Tables

- Figure 1 and Figure 5 need clearer explanations regarding their significance.

- Table 1 and Table 3 present error values but lack confidence intervals or standard deviations.

- Adding error bars or variability measures would enhance statistical rigor.

Minor Suggestions

- Ensure all mathematical notations are consistently defined before use.

- Expand on potential real-world applications where inverse evolution augmentation could be impactful (e.g., climate modeling, fluid mechanics).

- Improve clarity in algorithm descriptions for easier reproducibility.

Final Recommendation

The paper presents a novel and promising augmentation technique, but stability concerns, initialization challenges, and computational trade-offs need further discussion. Addressing these points—especially regarding scalability, error analysis, and comparative studies—will significantly improve the paper's quality and impact.

I recommend **major revisions** before acceptance.

Declarations

Potential competing interests: No potential competing interests to declare.