Review of: "Trace anomaly Redefined in a Convention Leading to the Pontryagin Resolution"

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Potential competing interests: No potential competing interests to declare.

The thesis of the paper is interesting: it consists in showing that in order to compute chiral anomalies (in particular the trace Pontryagin anomaly), it is not necessary to impose the request of unitarity. I personally agree with the conclusion, but I must confess that I cannot follow all the passages and arguments the author uses: some of the symbols and equations are not clearly defined or explained; the language of the article may be familiar to mathematicians, but it is not very familiar to field theorists. I would suggest to the author a further effort, a more detailed exposition with more definitions and explanations (also, the English should be improved to avoid ambiguities).

For instance:

- last sentence of $\&$1: the sentence ``only the Pontryagin term contributes to the anomaly'' is not true; beside the odd Pontryagin term, there is also an even-parity contribution. The fact that the theory of chiral fermions is not parity conserving does not mean that there are no parity-even contributions to the trace anomaly. In fact, there is one.

- where does eq.(4, A1b) come from?

- could the γ^0 regularization be better explained?

- what is the operator T^{3-space}? And what is the last equation of the Appendix meant to represent?

- what is the meaning of the line before (5b)? Is anything missing there?

- ``the result of deWitt is correct'': what is deWitt's result?

- also, the bibliography should be improved (there have been various different derivations of the Pontryagin anomalies for Weyl fermions, beside [5,6]).

But, most importantly, I would like the author to clarify one point: is the request for unitarity not necessary or not correct? Because, if it is not necessary but correct, it does no harm to impose it.
Instead, I believe it is not correct. All physicists know that while calculating physical quantities, one must respect unitarity. But the critical point here is that some anomalies are not physical quantities. To be more clear, there are two types of anomalies: the dangerous ones and the non-dangerous or physical ones. The dangerous ones are, for instance, the chiral gauge anomalies in theories of Weyl fermions, those that have to be canceled in order to guarantee unitarity of the theories. These anomalies break unitarity, and the Pontryagin trace anomaly does the same.

The other anomalies, such as the ABJ anomalies in theories of Dirac fermions, are not dangerous, they respect unitarity, and can be used to describe physics (for instance, the decay of a π^0 into two gammas). Now, if we search for the first type of anomalies (those that may impair unitarity), we cannot impose unitarity, otherwise we will never find them. The first type of anomalies signals the existence of a contradiction between mathematical consistency (a correct mathematical derivation) and physical consistency (unitarity). It is obvious that if we impose the second, we will never be able to identify the sick theories, those that give rise to violations of unitarity. This is precisely what happens in relation to ref.[4]. The request for unitarity leads to a squared Dirac operator which is symmetric in the exchange γ_5 with $-\gamma_5$, so cannot detect any chiral anomaly of the first type (not only the Pontryagin trace anomaly). But one cannot cheat mathematics; in fact, such a request brings about a violation of diffeomorphism invariance. The truth is that these anomalies exist, they correspond precisely to obstructions to the existence of the fermion propagator, as was predicted long ago by the Atiyah-Singer family's index theorem (this issue is fully treated in the book by L. Bonora, ``Fermions and Anomalies in Quantum Field Theories," Springer, 2023).