Review of: "On the Application of the Rayleigh-Ritz Method to a Projected Hamiltonian"

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The main result in this paper is a numerical calculation of eigenvalues on an elementary example: the Schrodinger operator H without potential on a bounded interval with Dirichlet boundary conditions. The author first projects this operator onto the space of its D first eigenfunctions and calls H_D the projected operator. Then he computes the eigenvalues by performing Galerkin approximations of larger and larger dimension N associated with a nonorthogonal complete sequence of functions (u_j). The author notices that the eigenvalues obtained by the Galerkin approximations converge from below to the eigenvalues of H_D. He calls this a “most curious fact” and claims that it cannot be explained by the classical Raighley-Ritz principle since H_D is defined on a space of dimension D spanned by the D first eigenfunctions of H, while the u_j are not in this space but in a larger Hilbert space (the one in which H is defined). He also mentions the fact that usually, the Raighley-Ritz principle predicts convergence of eigenvalues from above, and here, the experimental fact is just the opposite. To me, there is no surprise: the explanation is the classical Raighley-Ritz principle applied to -H_D considered as an operator acting on the infinite-dimensional space on which H is defined. Indeed, if H_D is defined on this infinite-dimensional space, it admits O as an eigenvalue of infinite multiplicity: all the eigenvectors of H associated with the eigenvalues E_k, k>D are eigenvectors of H_D with eigenvalue 0. Thus, the positive eigenvalues E_1,…E_D of H_D are in fact its D largest eigenvalues, and below them, there are infinitely many eigenvalues equal to zero. Thus, I agree with the author that the Raighley-Ritz principle cannot be applied directly to H_D. However, it can be applied to -H_D. The D smallest eigenvalues of -H_D are negative and equal to -E_1,…-E_D. The Galerkin approximations of -H_D give negative eigenvalues -W_1,…,-W_D that approximate -E_1,…-E_D from above, as they should. This explains the observed convergence from below of W_k to E_k.