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Peer Review

Review of: "The Measurement of Superpositions"

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The author gives a very interesting analysis of the process of measurement of a physical quantity. First, he considers measurement as a relative process, i.e., comparing the quantity with a basic unit. When the unit is made smaller and smaller, the result of the measurement becomes more and more accurate. However, as correctly pointed out by the author, in quantum mechanics, the unit cannot be made arbitrarily small because of the Heisenberg uncertainty principle. This is, of course, true only when the measurement of two non-commuting observables like position and momentum is considered. This is one of the reasons why one cannot define the joint probability density of two noncommuting observables in a given state. Of course, a related phenomenon is that the joint moment generating function or the joint characteristic function of two noncommuting observables like two Pauli spin matrices will generally not have the positive definite property as required by Bochner's theorem in classical probability for the inverse Fourier transform to be a joint probability density. However, one can still define a joint Wigner density of two non-commuting observables in a state using the Wigner transform, and although this joint Wigner density would generally be complex, i.e., have a non-zero imaginary part, its first marginals would still be classical probability densities. The author then considers sums or time averages of independent identical measurements and, arguing by means of the law of large numbers, he says thus such time averages would have smaller and smaller variances and hence be less random and then compares this phenomenon to the wave function/state collapse in quantum measurement theory wherein, following a measurement, the state of the system collapses to the state corresponding to the measurement outcome and hence its variance reduces. However, in quantum mechanics, one can perform a measurement without noting its outcome, and then the resulting collapsed state would be obtained by applying a projective measurement to the original state, which would have more entropy than the state prior to the measurement. Perhaps such phenomena can be pointed out by the author. The

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author also correctly mentions the non-locality of quantum measurements based on the EPR problem wherein measurement of an entangled state at one end causes collapse of the state at the other end, which means that any measurement taken anywhere in an entangled network would cause the state at all the other points in this network to collapse and consequently affect the probability distribution of measurements taken at these other points. The author may also like to emphasize that, owing to the wave function collapse in quantum mechanics, it is meaningful in quantum mechanics to speak only of the probability distribution of sequential measurements taken in a specified order. In short, if two members sharing an entangled state make measurements in different orders, the joint probability distribution of their measurements would be different. This is similar to saying that the product of two or more non-commuting spectral measures is generally different if the order in which these spectral measures are multiplied is permuted. The joint probability distribution of N observables in a given joint state is simply obtained by multiplying N spectral measures in a given order with the joint state and then taking the trace. This trace would change if the order of multiplying these N spectral measures is changed, provided that they do not all mutually commute. Another interesting aspect of measurement pointed out by the author is based on Cantor's theory of infinite sets. Specifically, Cantor suggested that one can give meaning to orders of infinity. For example, the set of integers and natural numbers have the same cardinality, but this cardinality differs from that of the unit interval on the real line. In other words, one can construct an injective map from the set of integers or natural numbers into the unit interval, but not from the unit interval into the set of integers or natural numbers. The set of all infinite binary sequences has the same cardinality as the unit interval, but the set of all binary sequences that have only a finite number of ones and the remaining zeros has the same cardinality as the set of integers or natural numbers. Now, the set of all binary sequences of finite length N is in one-to-one correspondence with the set of all subsets of a set having N elements. However, when one admits infinite binary sequences of zeros and ones with ones occurring at arbitrarily large places, then to describe the notion of cardinality amounts to including objects like the set of all sets. In other words, if a set E is countably infinite, then the sequence of all finite subsets of this set is again countable, but the set of all infinite subsets of this set is not countable. Some light must be thrown on the importance of this idea in measurement theory. I think one way to see its importance is in measure theory, wherein there exist uncountably infinite sets of measure zero on the real line. When we throw off the middle third of the unit interval and then successively throw off middle thirds of each remaining interval, then after a countably infinite number of such steps, we arrive at the Cantor set consisting of real numbers in the unit interval which are ternary expansions with only zero or two occurring, with two occurring at arbitrarily large places. Such numbers

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have Lebesgue measure zero, but they can obviously be put into a one-to-one correspondence with infinite binary sequences, and hence this set of measure zero has uncountable cardinality.

I think that the author, while discussing measurements in superpositions of pure quantum states $\psi_1+\dots+\psi_n$, should emphasize that the probability of measuring a given by $|\langle a|\psi_1+\dots+\psi_n\rangle|^2$ gives interference terms as in Young's n-slit experiment, thereby violating the law of total probabilities in classical probability theory. This violation makes it impossible to detect which slit the particle on the screen arrived from. Moreover, the original way that Heisenberg's uncertainty principle was explained was based on the interference pattern in Young's double-slit experiment, namely, the interference pattern generated on the screen combined with the fact that the effective resolution limit was the distance between an intensity maximum and an intensity minimum, which was given by $\lambda/2$, where λ was the wavelength of light. Combined with De-Broglie's matter wave duality, this could be used to provide an experimentally oriented theoretical proof of the Heisenberg uncertainty principle.

Cantor's theory implies that if we can construct projective limits of sequences of sets wherein the projective limit is an infinite sequence of increasing sets E_n, $n=1,2,\dots$, then if our units of measurement were to be made smaller and smaller in order to improve the accuracy of the measurement, we could give meaning to measurement at each finite resolution using the projection of the limiting set onto each E_n. Even though we cannot define infinitesimally small resolutions, we can give mathematical meaning to the limit as the projective limit. Specifically, if for each $l>m$, we have a projection $p_{lm}:E_l\longrightarrow E_m$, that satisfies the consistency condition $p_{mk}\circ p_{lm}=p_{lk}$ for $l>m>k$, then we can define the inverse limit E with projections $p_l:E\longrightarrow E_l$, $l=1,2,\dots$, in a consistent way. This enables us to determine measurements at all resolutions using the limiting set E having the smallest resolution.

In quantum theory, however, for commuting observables, we can apply the projective limit theory based on Cantor's ideas, but for non-commuting observables, owing to Heisenberg uncertainty, we cannot do so; indeed, we cannot resolve two noncommuting observables to arbitrarily small levels.

The probability of the particle being in the state a given that the state is the superposition

$\psi_1+\dots\psi_n$ is given by $|\langle a|\psi_1+\dots+\psi_n\rangle|^2=|\langle a|\psi_1\rangle|^2+\dots+\langle a|\psi_n\rangle|^2+2\sum_{k<l}\mathrm{Re}(\langle a|\psi_k\rangle\langle a|\psi_l\rangle^*)$. The second sum is the interference term that is a purely quantum phenomenon. For small wavelengths compared to the linear dimensions of the apparatus, this interference term is small. This can be verified by expressing the ψ_k's as plane waves of definite wavelength and showing that the interference term spatially averages out to zero very fast using an idea similar to the vanishing of the Fourier transform of an integrable function at large frequencies.

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Declarations

Potential competing interests: No potential competing interests to declare.