Research Article

# Parameter Calibration for Johnson Cook and Preston-Tonks-Wallace Material Strength Models with Uncertainty Quantification

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**In this study, we perform a Bayesian calibration of the parameters in the Johnson Cook (JC) material strength model, with uncertainty, using experiments for a range of low and medium strain rates.**
**For the parameter calibration, we used a variational Bayesian approach with experimental data from Hopkinson bar and quasi-static tests conducted on Oxygen Free High Conductivity (OFHC) copper. The estimated parameter values matched well with experimental data for a modest range of strain rates and temperatures. Through this method, we also recovered uncertainty and correlation information for the estimated parameter values.**
**We also compared our results to a calibration of parameters in the Preston-Tonks-Wallace (PTW) material strength model, using the same variational Bayesian method. The parameters estimated for both models provided good agreement with the experimental data.**

## Models

### *Johnson-Cook*

The Johnson Cook (JC) material strength model predicts the relationship between stress, strain, strain rate, and temperature.

$$\sigma = (A + B\varepsilon^n)(1 + C\ln\dot{\varepsilon}^*)(1 - T^{*m})$$

 doi.org/10.32388/755I45 

$$
\begin{aligned}
\sigma &= \text{stress} \\
\varepsilon &= \text{equivalent plastic strain} \\
\dot{\varepsilon}^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} &= \text{dimensionless plastic strain rate for } \dot{\varepsilon}_0 = 1\text{s}^{-1} \\
T^* = \frac{T_{test} - T_{ref}}{T_{melt} - T_{ref}} &= \text{non-dimensional temperature} \\
A &= \text{parameter, the yield stress} \\
B &= \text{parameter related to the effect of strain hardening} \\
C &= \text{parameter, the strain rate constant} \\
n &= \text{parameter related to the effect of strain hardening} \\
m &= \text{parameter related to the effect of thermal softening}
\end{aligned}
$$

It is a relatively simple model that has been implemented in various codes and has many existing calibrations.

### *Preston-Tonks-Wallace*

The Preston-Tonks-Wallace (PTW) material strength model predicts the relationship between stress, flow stress, yield stress, strain, strain rate, temperature, and density. It also uses sub-models for melt temperature and shear modulus. For additional details on the PTW model form and variables, see[1] and[2].

This is a newer model with more degrees of freedom than the Johnson-Cook. It also has not been implemented in as many codes yet and does not have as many available calibrations.

## Experimental Data

We used data from eleven Hopkinson bar experiments and five quasi-static experiments for the calibration. The test temperatures range between 77 K and 873 K, where the Hopkinson bar experiments were conducted at between 293.15 K and 296.15 K. The strain rates captured in this set of experiments range from 0.001 s-1 to 9000 s-1, or log 10 strain rates ranging from -3 to 4.

## Bayesian Calibration

To identify the optimal model parameters given the experimental data, we used a variational Bayesian method. The method assumed multivariate normal distributions for the priors and likelihood. The means of the priors were set to previously calibrated values of the parameters for each model (see below), and the variance was assumed to be uncorrelated and 10% of the prior mean value. The posterior mean value is estimated from the variational Bayesian method, and the covariance comes from a Laplace approximation. This method also allows for the implementation of constraints on the parameter values. For the Johnson-Cook model, we enforced that the parameters be positive. In the Preston-Tonks-Wallace model, we required that $\theta > 0$, $\kappa > 0$, $y_o > y_\infty > 0$, $s_o > s_\infty > 0$, $s_o \geq y_o$, and $s_\infty \geq y_\infty$[3]. For additional details on the method, see[4].


doi.org/10.32388/755I45

### *Johnson-Cook*

For the mean of the prior distributions, we used the parameter values for OFCH copper from the original Johnson-Cook paper[5]:

$$\begin{aligned} A &= 90 \text{ MPa} \\ B &= 292 \text{ MPa} \\ C &= 0.025 \\ n &= 0.31 \\ m &= 1.09 \end{aligned}$$

We also specified a melt temperature of 1356 K[6] and a reference temperature of 293 K.

### *Preston-Tonks-Wallace*

For the mean of the prior distributions, we used the parameter values from the original paper by Preston, Tonks, and Wallace[1]:

$$\begin{aligned} p &= 2.0 \\ \gamma &= 0.00001 \\ \theta &= 0.025 \\ s_o &= 0.0085 \\ s_\infty &= 0.00055 \\ y_o = y_\infty &= 0.00011 \\ \kappa &= 0.11 \\ y_1 &= 0.094 \\ y_2 &= 0.575 \\ \beta &= 0.25 \end{aligned}$$

We also specified the following constant model parameters:

$$\begin{aligned} \rho_0 &= 8.993 \text{ g cm}^{-3} \\ M &= 63.54 \text{ g mol}^{-1} \\ \nu &= 0.23 \\ G_0 &= 518 \text{ kbar} \\ \alpha &= 0.2 \\ a &= 1.0 \\ T_{melt} &= 1359 \text{ K} \end{aligned}$$

## Results

We found the optimized parameters for the JC model to be:

$$\begin{aligned} A &= 0.0499 \text{ MPa} \\ B &= 688.53 \text{ MPa} \\ C &= 0.039 \\ n &= 0.636 \\ m &= 0.964 \end{aligned}$$

The three dimensionless parameters, $C$, $n$, $m$ all stayed about the same as their prior values, thus the sensitivity to strain, strain rate, and temperature stayed about the same. The two dominant parameters, $A$, $B$,


doi.org/10.32388/755I45

changed more significantly. The additive parameter $A$ went down by three orders of magnitude, whereas $B$, the multiplicative term on the strain, more than doubled. The parameter $A$ is related to the yield stress of the material, which for OFE copper is 69-365 MPa[7]. This smaller posterior value of $A = 0.05$ MPa falls far outside the range of yield stress values for OFE copper and is less physically reasonable compared to the prior value of 90 MPa. However, it contributes to a better fit to the experimental data than the prior values.

The uncertainty in all parameter estimates reduced from the initial 10% assumption. The highest uncertainty is in $B,\ C$, at about 1.4%, and $A$ has the lowest uncertainty at about 0.2% (Figure 1).

Focusing on the correlation of the JC parameters (Figure 2), we see that $B$ and $n$, the two parameters on the strain term, are highly positively correlated. There is also a strong positive correlation between $A$ and $n$, the two terms relating to the strain effects. Finally, there is a strong negative correlation between $B$ and $C$, the two multiplicative parameters, and $B$ and $m$, the multiplicative parameter on strain and the temperature sensitivity.

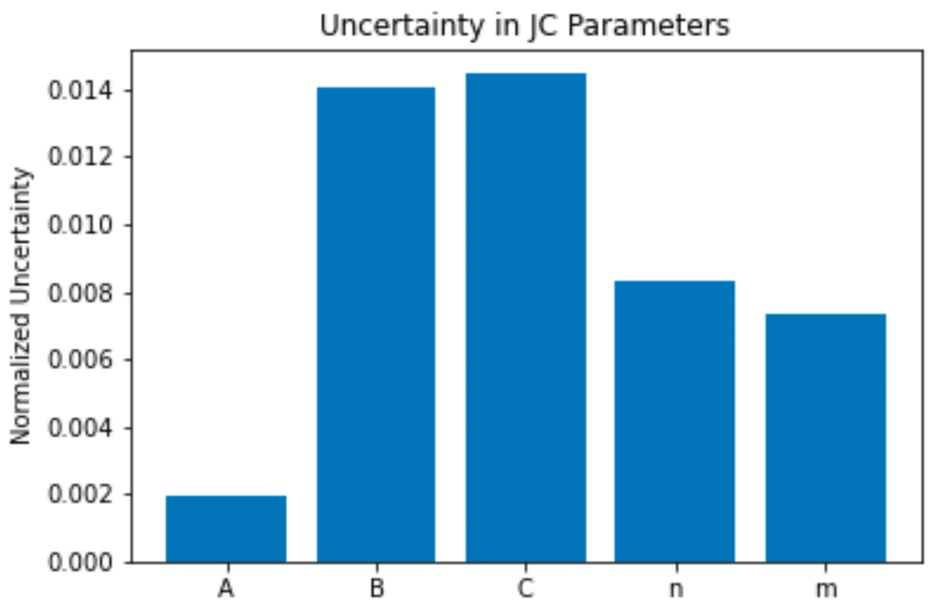


**Figure 1.** Uncertainty in JC parameters

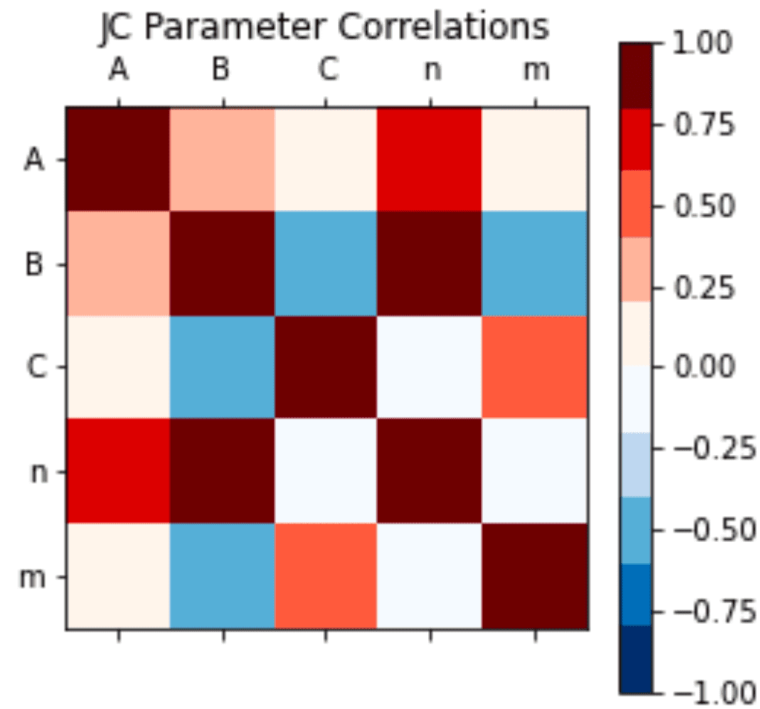


**Figure 2.** Posterior correlation matrix for JC parameters

To compare the performance of the JC and PTW models, we look at how well the models capture the behavior of the experimental results with the prior and posterior parameter values. In the quasi-static case (Figure 3), the priors for both the JC and PTW models do not match the experimental results well. However, both models do a better job with the posterior parameter values. The PTW model does slightly better at matching the experimental data, but this is expected since it is a more complicated model with more degrees of freedom, and the JC model still captures the general behavior. In the Hopkinson bar case (Figure 4), the prior parameter values for the JC model do not capture the experimental results well, but the posterior values provide good agreement. With the PTW model, both the prior and posterior values match the experiments well, but the posteriors still provide an improvement over the priors.


doi.org/10.32388/755I45

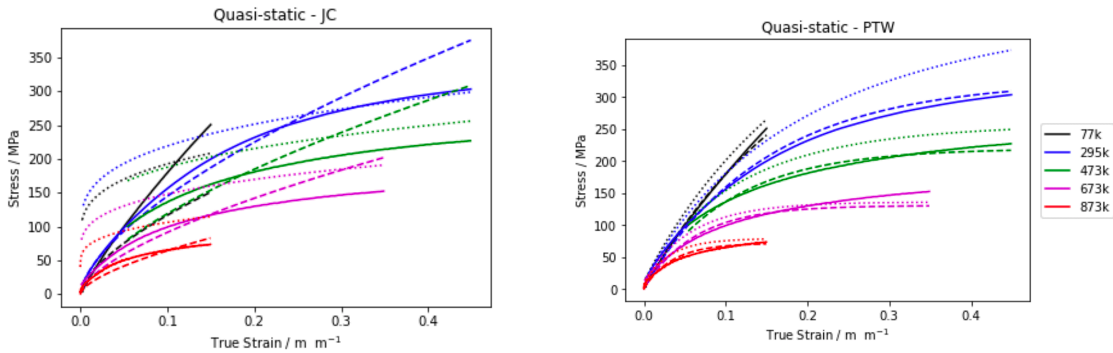


**Figure 3.** Quasi-static results. Solid lines are experimental data, dotted lines are priors, dashed lines are posteriors

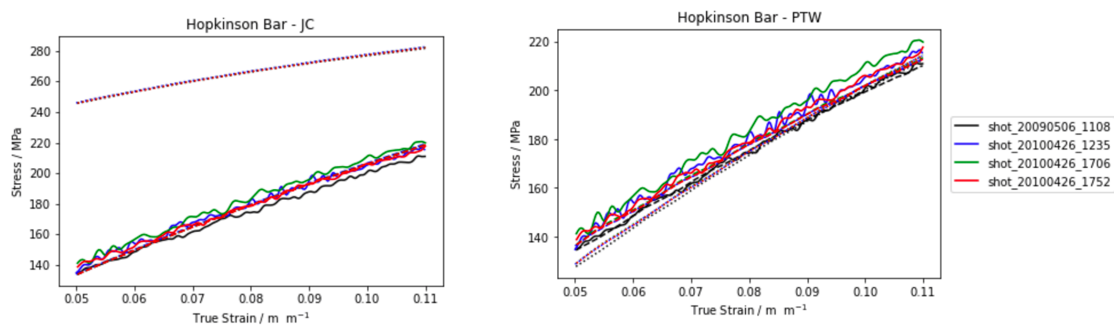


**Figure 4.** Selection of Hopkinson bar results. Solid lines are experimental data, dotted lines are priors, dashed lines are posteriors. Shots *20090506-1108* and *20100426-1235* were conducted at 293.15 K, and shots *20100426-1706* and *20100426-1752* were conducted at 296.15 K.

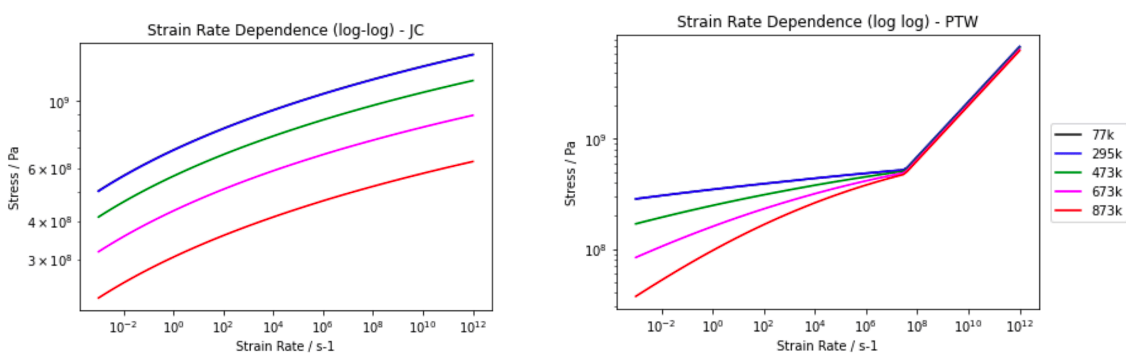


**Figure 5.** Stress at various strain rates, for five temperatures. Strain = 1


doi.org/10.32388/755I45

In Figure 5, we can see that the PTW model does a better job of capturing the strain rate effect than the JC model, particularly in the range of strain rates captured by the experiments used in the calibration, between Log 10 -3 and 4.

## Results Removing 77k Quasi-static Test

In the quasi-static data, the 77 K experiment is more linear and has a steeper slope than the other tests at higher temperatures (Figure 3). This difference in behavior could impact the parameter calibration, so here we repeat the calibration for the JC and PTW models, omitting the 77 K quasi-static test.

The optimized parameters for the JC model are

$$\begin{aligned} A &= 6.201 \text{ Mpa} \\ B &= 547.799 \text{ Mpa} \\ C &= 0.047 \\ n &= 0.573 \\ m &= 1.091 \end{aligned}$$

Like the previous calibration, the parameters $C,\ n,\ m$ did not change significantly from the prior values, but $A,\ B$ did. In this case, $A$ went down by two orders of magnitude, and $B$ almost doubled. This posterior value of $A$ still does not fall in the range of known yield stresses for OFE copper (69-365 MPa[7]), but it is closer than the previous calibration.

The uncertainty in all parameter estimates reduced from the initial 10% assumption. The highest uncertainty is in $C$, at about 3.0%, and $A$ has the lowest uncertainty at about 0.5% (Figure 6). Overall, the uncertainties are higher here than those in the previous calibration.

The correlations in the parameters changed in magnitude from the previous calibration (Figure 7). There is still a positive correlation with $B$ and $n$, but it is not as strong. This positive correlation is as strong as the positive correlation between $C$ and $m$. The negative correlations between $B$ and $C$, and $B$ and $m$ are still present, but not as strong compared to the previous calibration.


doi.org/10.32388/755I45

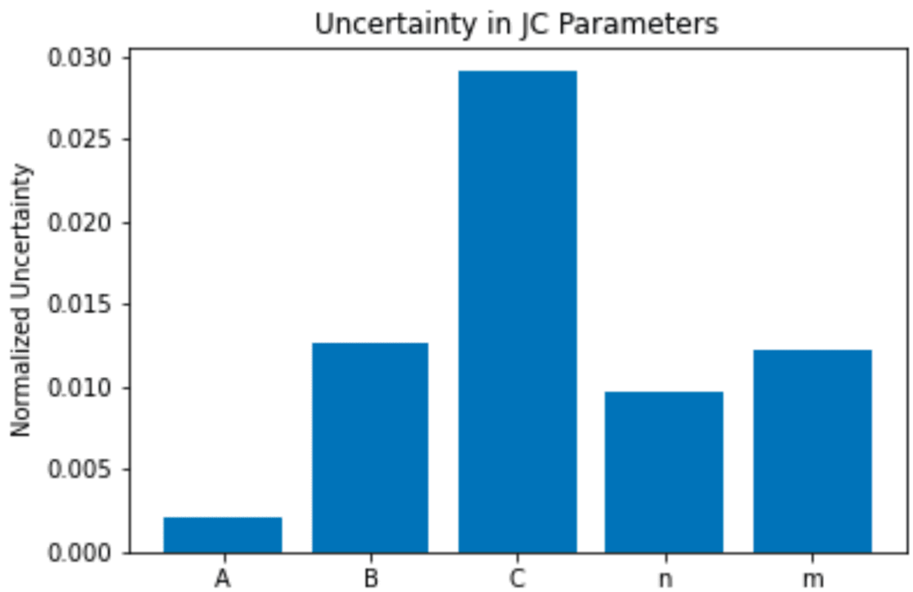


**Figure 6.** Uncertainty in JC parameters, omitting 77 K

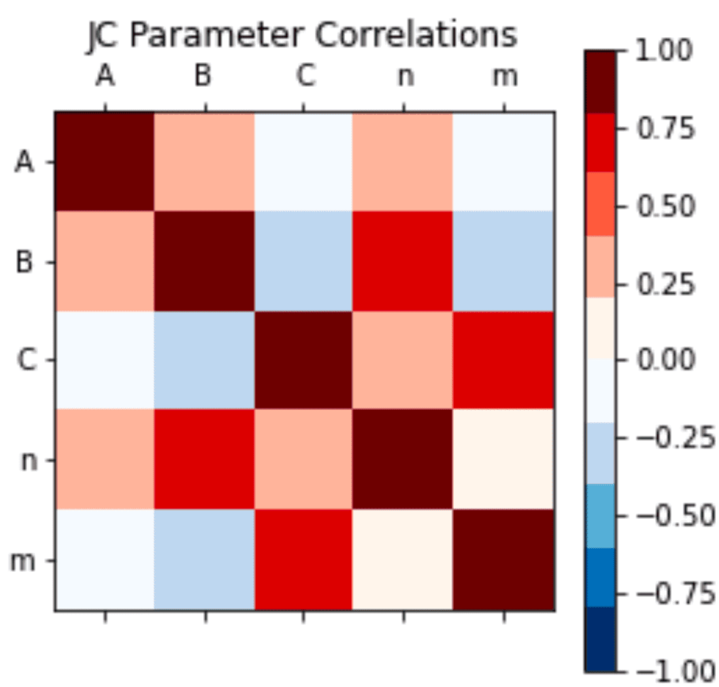


**Figure 7.** Posterior correlation matrix for JC parameters, omitting 77 K


doi.org/10.32388/755I45

As before, both the JC and PTW models match the experimental data better with the posterior parameter values than with the prior values (Figures 8 and 9).

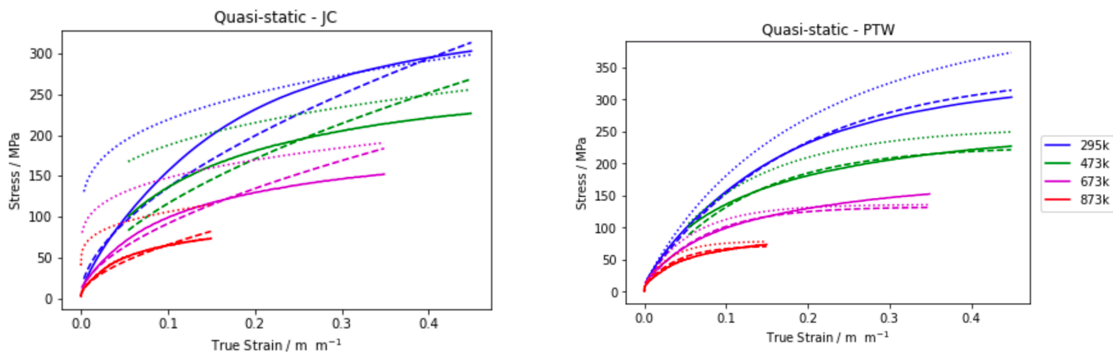


**Figure 8.** Quasi-static results for calibration omitting 77 K. Solid lines are experimental data, dotted lines are priors, dashed lines are posteriors

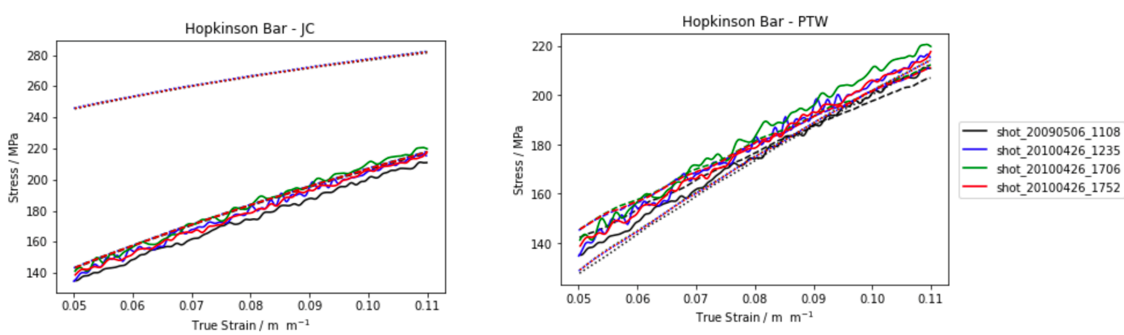


**Figure 9.** Selection of Hopkinson bar results for calibration omitting 77 K quasi-static test. Solid lines are experimental data, dotted lines are priors, dashed lines are posteriors. Shots *20090506-1108* and *20100426-1235* were conducted at 293.15 K, and shots *20100426-1706* and *20100426-1752* were conducted at 296.15 K.


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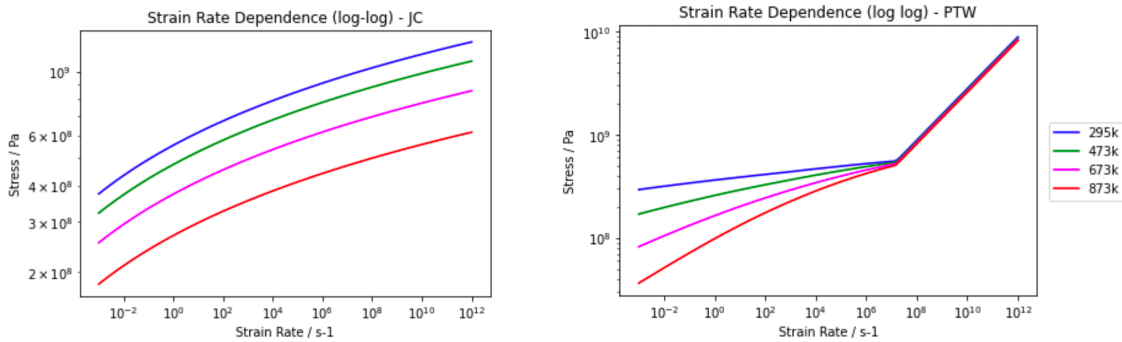


**Figure 10.** Stress at various strain rates, for four temperatures. Strain = 1

Like the previous calibration, in Figure 10 we can see that the PTW model does a better job of capturing the strain rate effect than the JC model.

## Conclusion

The calibrations provided here for the Johnson-Cook model match experimental data well for a modest range of temperatures and strain rates. We also captured uncertainty and correlation information for the model parameters.

For the best agreement with the experimental data, one should use the PTW model; however, we found calibrations for the JC model that give qualitatively good agreement with the experimental data. Thus, if the application requires a simpler model than PTW, JC can also be used.

## Statements and Declarations

### *Acknowledgments*

This work was supported by the U.S. Department of Energy through the Los Alamos National Laboratory. Los Alamos National Laboratory is operated by Triad National Security, LLC, for the National Nuclear Security Administration of the U.S. Department of Energy (Contract No. 89233218CNA000001).

Approved for public release LA-UR-22-29965.

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## Declarations


**Funding:** U.S. Department of Energy’s Advanced Simulation and Computing (ASC) program. This work was performed at the Los Alamos National Laboratory. Los Alamos National Laboratory is operated by Triad National Security, LLC, for the National Nuclear Security Administration of U.S. Department of Energy (Contract No. 89233218CNA000001).


**Potential competing interests:** No potential competing interests to declare.