

Review Article

Visualization of Graphic Data Allied to Nomographs: Surface Plots, Contour Maps, Slices, Projections

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The classical nomograph is a two-dimensional plot that expresses a three-variable equation such as the van der Waals equation for a non-ideal gas which can also be represented in three dimensions. Nomographs are useful for visualising and communicating the behaviour of equations. Traditionally they were used for calculation but this use has become limited with the development of capable computers. They are, however, currently re-appearing in educational contexts and for time-critical data use.

The nomograph is principally used to obtain numeric values from the graphical construction which are otherwise difficult to determine. We show how the projections of van der Waals equation onto planes such as pV (pressure–volume) or pT (pressure–temperature) can be used to represent this equation; other cubic equations will be similarly related to such nomographs.

While traditional methods of preparation of nomographs require considerable expertise, artificial intelligence software focused on coding, such as Claude Sonnet, is now capable of creating nomographs from simple prompting as here demonstrated.

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Introduction

Nomographs (also known as nomograms, alignment charts, or even abagues/abacs – words related to ‘abacus’ – are graphical devices where curves on a 2D surface relate three variables^{[1][2][3]} in an equation. It is the purpose of the current manuscript to relate the various common projections of 3D graphic data to the 2D nomograph.

Fig. 1 is a modern nomograph for van der Waals equation of state^[4] for one mole of carbon dioxide. The outer scales, or axes, denote pressure, p (atmospheres), and temperature, t (Celsius) while the interior curve denotes volume, v (litres). It was prepared by Claude Sonnet 4.6 with reference to Fig. 7 of P. Martínez Pagán, and L. Roschier (2022). "Nomography: A renewed pedagogical tool to sciences and engineering high-education studies." *Heliyon*, 8(6), e09731.^[4]

A straight-edge (termed an isopleth) laid across the scales connects the values at one or three crossing points, two of which then represent the values of the independent variables while the third (or single) is regarded as the value of the dependent variable since this is a symmetric relation among the variables. The data along the scales may be mathematical or based upon experiment.^{[1][5]}

The example isopleth in red in Figure 1 shows that when pressure is 4 atm. and temperature is 50°C, the volume is 0.0248 L/mol.

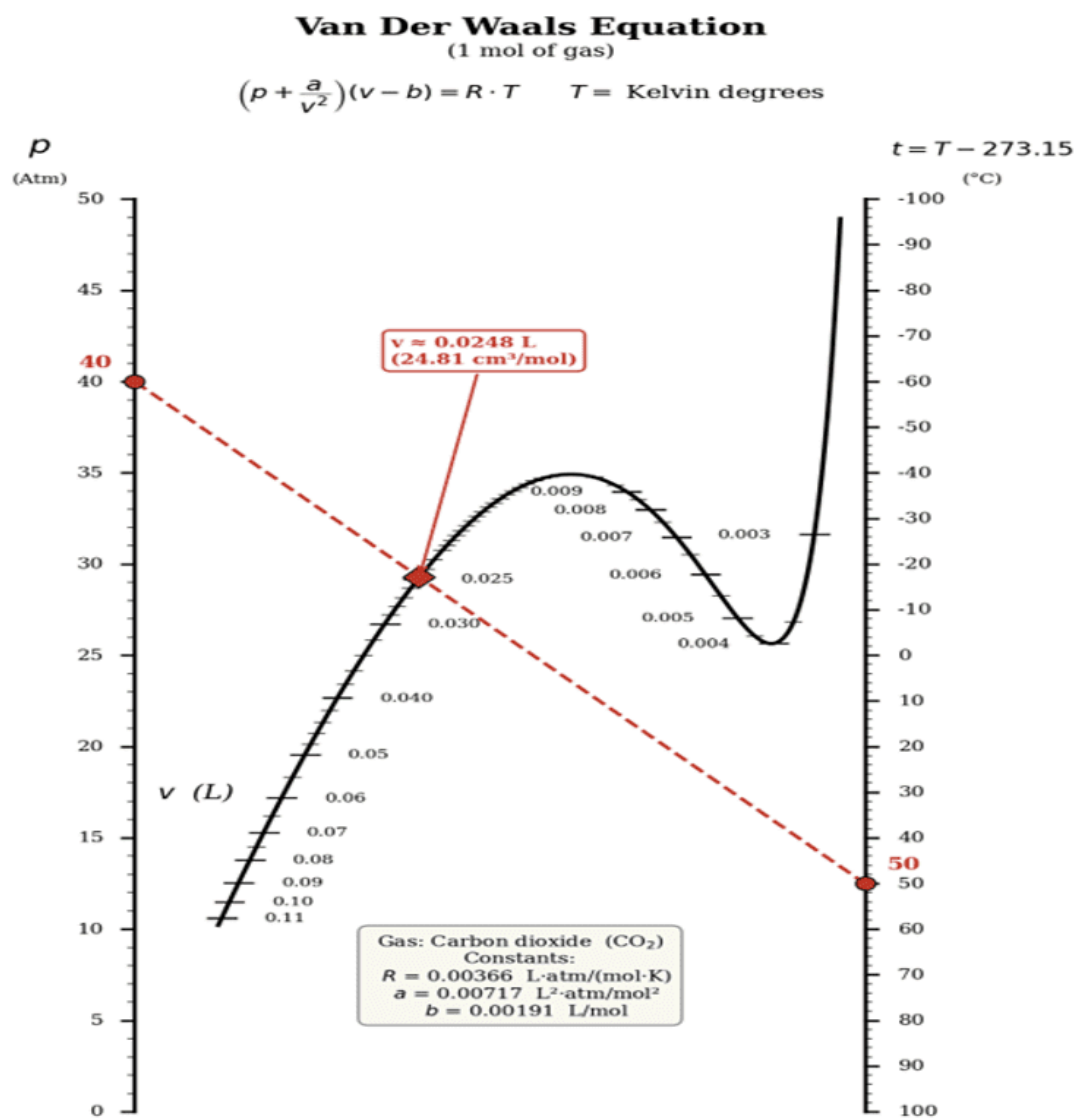


Figure 1. A nomograph of an experiment-based equation of state for carbon dioxide gas.^[6] the red isopleth connects pressure and temperature to molar volume.

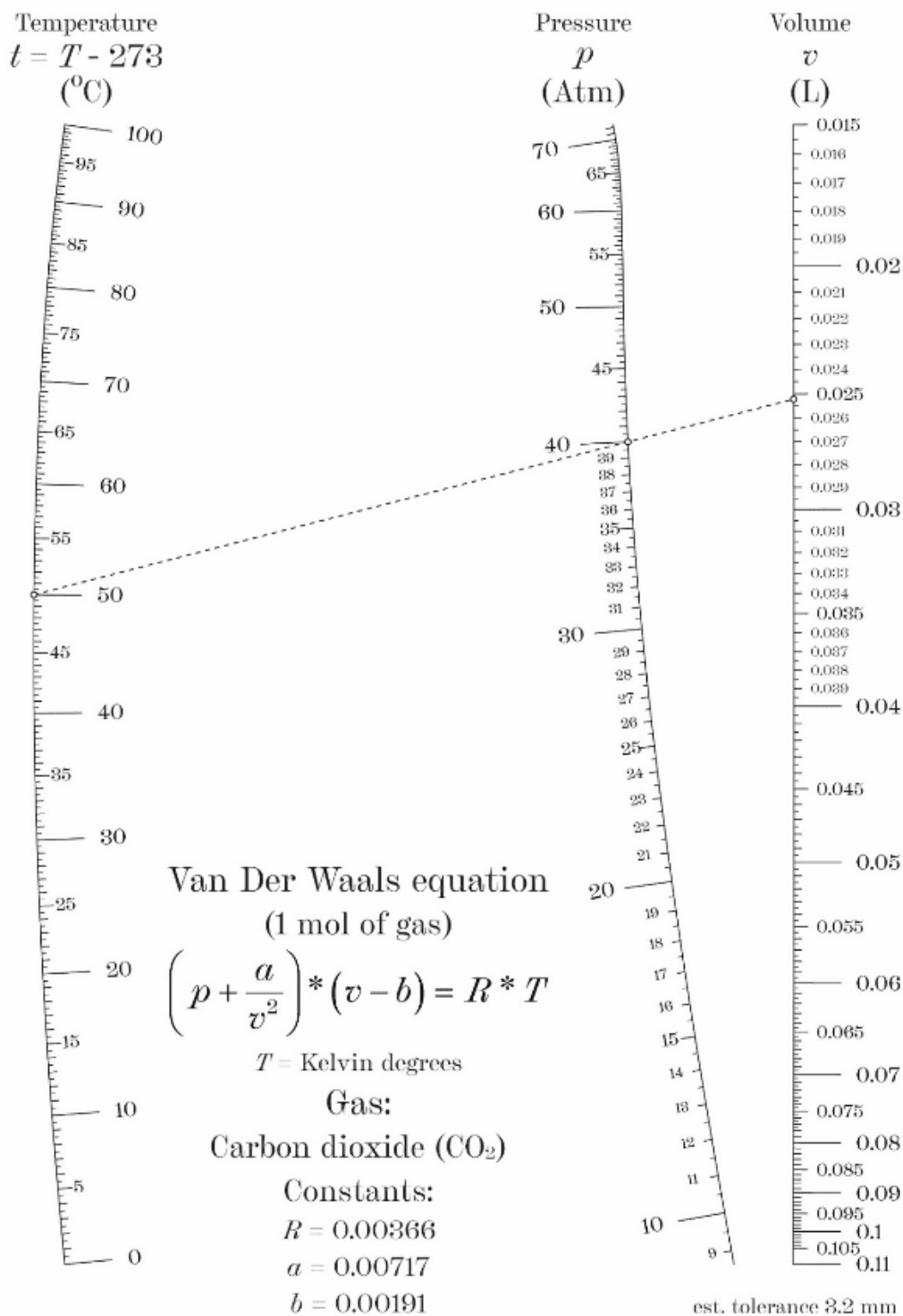


Figure 2. A linearized portrait format version of the van der Waals equation of Fig. 1 suitable for easy reading and interpolation.

Nomographs (named and much applied by Maurice d'Ocagne^[7] from 1884) were important tools in many branches of science, engineering, even in finance, from the 1880's until about the 1960's when they were largely superseded by the burgeoning computing devices.^[8] However, they remain useful and important for their immediate access and rapid application with little chance of serious scale error^{[9][10]} as well as for their demonstration of the relations among the variables.

Early nomographs were constructed in a tortuous manual process^{[11][12][13][14][15][16][17][18]} by laying out the initial pair of scales on coordinate paper then triangulating points successively to produce the third scale. The triangulation consists of drawing isopleths for at least three pairs of the independent variables for a common dependent variable and then placing the dependent variable at the crossing point. With experimental data rather than an explicit mathematical relationship the crossings may cover a small area rather than a single point, when the area centre is used for the dependent variable. The dependent scale then follows the set of selected crossing points and is progressively refined to a smoothed curve. This complex procedure is no longer required since two important Python programs are now available for nomograph preparation: Nomogen^[19] is the automation tool which develops the equations for the scales while Pynomo^{[20][21][22][23]} generates the vector graphic PDFs of the nomographs.

There appears to be increasing recognition of the utility of nomographs in education,^{[4][19][24][25]} because of the simplicity of reading of numeric data. However, the graphic relation of the nomograph to the generating equation may not be clear.

It should be noted that the so-called "clinical nomographs" are not for three-variable systems but rather are used for statistical summations and hence not further discussed here.

Graphic Views and Dimensional Reduction

Graphic views of experimental data and in coordinate space are essential components of science and related subjects.^[26] Indeed, J. Willard Gibbs, the doyen of thermodynamics regarded graphic views as important considerations in thermodynamic treatments;^[8] his first essay is entitled "Graphical Methods in the Thermodynamics of Fluids" while the title of his second is "A Method of Geometrical Representation of the Thermodynamic Properties of Substances by Means of Surfaces."

In current software, textbooks and the literature three-variable equations are usually provided as projections along orthogonal axes and as contour maps. As an example, Fig. 3 shows a map of the van der Waals equation^[27] as the projection of the isotherms in the vicinity of the critical point onto the

pressure-volume (pV) plane. The red isotherm represents values of the independent variables at the critical temperature and the critical point, where this temperature intersects both the critical pressure and volume, is labelled D. The curved section labelled A – B – C represents the Maxwell construction to determine the one- to two-phase transition.^{[28][29]} from supercritical gas to gas plus equilibrium liquid.

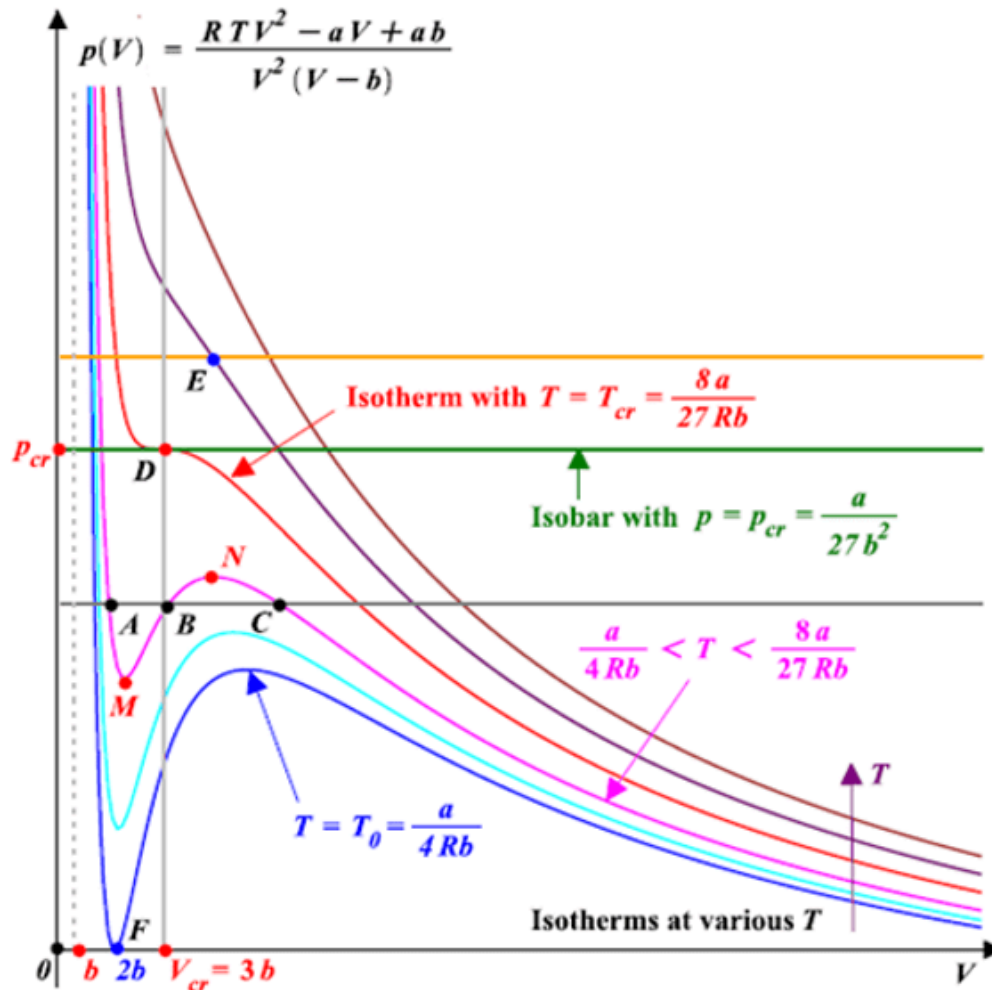


Figure 3. Projections of the van der Waals isotherms which are slices of the equation of state for various temperatures^[30] above, at, and below the critical temperature. Republished under Open Access Licence “CC BY 4”.

Equation for an isotherm:

$$p(v, T) = \frac{RT}{v - b} - \frac{a}{v^2} = \frac{RTv^2 - av + ab}{v^2(v - b)}$$

The loops generated by the van der Waals equation (such as M - B - N) are unphysical (i.e., they exist mathematically but not physically) representing increasing pressure yielding increasing volume. In the nomogram in Fig 1, this corresponds to the reverse slope of the central curve where the volume ranges from 0.004 to 0.008 litres per mole.

In Fig. 4, we note that these loops relate to the liquid-gas phase relations in the van der Waals system. The spinodal curve^[31] encloses the two-phase dome of liquid plus gas, connected across the region by Maxwell tie lines. The spinodal curve is determined alternatively by mathematical and thermodynamic relations:

(i) the spinodal points are the local maxima and minima of the pressure-volume isotherm where

$$\left(\frac{\partial p}{\partial v} \right)_T = 0;$$

(ii) Alternatively, the spinodal points are where the second derivative of the Helmholtz free energy, A , changes sign: $\frac{(\partial^2 A)}{(\partial x^2)} = 0$ at the spinodal point, and the single-phase system becomes unstable, separating

into equilibrium liquid and gas phases. The equation of the spinodal curve is $T_{spin} = \frac{2a(v-b)^2}{Rv^3}$.

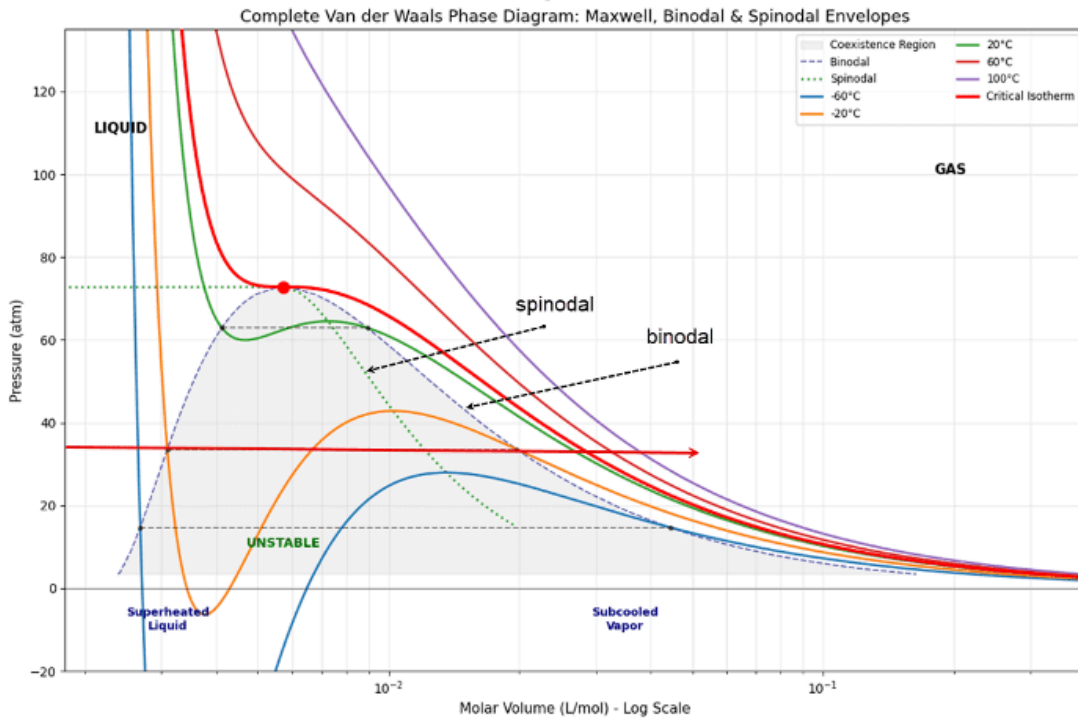


Figure 4. Projection onto the pressure-volume plane for a range of temperatures of the van der Waals CO_2 equation used for Fig. 1. The critical isotherm, based on an inflection point of the isotherm, is dark red, and the critical point is marked with a red dot. The spinodal curve surrounds the limit of the two-phase gas-liquid region while the outer binodal curve is the limit of the equilibrium state of the system. The area between these two curves represents metastable conditions.

The binodal dome^[32] surrounding the spinodal dome represents the region of stability in terms of the Gibbs free energy when G is negative. The curve may be determined by using the Maxwell equal area criterion.^{[30][33][34]}

Critical Values of the Van der Waals Equation constants using^[6] $R = 0.00366 \text{ L atm K}^{-1} \text{ mol}^{-1}$

In Fig. 5, we reproduce a landscape format van der Waals equation nomograph^[4] to be compared with the portrait format version of Fig. 1. The basic diagram is identical to that of Fig. 7 of the earlier mentioned publication.^[4] The critical isopleth is shown in red and it is notable that this line meets the volume scale at a tangent. This corresponds to the convergence of the gas and liquid properties, corresponding to top of hill in a pressure-enthalpy diagram. (https://people.ohio.edu/urieli/thermo/property_tables/CO2/ph_CO2.html).

A fan of constant pressure isopleths is here added to Fig. 5: that is, each isopleth starts from the same pressure with successive isopleths rotating towards increasing temperature, t . The sequence of intersections with the central curve represent increasing volume values as temperature rises.

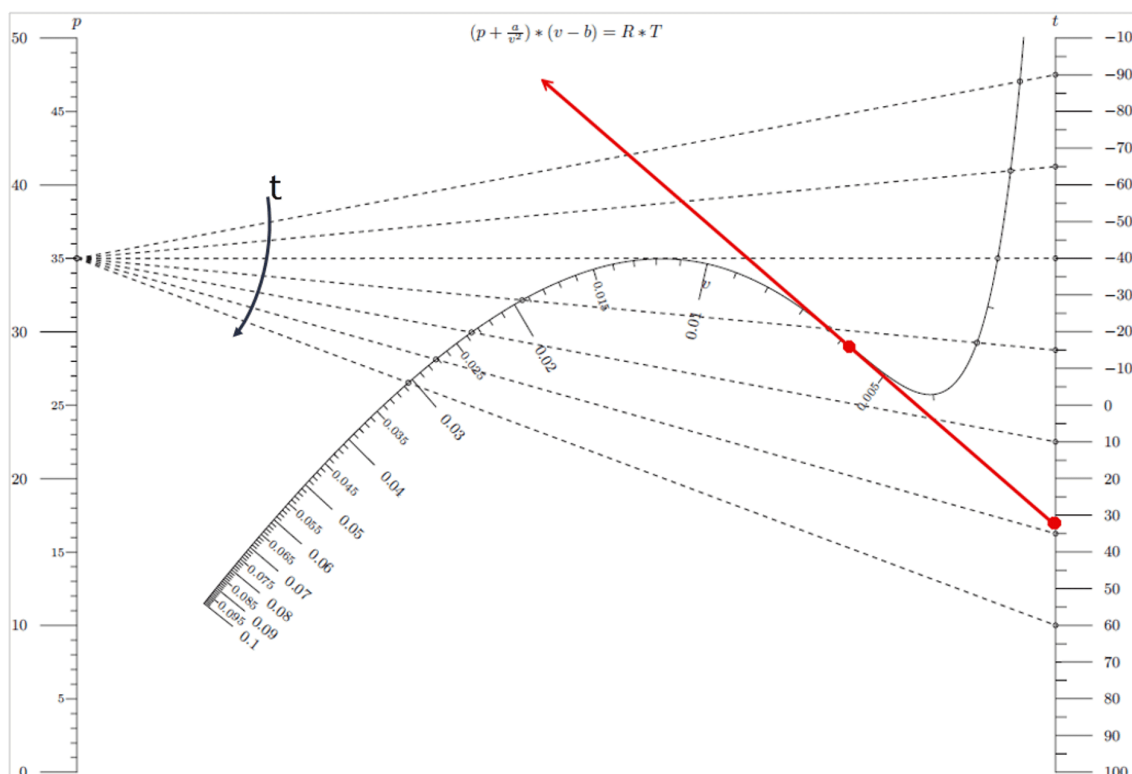


Figure 5: A landscape format version of the van der Waals carbon dioxide nomograph with the critical isopleth indicated in red and a fan of constant pressure isotherms of increasing temperature overlaid.

Fig. 6 Is a contour map of volumes from a projection onto the pT plane of the van der Waals equation. The horizontal plane in the upper-left (that is, the area lacking contours) represents the condensed phase. The somewhat broken dark curve stretching from lower left to the critical point on the upper right represents the projection of the dome of the spinodal curve.

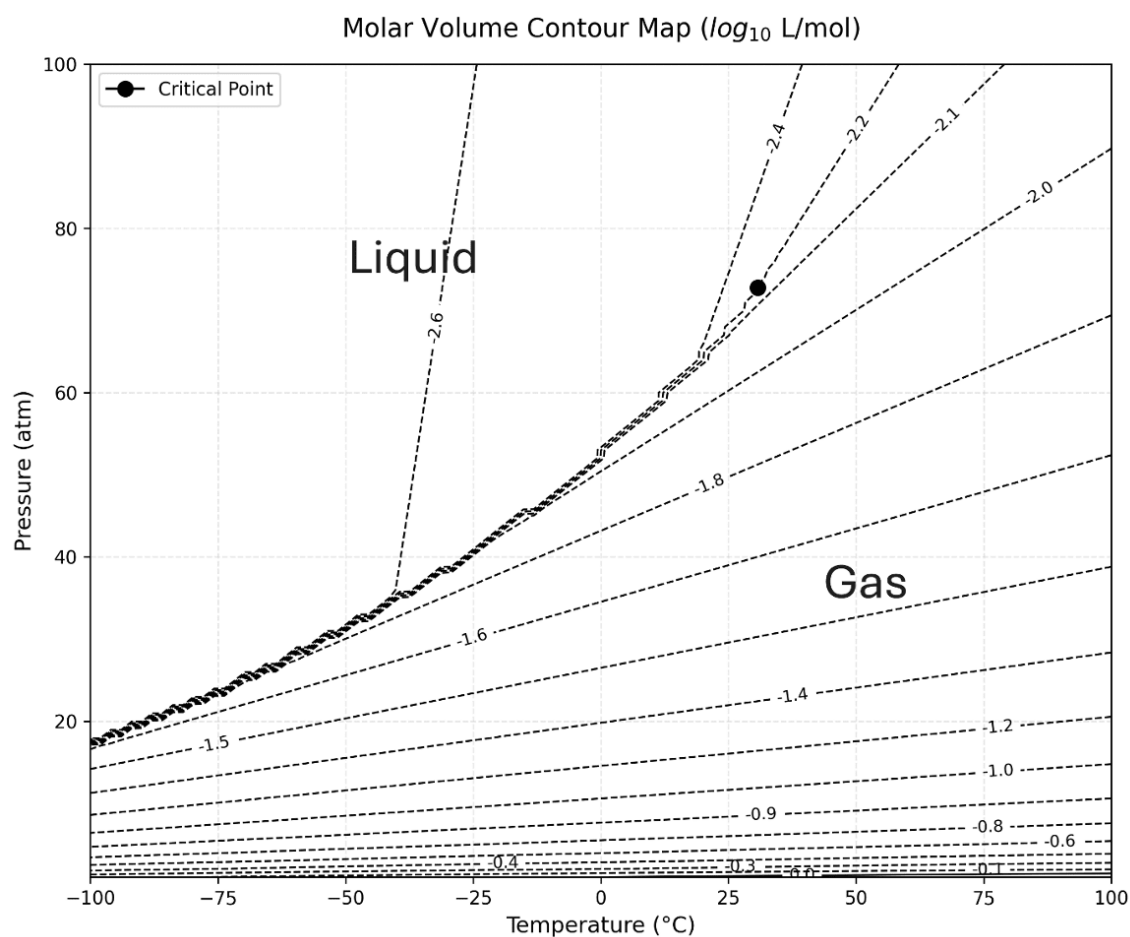


Figure 6. Contour map of the van der Waals equation projected onto the pT plane.

A full 3D surface plot of the van der Waals equation appears in Fig. 7. The projections of the isotherms can be seen in the pV base of this diagram. (cf. Fig. 3).

3D van der Waals Surface (Viridis Colour Mapping)

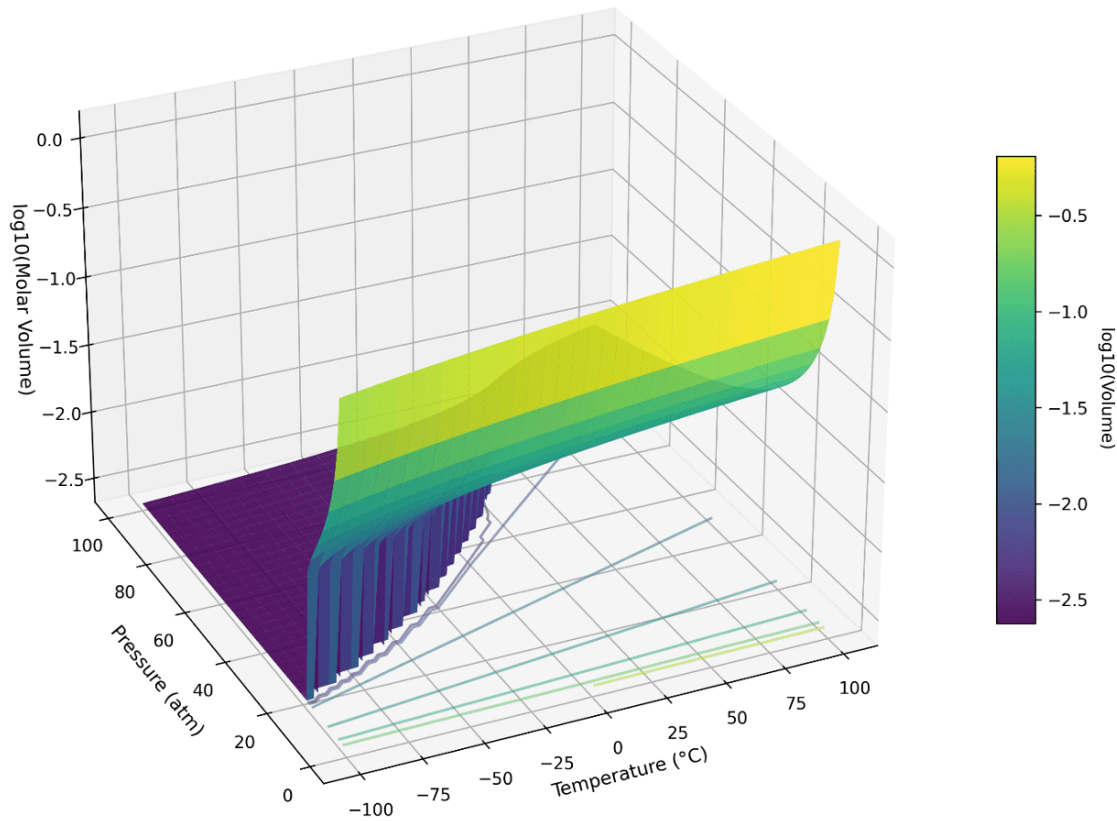
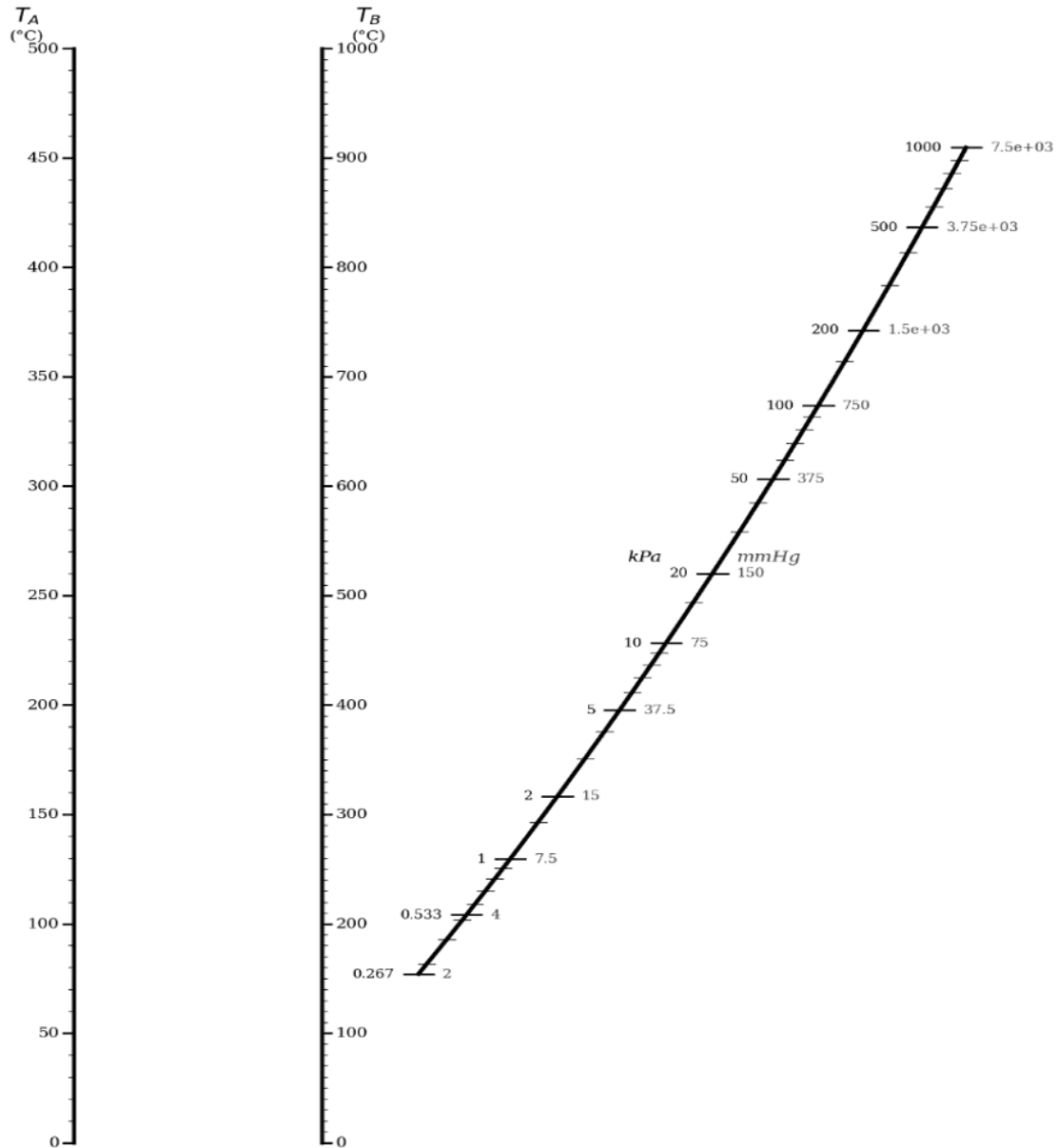


Figure 7. Surface plot of volumes of the van der Waals equation. The dark purple base area represents the condensed phase. It will be noted that the isotherms in the pV plane in Fig. 4 and 6 also appear in this Figure. The projections onto the pT plane are simply vertical lines.

Reverse Engineering of Nomographs

It is sometimes useful to recover an equation from a nomograph which may have been prepared using experimental data or an unknown equation. Procedures for such purposes have been reported.^{[35][36]}

Pressure-Temperature Vapour Pressure Nomograph



Scale A: normal boiling point T_A (°C) · Scale B: operating temperature T_B (°C) · Scale C: vapour pressure P

Figure 8. An empirical pressure-temperature nomograph. The curvature of the vapour pressure curve is clear. This nomograph has been reverse-engineered and prepared by using Claude Sonnet AI. It provides a smoother regularised version of a commercial nomograph.^[37]

Nomographs as Reduced Dimension Transforms: 1D <-> 3D

While three-dimensional graphics are useful for viewing functions, they do not permit easy reading of the data from which they are composed. On the other hand, nomographs are designed for simplicity of data reading of the isopleth point on the dependent variable scale. In this sense, the nomograph may be described as a transform from 3D to a 2D plane on which the desired 1D data point lies. Thus, “using the principle of duality from projective geometry,^[38] d’Ocagne showed that a point on a graph with Cartesian coordinates transformed into a line on an alignment chart, that a line transformed into a point, and, finally, that a family of lines or a surface transformed into a single line.”^[8] So, referring to Fig. 4 and 5 for example, the critical point on the pT plot transforms to a line on the nomograph and consequently, the set of all isopleths that lie above this in the supercritical region. {Span, 1996 #5479} Mathematically, the preparation of a nomograph involves an affine transformation together with a projective transformation. ^[14]

To understand the relation between the 3D and 2D graphics, we use the van der Waals nomograph of Fig. 5. The central scale for volume lies roughly at an angle to the van der Waals scales for pressure and temperature, which is the nomographic style for a ratio of values.^[21] Indeed, if the a and b parameters of the van der Waals equation are each set to zero the central scale becomes an angled straight line, corresponding to the ideal gas hyperbolic equation $V = p/nRT$ as a ratio of variables p and T . Alternatively, the v scale approximates a straight line as the volume per mole increases sufficiently (ie the density becomes sufficiently small). Similarly, in fig 3, the isotherms approximate hyperbolas as the temperature becomes sufficiently large.

The curved region of small volumes in the upper right of the central scale of Fig. 5 diverging from the main trend of the curve is reached by isopleths which correspond to points near the critical point. The clustering of values (see Fig. 4) results from the steep “wall” of values near that critical point.

It is possible to construct the projections, such as those of Figure 3, from the nomograph by generating a fan of isopleths starting from some given pressure across a range of temperatures as in Fig. 5. The molar volumes thus obtained correspond to a slice through the surface plot from which one would be able to plot the isotherm. Further isotherms are generated by selection of other starting pressure points.

The van der Waals equation, as a cubic equation in pT space, exhibits three contiguous regions over its full temperature range.

Supercritical region ($T > T_{c, upper}$): The material exists as a low-density gas at the highest temperatures down to the (upper) critical temperature. The inflection point of an isotherm defines the end of this region, termed the (upper) critical temperature, corresponding to the convergence of the three roots to a single root as temperature rises. The single root to the equation is given by the crossing of an isobar across the inflection of the van der Waals curve.

Intermediate region ($T_{c, upper} > T > T_{c, lower}$): From the (upper) critical temperature towards lower temperatures gas and liquid coexist and the graph of the curve exhibits loops. There are three roots to the equation in this coexistence region with the largest and smallest roots respectively representing gas and liquid, while the intermediate root is unphysical, not representing a stable state. The part of a curve above the Maxwell equal area line represents a supercooled vapour, while the part below is described as a superheated liquid as observed experimentally.^[39]

Subcritical region ($T > T_{c, lower}$): There is also a lower critical temperature which corresponds to the bottom end of the loop. From this temperature down the material is a dense liquid. There is only a single root to the equation in this region corresponding to the molar volume or density of the liquid.

Each of these critical points represents a convergence of the volumes (or densities) of liquid and gas to a double root plus a single root. On a nomograph of the van der Waals equation, these points are represented by an isopleth which is tangent to the volume curve. These tangent points lie on the spinodal curve (which represents the limits of stability of the two-phase system with the stable phase merging with the unstable phase) and do not represent solutions of the van der Waals equation.

Conclusions

Visualising the behaviour of variables in an equation or process provides both insight and a communication tool. We have considered the features of 3D graphics and 2D Nomography which allow them to provide complementary information. 3D graphics has the greater versatility but nomographs are more focused and numerically accessible. While nomographs were once widely distributed and important tools they were superseded by the versatility of ever more powerful computer software. Significant software tools, Pynomo and Nomogen, are now freely available to overcome the onerous tasks involved in the preparation of new nomographs and there is increasing interest in the educational and engineering fields. Together with this, artificial intelligence software specializing in coding, such as Claude Sonnet, is now capable of producing usable nomographs as here also demonstrated.

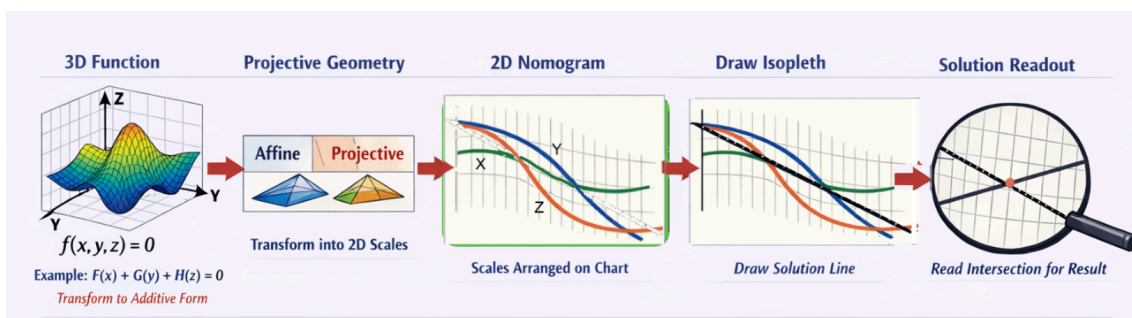
We present only a small sample of the wide range of nomographs which may be found in the literature, online or prepared.

Appendix

Python scripts

The contour map of Fig. 6 and the surface plot of Fig. 8 as well as projections of the sliced isotherms of van der Waals equation (Fig. 4) were prepared using Python scripts developed by Google in prompted AI mode, based on Pynomo and Nomogen software. The nomograph in Fig. 1 was developed using Claude Sonnet 4.6 with reference to Fig. 7 of the principal source of the van der Waals equation nomograph.^[4]

Table of Contents Graphic



Nomographs are two-dimensional plots of ternary (cubic) equations. These can be simply used to find numeric solution values by laying a straight line (an isopleth) across the three curves, with the intersection points providing the numerical data sought. This publication explains the graphic relation among the three-dimensional function, the two-dimensional nomograph, and the one-dimensional data point read from the nomograph. This is illustrated using the van der Waals equation of state for real gases.

The Python scripts used to generate the diagrams were prepared directly by prompted AI.

Statements and Declarations

Data Availability

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Use of Generative AI

Except for Figure 2, prompted Generative AI (both online Claude Sonnet 4.5 and Google Gemini) was used in the preparation of the nomographs that are here published, The mathematical relations discussed were confirmed using online Google Gemini.

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References

1. ^{a, b}Doerfler R. "The Art of Nomography I." *deadreckonings.com*. https://deadreckonings.com/author/rondoe_rfler1/page/3/ (accessed April, 2025).
2. ^ΔDoerfler R. "The Art of Nomography II: Designing with Determinants." *deadreckonings.com*. https://deadreckonings.com/author/rondoe_rfler1/page/3/ (accessed April, 2025).
3. ^ΔDoerfler R. "The Art of Nomography III: Transformations." *deadreckonings.com*. https://deadreckonings.com/author/rondoe_rfler1/page/3/ (accessed April, 2025).
4. ^{a, b, c, d, e, f}Martínez-Pagán P, Roschier L (2022). "Nomography: A Renewed Pedagogical Tool to Sciences and Engineering High-Education Studies." *Heliyon*. 8:e09731.
5. ^ΔTournès D. "Mathematics of Nomography." *academia.edu*. https://www.academia.edu/30868186/Mathematics_of_Nomography (accessed April, 2025).
6. ^{a, b}Rey Pastor J, Pi Calleja P, Trejo CA (1960). *Análisis matemático. Análisis funcional y aplicaciones [Mathematical Analysis. Functional Analysis and Applications]*. Buenos Aires: Editorial Kapelusz, S.A. Vol. 3, pp. 718.
7. ^ΔWikipedia. "Philbert Maurice d'Ocagne." *Wikipedia*. https://en.wikipedia.org/wiki/Philbert_Maurice_d'Ocagne (accessed August, 2018).
8. ^{a, b, c}Hankins TL (1999). "Blood, Dirt, and Nomograms: A Particular History of Graphs." *Isis*. 90:50–80.
9. ^ΔMottola G, Cocconcelli M (2024). "Nomograms in the History and Education of Machine Mechanics." *Found Sci*. 29:125–155.
10. ^ΔSun GJ, Xu LQ, Wang Q, Sun G, Xu L, Wang Q (2026). "Nomograph-Based Precision Engineering: A Framework for Efficient Optimization of 42CrMo Steel Tempering Parameters in Industrial Heat Treatment." *J Mater*

r Eng Perform.

11. [△]Doerfler R. "The Lost Art of Nomography." *deadreckonings.com*. <https://deadreckonings.com/2010/04/18/nomography-article-in-the-umap-journal/#more-44> (accessed April, 2025).
12. [△]Rybner J (1930). "Nomograms." *Gen Electr Rev*. 3:164–179.
13. [△]Cole-Baker BM (1947). "The Construction of Nomograms." *N Z Eng*. 2:1024–1029.
14. [△][‡]Allcock HJ, Jones JR, Michel JGL (1963). *The Nomogram: The Theory And Practical Construction Of Computational Charts*. 5th ed. Pitman. <https://babel.hathitrust.org/cgi/pt?id=mdp.39015000960115&seq=9> (accessed April, 2025).
15. [△]Evesham HA (1986). "Origins and Development of Nomography." *Ann Hist Comput*. 8:324–333.
16. [△]Winchell H (1948). "Alignment Chart for Calculation of Specific Gravity." *Am Miner*. 33:353–359.
17. [△]Winchell H (1951). "Alignment Chart for Calculation of Refractive Index from the Deviation of Light by a Prism." *Am Miner*. 36:287–290.
18. [△]Brodetsky S (1920). *A First Course in Nomography*. G. Bell and Sons. <https://ia804507.us.archive.org/0/items/firstcourseinnom00broduoft/firstcourseinnom00broduoft.pdf>.
19. [△][‡]Blight T, Martínez-Pagán P, Roschier L, Boulet D, Yepes-Bellver L, Yepes V (2025). "Innovative Approach of Nomography Application into an Engineering Educational Context." *PLoS One*. 20:e0315426.
20. [△]Roschier L. "PyNomo - Nomographs with Python." *lefakkomies.github.io*. <https://lefakkomies.github.io/py-nomo-doc/index.html#> (accessed August, 2018).
21. [△][‡]Doerfler R. "Creating Nomograms with the PyNomo Software." *deadreckonings.com*. <https://deadreckonings.com/2009/07/31/creating-nomograms-with-the-pynomo-software/> (accessed April, 2025).
22. [△]Roschier L. "PyNomo Playground (Beta)." 46.101.125.20:8080. <http://46.101.125.20:8080/> (accessed April, 2025).
23. [△]Uzoigwe L, Udeorji CK, Ekwughonu J (2017). "Development of Nomographs as Quick Estimation for Engineering Solutions." *Int J IT Eng*. 5:7.
24. [△]Mottola G, Cocconcelli M (2022). "Nomograms: An Old Tool with New Applications." In: Ceccarelli M, López-García R, editors. *Explorations in the History and Heritage of Machines and Mechanisms*. Cham: Springer International Publishing. pp. 314–329.
25. [△]Martínez-Pagán P, Roschier L (2022). "PyNomo Software Dataset for Sciences and Engineering Nomogram Construction." *Data Brief*. 45:108661.
26. [△]Glasser L (2014). "Illusions of Space: Charting Three Dimensions." *J Chem Educ*. 91:2209–2212.

27. [△]Wikipedia. "Van der Waals Equation." Wikipedia. https://en.wikipedia.org/wiki/Van_der_Waals_equation (accessed April, 2025).
28. [△]LibreTexts. "Maxwell Relations." chem.libretexts.org. [https://chem.libretexts.org/Bookshelves/Physical and Theoretical Chemistry Textbook Maps/Physical Chemistry \(LibreTexts\)/22%3AHelmholtz and Gibbs Energies/22.03%3A_The_Maxwell_Relations](https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Physical_Chemistry_(LibreTexts)/22%3AHelmholtz_and_Gibbs_Energies/22.03%3A_The_Maxwell_Relations).
29. [△]Wikipedia. "Maxwell Relations." Wikipedia. https://en.wikipedia.org/wiki/Maxwell_relations.
30. [△]Prodanov EM (2022). "Mathematical Analysis of the Van der Waals Equation." *Physica B*. **640**:414077.
31. [△]Lienhard JH, Shamsunder N, Biney PO (1986). "Spinodal Lines and Equations of State: A Review." *Nucl Eng Des*. **95**:297–314.
32. [△]Lekner J (1982). "Parametric Solution of the Van der Waals Liquid–Vapor Coexistence Curve." *Am J Phys*. **50**:161–163.
33. [△]David CW (2015). "Plotting the Van der Waals Fluid in Pseudo–3D and the Maxwell Construction." *Chem Educ Mater*. **89**.
34. [△]Rajendran K, Ravi R (2010). "Critical Analysis of Maxwell's Equal Area Rule: Implications for Phase Equilibrium Calculations." *Ind Eng Chem Res*. **49**:7687–7692.
35. [△]Eldredge B (2003). "Reverse Engineering a Nomograph to Equations." scribd.com. <https://www.scribd.com/document/827741978/Reverse-Engineering-Nomographs-Paper> (accessed April, 2025).
36. [△]Holmes D. "You Can Recover the Triple of Nomographic Curves Uniquely from Their Isopleth Lines." *logical.ai*. <http://logical.ai/nomo.guide/from-slopes.pdf> (accessed April, 2025).
37. [△]TCI (Tokyo Chemical Industry). "Pressure-Temperature Nomograph." tcichemicals.com. <http://www.tcichemicals.com/pdf/pressure-temperature-nomograph.pdf> (accessed August, 2018).
38. [△]MathWorld W. "Duality Principle." mathworld.wolfram.com. <https://mathworld.wolfram.com/DualityPrinciple.html#:~:text=All%20the%20propositions%20in%20projective%20geometry%20occur%20in,parts%20played%20by%20the%20words%20%22point%22%20and%20%22line.%22>.
39. [△]Liu H (2020). "The Maxwell Crossover and the Van der Waals Equation of State." PREPRINT (Version 1). doi:10.21203/rs.3.rs-103639/v1.

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