Research Article

Possible Tetraquark Explanations for the Proposed X(4630) and X(4685)

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The recently proposed X(4630) and X(4685) structures are investigated using a first-order tetraquark mass formula. This mass relationship is based on weakly bound J/Ψ(c c-bar) + φ(s s-bar) meson clusters and their excited states. The first-order tetraquark mass formula provides a reasonable prediction (within about 9%) of the measured masses.

1.0 Introduction

The LHCb Collaboration recently reported the first observation of two charged $Z_{cs}{}^{+} \rightarrow J/\Psi K^{+}$ tetraquark states from an updated amplitude analysis of the $B^{+} \rightarrow J/\Psi\phi K^{+}$ decay[1]. Ref. 1 reported these exotic states with the unique quark content of c c-bar u s-bar that decay to the $J/\Psi K^{+}$ final state. The LHCb Collaboration's $J/\Psi K^{+}$ result[1] is based on the combined proton-proton collision data at center-of-mass energies of 7, 8, and 13 TeV.

The LHCb Collaboration also reported the first observation of two new X states decaying to $J/\Psi\phi$[1]. These states, the X(4630) and X(4685), are addressed in this paper. The 1^{+} X(4685) state is observed with relatively narrow width of about 125 MeV with high significance. An additional 1^{-} X(4630) state is observed with a 5.5σ significance[1], with the 1^{-} assessment preferred over a 2^{-} spin-parity assignment at the 3σ level.

In this paper, the first-order tetraquark mass formulas of Refs. 2 - 11 are applied to evaluate the possible mass and J^{π} values of the X(4630) and X(4685) tetraquarks. This mass relationship is based on weakly bound X(4630) and X(4685) structures based on J/Ψ(c c-bar) + ϕ(s s-bar) meson clusters and their excited states. Although the ϕ meson is a superposition of u, d, and s quark-antiquark pairs,

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it is dominantly s s-bar[15]. Accordingly, the first-order mass formula assumes a pure s s-bar configuration for the ϕ meson.

2.0 Model and Formulation

Zel'dovich and Sakharov[12,13] proposed a semiempirical mass formula that provides a prediction of mesons and baryons in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest 1S state. These meson mass formulas are used as the basis for deriving a first-order tetraquark mass formula. In particular, the model proposed in this paper assumes the tetraquark is partitioned into two meson clusters with the interaction between the clusters providing a minimal contribution to the tetraquark mass.

The meson mass (M_m) formula of Refs. 2 - 11 is:

$$\mathbf{M_m = \delta_m + m_1 + m_2 + b_m\,[m_0^2 / (m_1\, m_2)]\, \sigma_1 \cdot \sigma_2} \quad \mathbf{(1)}$$

where m_1 (m_2) are the mass of the first (second) quark comprising the meson, m_0 is the average mass of a first generation quark[12,13], and the σ_i (i = 1 and 2) are the spin vectors for the quarks incorporated into the meson. The parameters δ_m and b_m are 40 MeV/c^2 and 615 MeV/c^2, respectively[13].

The last term in Eq. 1 represents the spin-spin interaction of the quarks and $\boldsymbol{\sigma_1 \cdot \sigma_2}$ is the scalar product of the quark spin vectors. $\boldsymbol{\sigma_1 \cdot \sigma_2}$ has the value -3/4 and +1/4 for pseudoscalar and vector mesons, respectively[12].

In formulating the tetraquark mass formula, effective quark masses provided by Griffiths[14] are utilized. These effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively. The effective masses are utilized in Eq. 1.

These six quarks are arranged in three generations: [d(-1/3), u(+2/3)], [s(-1/3), c(+2/3)], and [b(-1/3), t(+2/3)] [15]. The three generations are specified by the square brackets and the quark charges [in elementary charge units (e)] are given within parentheses.

3.0 First-Order Mass Formula for the X(4630) and X(4685)

The spin of a tetraquark within the first-order mass formula is determined by coupling the two meson clusters

$$\mathbf{J^\pi = J^\pi(1) \times L \times J^\pi(2)}$$

where the first-order mass formula assumes a minimally interacting L=0 configuration[2,3] between the meson clusters. Eq. 2 provides a primitive J^{π} assignment using the possible meson clusters noted in Table 1.

The first-order mass formula used in this paper partitions the tetraquark into two meson clusters. These clusters include the J/Ψ(1S) vector meson (vm), Ψ(2S) vector meson, ϕ(1020) vector meson, and ϕ(1680) vector meson. Using these structures, three tetraquark configurations are evaluated: J/Ψ(1S)ϕ(1020), Ψ(2S)ϕ(1020), and J/Ψ(1S)ϕ(1680). These configuration limitations are incorporated to minimize model complexity which is consistent with an initial first-order mass formulation. In addition, the general X(4630) and X(4685) tetraquark mass formula involving ground state meson clusters is assumed to have the form[2,3]

$$\mathbf{M\,(J/\Psi(1S)\phi(1020)) = M_{vm} + M_{vm} + \Phi} \quad (3)$$

where **Φ** defines the interaction between the meson clusters. Within the scope of this mass formula, the meson-meson cluster interaction is assumed to be weak and sufficiently small to be ignored. Accordingly, Eq. 3 represents the X(4630) and X(4685) structures as quasimolecular four quark systems whose basic character is a weakly bound meson-meson system where the mesons reside in their ground states.

For tetraquarks involving an excited state meson cluster, the following modifications to Eq. 3 are required using the formulation of Refs. 2 – 11:

$$\mathbf{M\,(\Psi(2S)\phi(1020)) = M\,(J/\Psi(1S)\phi(1020)) + \Delta_1(\Psi(2S) - J/\Psi(1S))} \quad (4)$$

$$\mathbf{M\,(J/\Psi(1S)\phi(1680)) = M\,(J/\Psi(1S)\phi(1020)) + \Delta_2(\phi(1680) - \phi(1020))} \quad (5)$$

where Δ_1 is the Ψ(2S) – J/Ψ(1S)) mass difference (589.2 MeV/c^2)[15] and Δ_2 is the ϕ(1680) – ϕ(1020) mass difference (660.54 MeV/c^2)[15].

4.0 Results and Discussion

The angular momentum coupling from Eq. 2 and the first-order mass formula of Eqs. 1 and 3-5 are used to construct the X(4630) and X(4685) states. First-order mass formula model results are summarized in Table 1.

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Table 1 Summary of Candidate J/Ψ ϕ Tetraquarks					
Energy (MeV/c^2)	Experiment		First-Order Model		
	State	J^π	Configuration	Energy (MeV/c^2)	J^π
4630	X(4630)[a]	1^-	J/Ψ(1S)ϕ(1020)	4234	$0^+, 1^+, 2^+$
4685	X(4685)[a]	1+	Ψ(2S)ϕ(1020)	4823	$0^+, 1^+, 2^+$
4685	X(4685)[a]	1+	J/Ψ(1S)ϕ(1680)	4894	$0^+, 1^+, 2^+$
[a] Ref. 1.					

The spin and parity assignments for these states are derived from Eq. 2. All first-order mass formula states have the primitive $J^\pi = 1^- \times 0 \times 1^- = 0^+, 1^+, 2^+$ configurations.

The J/Ψ(1S)ϕ(1020) configuration first-order mass formula leads to a predicted mass of 4234 MeV/c^2 for the X(4630). This is about 8.6% smaller than the measured value[1]. The model predicts a 0^+, 1^+, or 2^+ assignment for this state that is not in agreement with the 1^- value of Ref. 1.

The Ψ(2S)ϕ(1020) configuration first-order mass formula leads to a predicted mass of 4823 MeV/c^2 for the X(4685). This is about 2.9% larger than the measured value[1]. The model also predicts a 1^+ assignment for this state that is in agreement with Ref. 1. However, the 0^+ and 2^+ configurations are not representative of the data. As noted in Refs 2 – 11, the first-order mass formula only provides a primitive spin and parity assignment for the meson-meson cluster configuration.

In a similar manner, the J/Ψ(1S)ϕ(1680) configuration first-order mass formula leads to a predicted mass of 4894 MeV/c^2 for the X(4685). This is about 4.5% larger than the measured value[1]. The model also predicts a 1^+ assignment for this state that is in agreement with Ref. 1. However, the 0^+ and 2^+ configurations are not representative of the data.

Although these results are encouraging, they are based on a first-order mass formula with a number of uncertainties including the assumed quark masses[14] and the magnitude of the meson-meson cluster interaction. However, the model does provide an initial description of the X(4630) and X(4685) states that is reasonable agreement with the experimental data[1].

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The reader should also note that the X states summarized in Table 1 and those noted in Ref. 15 are varied in structure. This complexity is not completely represented by a first-order mass formula.

5.0 Conclusions

The recently proposed X(4630) and X(4685) structures[1] are investigated using a first-order tetraquark mass formula. This mass relationship is based on weakly bound J/Ψ(c c-bar) + φ(s s-bar) meson clusters and their excited states.

The assumed J/Ψ(1S) + ϕ and their excited state configurations lead to mass results that are within about 9% of the measured X(4630) and X(4685) values[1]. However, the X states summarized in Table 1 and Ref. 15 are varied in structure. This complexity is not completely represented by a first-order mass formula.

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