Variability of the Speed of Light in a Weak Gravitational Field

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Funding: No specific funding was received for this work.
Potential competing interests: No potential competing interests to declare.

Abstract

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General relativity suggests that the speed of light is influence by the gravitational potential. Using the Schwarzschild metric, a previous paper noted that the influence of gravity on the speed of light could occur most strongly in the vicinity of a massive object (e.g., in the vicinity of a black hole). No temporal dependence was noted for the time-independent Schwarzschild metric.

This paper evaluates the variability of the speed of light in a weak gravitational field. The solution of the resulting weak gravity field equations results in plane waves for the two polarization components of the gravitational wave. These solutions suggest that the speed of light varies with respect to time with a magnitude that is proportional to the polarization amplitude. Since this amplitude is very small, the effect on the speed of light is likely not measurable given current technology.

Keywords: General Relativity, Weak Gravity, Variability of the Speed of Light, Plane Wave Solutions, Two Polarization Components.

1.0 Introduction

Unlike the special theory of relativity, the general theory admits the possibility of the variability of the speed of light. Within the scope of general relativity, this variability is not definitively established or universally accepted.

Additional considerations involving the variability of the speed of light were provided by Santilli[1-5]. These include the importance of evaluating the local spacetime geometry and associated perturbations[1], incorporating inherent symmetries and conservation laws[2], variability of the speed of light with local physical conditions[3], arbitrary speeds for interior dynamical problems that are compatible with the abstract axioms of special relativity[4], and interior dynamics problems encountered in general relativity[5]. Santilli also established the variability of the speed of light within physical media utilizing new isomathematics[1]. Ahmar et al.[6] provide additional experimental confirmations of Santilli's assertions.

One of the key postulates of the special theory of relativity is that the speed of light c is a constant that is independent of the relative motion between inertial reference frames. A second postulate is the concept that the laws of physics are the same, and have the same form in all inertial reference frames. These postulates lead to a one to one correspondence between the metric tensor coordinates and the physical quantity (i.e., length and time)[7]. For example, the coordinate time t and proper time τ are the same.

This is illustrated by considering the coordinates utilized in special relativity in which spacetime is flat. Each inertial reference frame is distinguished from others by its relative uniform motion. There are no matter or energy perturbations in flat spacetime, but these occur in general relativity (e.g., mass distributions or sources of energy). These mass and energy sources alter the flat spacetime metric tensor that has the form[8] of Eq. 1.

$$\eta_{\mu\nu} = \mathrm{diag}[-1,+1,+1,+1] \tag{1}$$

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{2}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} + \lambda' g_{\alpha\beta} = 8\pi G\, T_{\alpha\beta} \tag{3}$$

$$g_{\alpha\beta}(x) = \eta_{\alpha\beta} + h_{\alpha\beta}(x) \tag{4}$$

$$g_{\alpha\beta}(x) \to \overline{g}_{\gamma\delta}(\overline{x}) = \frac{\partial x^\alpha}{\delta \overline{x}^\gamma} \frac{\partial x^\beta}{\delta \overline{x}^\delta} g_{\alpha\beta}(x(\overline{x})) \tag{5}$$

$$\overline{x}^\alpha(x) = x^\alpha + \lambda^\alpha(x) \tag{6}$$

$$\overline{h}_{\alpha\beta} = h_{\alpha\beta} + \partial_\alpha \lambda_\beta + \partial_\beta \lambda_\alpha \tag{7}$$

$$\partial^\alpha h_{\alpha\beta} = 0,\ \eta^{\alpha\beta} h_{\alpha\beta} = 0,\ h_{0\alpha} = 0 \tag{8}$$

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} \eta^{\alpha\delta} \left(\partial_\beta h_{\delta\gamma} + \partial_\gamma h_{\delta\beta} - \partial_\delta h_{\beta\gamma} \right) \tag{9}$$

$$R^\alpha{}_{\beta\varepsilon\gamma} = \partial_\varepsilon \Gamma^\alpha_{\beta\gamma} - \partial_\gamma \Gamma^\alpha_{\beta\varepsilon} \tag{10}$$

$$R^\alpha{}_{\beta\alpha\gamma} = \frac{1}{2} \left(\partial^\delta \partial_\beta h_{\delta\gamma} - \partial^\delta \partial_\delta h_{\beta\gamma} - \eta^{\delta\alpha} \partial_\gamma \partial_\beta h_{\delta\alpha} + \partial_\gamma \partial^\delta h_{\beta\delta} \right) \tag{11}$$

$$\partial^\delta \partial_\delta h_{\alpha\beta} = 0 \tag{12}$$

$$h_{\alpha\beta}(\vec{x}, t) = \varepsilon_{\alpha\beta} \exp\left(i\left[\vec{k} \cdot \vec{x} - \omega t\right]\right) \tag{13}$$

$$h_{\alpha\beta}(\vec{x}, t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \varepsilon_+ & \varepsilon_x & 0 \\ 0 & \varepsilon_x & -\varepsilon_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sin(k[z-t]) \tag{14}$$

$$ds_+^2 = -dt^2 + (1 + \varepsilon_+ \sin(k[z-t]))dx^2 + (1 - \varepsilon_+ \sin(k[z-t]))dy^2 + dz^2 \tag{15}$$

$$ds_x^2 = -dt^2 + dx^2 + dy^2 + dz^2 + 2\varepsilon_x \sin(k[z-t])dxdy \tag{16}$$

For a more general metric $g_{\mu\nu}$, an associated line element is defined in Eq. 2, where x^{μ} are the metric coordinates. Matter and energy sources create curved spacetime and fall within the scope of general relativity.

This situation is considerably more complex within the framework of general relativity that is illustrated by considering the field equations summarized in Eq. 3, where $G_{\alpha\beta}$ is the Einstein tensor, $R_{\alpha\beta}$ is the Riemann tensor, R is the Ricci scalar, λ' is the cosmological constant, G is the gravitational constant, and $T_{\mu\nu}$ is the energy momentum tensor[8]. The existence of mass and various source terms perturb flat spacetime and impact the postulates of special relativity.

Variability of c is a controversial topic. Magueijo[9] outlines the various arguments supporting and challenges the variable c concept, and also describes the various theoretical arguments that lead to a variation in the speed of light within the scope of general relativity.

A previous paper reviewed the various experiments supporting the gravitational time delay that support the variability of the speed of light, and offers additional theoretical arguments to support the variability[10]. Using the Schwarzschild metric, the gravitational potential impact on the speed of light was quantified[10], and found to be most significant in a strong gravitational field (e.g., in the vicinity of a massive body such as a black hole).

In contrast, this paper investigates the variability of the speed of light in a weak gravitational field. Since the deviation from flat spacetime is very small, any impact on the speed of light is also expected to be very small. It should be noted that although these assertions are speculative[9], the variability of the speed of light is an important concept that has astrophysical implications. These implications will also be reviewed.

2.0 Weak Gravity

Assuming a nearly flat metric, the metric $g_{\alpha\beta}$ can be written in Eq. 4, where $\eta_{\alpha\beta}$ is the Minkowski metric, and the tensor $h_{\alpha\beta}$ has elements that are small compared to unity. Accordingly, terms of quadratic of higher order in $h_{\alpha\beta}$ are sufficiently small to be neglected. The gauge transformation of Eq. 5 preserves the form of Eq. 4. For small λ, the metric transforms under this gauge transformation is noted in Eq. 6.

To first order in λ, the metric transforms under this gauge transformation as Eq. 7. Using Eq. 7, $h_{\alpha\beta}$ satisfies the three gauge conditions summarized in Eq. 8. Inserting Eq. 4 into the vacuum Einstein equations, and neglecting quadratic terms in $h_{\alpha\beta}$, a second-order partial differential equation is obtained[11].

These transformations result in the form for the Christoffel symbols and Riemann tensor noted in Eqs. 9 and 10, respectively. Accordingly, the vacuum equations without the cosmological constant are noted in Eq. 11.

Upon utilizing the gauge conditions, the equations reduce to the simple wave equation of Eq. 12 results. This equation is solved by a linear combination of plane waves summarized in Eq. 13.

For gravity waves, $\omega^2 = | k^2 |$. The amplitude $\varepsilon_{\alpha\beta}$ is (1) symmetric, (2) has only space components, (3) is traceless ($\eta^{\alpha\beta}$

$\varepsilon_{\alpha\beta} = 0$), and (4) is transverse ($k^{\alpha}\, \varepsilon_{\alpha\beta} = 0$)[11]. Accordingly, a gravitational wave traveling in the z direction has the form of Eq, 14, where ε_{+} is the + polarization mode and ε_{x} is the x polarization mode. The line elements of the two polarizations are provided in Eqs. 15 and 16.

These line elements correspond to two plane gravitational waves. The second corresponds to the first rotated by π/4 degrees around the z axis. Each gravitational wave transforms into itself for a rotation by π/2 degrees. This description suggests the waves have a spin of two that characterizes a gravitational wave and its associated quanta (i.e., the graviton).

3.0 Implications for the Variability of the Speed of Light

Using the basic light ray equation $ds^2 = 0$, and a path following a y = z = 0 trajectory in Eq. 15, the local variation in the speed of light is determined. Following the arguments of Ref. 10, dx/dt corresponds to the local speed of light (c'). Eq. 17 illustrates that the speed of light varies locally with time within the weak gravity approximation. Globally, the speed of light (c) is a constant as postulated in the special theory of relativity.

$$\frac{dx}{dt} = c' = 1 - \frac{1}{2}\epsilon_{+} Sin(k[z - t])(17)$$

A previous approach based on the Schwarzschild metric incorporated both weak and strong gravitational components depending on the distance from a massive object[10]. The weak gravity approximation introduces a time dependence that was not observed using the time-independent Schwarzschild metric[10]. These results suggest that a metric having both weak and strong gravitational solutions and that has a time-dependence, offers the possibility that the speed of light may vary in both spatial as well as temporal coordinates. However, the temporal variation depends on the magnitude of the polarization. The variation is likely too small to measure using current technology.

The difficulty of detecting the temporal variation of the speed of light is analogous to current gravitational wave detection. This difficulty is attributed to the magnitude of the waves reaching the earth (i.e. $h \sim 10^{-21}$)[11]. For gravitational wave measurements, this requires detecting a change in length of one part in 10^{21}.

4.0 Conclusions

This paper evaluated the variability of the speed of light in a weak gravitational field. The solution of the weak gravitational field results in plane wave solutions for the two polarization components of the gravitational wave. These solutions suggest that the speed of light varies with respect to time with the magnitude of the polarization amplitude. Since this amplitude is very small, the effect on the speed of light may not be measurable given current technology.

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