On Probabilities in Quantum Mechanics

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Abstract

This is an attempt to clarify certain concepts related to a debate on the interpretation of quantum mechanics, a debate between Andrei Khrennikov on the one side and Blake Stacey and Rüdiger Schack on the other side. Central to this debate is the notion of quantum probabilities. I first take up the probability concept in the QBist school, and then refer to my own arguments for the Born formula for calculating quantum probabilities. In that connection, I also sketch some consequences of my approach towards the foundation and interpretation of quantum theory. I discuss my general views on QBism as a possible alternative interpretation before I give some final remarks.

1. Introduction

The current discussions on the foundation and interpretation of quantum mechanics may be rather intense. Quantum probabilities, as calculated by the Born formula, are central in many of these discussions. As a background for this, it may be useful to look at the various derivations of the Born formula; see Campanella et al. (2020) for some references. My own derivation is given in Helland (2021), and is related to my approach towards quantum foundation, which now has reached its assumed final form in Helland (2024a).

Taking a Theorem by Ozawa (2019) as his point of departure, Khrennikov (2024) has recently criticised the QBism interpretation of quantum mechanics. Based on an earlier version of Khrennikov's paper, this critique has been countered by Stacey (2023) and Schack (2023). The discussion is centered around the probability concept. The main purpose of this article is to look at this discussion as seen from my own standpoint.

2. Quantum probabilities according to a QBist

I will start with the brief paper by Stacey (2023), who approaches the probability concept from the point of view of a QBist. Here is a citation:

'According to this school of thought, a probability for an event is nothing more or less than a gambling commitment, a valuation by a specific agent of how much that agent would stake on that event occurring.'

He continues by referring to Khennikov's introduction of certain mathematical entities to describe the situation where two remote observers measure the same variable: Operators A, M_1 and M_2, a state vector $|\psi\rangle$ for the system and another $|\xi\rangle$ for the environment (Khennikov uses $|\xi_1\rangle$ and $|\xi_2\rangle$ for the states of the measurement apparata), plus a one-parameter family of unitaries $U(t)$ to represent time evolution.

'Any probability extracted from combining these quantities is necessarily, just like any other probability in personalist Bayesianism, the possession of the agent who commits to it. So, there is no way to mix the ingredients A, M_1, M_2 and so forth to arrive at a conclusion that the personal experiences of two agents will always agree, or that they will always disagree, or anything in between.'

I will approach these statements, and any other statements made by physicists, from the point of view of a statistician. The whole science of statistics is built upon probabilities. Bayesianism is one school within statistics, but there are also other schools.

Statistics is also a science that can be explained to intelligent people using fairly everyday terms. One of my own goals is that someday we will be able to do the same with quantum physics.

So concentrate on 'a probability for an event is nothing more or less than a gambling commitment'. QBists seem to think in terms of gambling all the time; every time we make a decision, we make an internal bet. I will claim that ordinary people neither think nor act in this way. We go through life making decision after decision, and very rarely we think in terms of gambling when we make these decisions. So also when we make statements of probabilities of events.

The concept of probability may be useful in many connections. Probabilities may be based upon symmetries, like when throwing a die or evaluating an opinion poll, they can be based upon subjective judgement, or they can be based on a lot of data and long experience, like when a meteorologist makes a probability statement. Only in the subjective case, it is possible at all to talk about some internal gambling procedure. I will claim that even in that case, people tend to make probability statements without having any bets in mind.

So, in my view, even in the case of quantum probabilities, the QBist probability concept as sketched above is unsatisfactory. For quantum probabilities, we in some way will have to take a closer look at the background for the Born theorem.

3. A possible route towards Born

In Chapter 5 of Helland (2021), I argue for a simple version of the Born formula using a variant of Gleason's theorem due to Busch (2013). It is all based upon a theory of theoretical variables (called e-variables in op. cit.). They may be accessible, possible to measure, and they may be maximal as such. Many physical examples exist. From a statistical point of view, the theoretical variables may be statistical parameters, but they can also be other things, like future data.

The simple version of Born's formula starts with a pure state $|\psi\rangle$, assumed to be an eigenvector of an operator A, which in my theory is associated with a maximal accessible variable θ, in such a way that $|\psi\rangle$ can be interpreted as '$\theta = u$' for some u. In this state, we seek the probability that $\eta = v$ for another variable η, and such that '$\eta = v$' is associated with another pure state $|\xi\rangle$. As is well known, this probability is given by the absolute value squared of the scalar product between these two state vectors.

I need two postulates in my derivation:

1. The likelihood principle of statistics holds. This is a principle that all statisticians now believe in; it can be derived from two fairly obvious principles; see Helland (2021).
2. The agent that is involved in the relevant measurement has ideals, and these ideals can be modelled by an abstract or concrete higher being, who is regarded by the agent to be perfectly rational, where rationality can be formulated in terms of the Dutch Book principle.

Any person who knows the two states can evaluate the quantum probability, and so the probability belongs to such a person. The use of such probabilities in connection to decisions is discussed in Helland (2023a).

4. Intersubjectivity and QBism

Consider two remote observers O_1 and O_2 who perform joint measurements on a system S. Let their observations at time t be $\theta_1(t)$ and $\theta_2(t)$, and let these correspond to operators $M_1(t)$ and $M_2(t)$. Khrennikov (2024) considers this situation, and assumes that $[M_1(t), M_2(t)] = 0$. Is this possible?

In my terminology it is only possible if $\theta_1(t)$ and $\theta_2(t)$, in some sense can be given meaning at the same time, and are both accessible. They can then not each be maximal, but one can imagine a situation where the vector $(\theta_1(t), \theta_2(t))$ is maximal. At least it has to be accessible to some agent.

Khrennikov then refers to a Theorem due to Ozawa (2019): Two observers performing the joint local and probability reproducible measurements of the same observable A on the system S should get the same outcome with probability 1. He says that this challenges QBism.

This last challenge is met by Schack (2023). His arguments are based on the quantum formalism, and the QBist interpretation of this formalism. I will not here go into his detailed mathematics, but only his interpretation of this mathematics. Here are two citations:

'The quantum formalism is a tool that any agent can use to optimize their choice of actions.'

'The quantum formalism does not describe nature in absence of agent, but instead is *normative*, i.e., answers the question of how one *should* act.'

I agree completely that quantum probabilities should be attached to an agent (or to a communicating group of agents), but here the agreement stops. First, I will allow any agent, not only one that is familiar with the quantum formalism. Next, I look upon quantum probabilities as descriptive, not normative. This is also my background for interpreting Ozawa's Theorem.

Here is a citation from Section 3 in Schack's article:

'As a mathematical result, Ozawa'stheorem says nothing about intersubjectivity or different observers. To arrive at their interpretation, both Ozawa and Khrennikov have to make the additional assumption that their scenario - two different observers interacting with a system followed by measurements on the meters - describes two different observers measuring the same system observable.'

He then goes on to argue that this assumption is incompatible with QBism. He says that from a QBist perspective, Ozawa's Theorem is about measurements that *a single agent*, say, Alice, contemplates performing on a system and two meters. The assumption that the theorem is about the measurement results of two different observers violates QBism's key tenet that the quantum formalism should be viewed as a single-agent theory.

If this is the case, I disagree with the main basis of QBism. In my view, one can well imagine two observers O_1 and O_2 measuring the same system. But then it must be done in such a way that the vector of results (θ_1, θ_2) is accessible to some agent. What does this mean? As I see it, it means that a third observer - you may well call her Alice - may be able to observe O_1 and O_2 during their measurements, and then be able to record their results all the time. So, one can well regard quantum mechanics as a single agent theory. From the point of Alice here, it can be taken to describe what she observes.

5. Interpretation and foundation of quantum mechanics

Unfortunately, there are many different, mutually incompatible interpretations of quantum mechanics. The relevant Wikipedia article mentions 16 different interpretations. QBism is one of them. There is a large literature on QBism, referred to in Schack's article. I agree with much that is written in this literature, but, as stated in the previous sections, I disagree with their views on quantum probabilities.

As I see it, any quantum interpretation should be coupled to a quantum foundation. My views on the quantum foundation are now described in Helland (2024a). This naturally leads to what I call a general epistemic interpretation of the theory. It is based on theoretical variables that are connected to an agent or to a group of communicating agents. Some of these variables are accessible to the agent, others are inaccessible. My first main theorem states that in a situation with two different accessible variables that in this sense are maximal to the agent, there can be defined a Hilbert space H such that all accessible variables are associated with self-adjoint operators in H. The eigenvalues of an operator A coincide with the possible values of the associated variable. An accessible variable is maximal if and only if the associated operator has only one-dimensional eigenspaces.

In general, these results require some symmetry assumptions, but in the discrete case, it seems as if these symmetry assumptions can be dispensed with, see Helland (2024a).

In addition to these basic results, I need arguments for the Born formula and for the Schrödinger equation. Both issues are addressed in Helland (2021). The assumptions behind a simple version of Born's formula are given above. More general versions of the formula may be derived under simple assumptions.

What are the prices paid for all this? First, some simple axioms are to be assumed. Most of them are rather obvious, but one should be mentioned: There exists an inaccessible variable ϕ such that all accessible variables are functions of ϕ. In several physical examples, ϕ can easily be constructed. One can also discuss purely statistical applications (Helland, 2024b). As a very general axiom, valid for all agents in all possible situations, one can take a religious perspective, see Helland (2023c).

A second price should be mentioned. The theory starts by constructing operators associated with all accessible variables. Pure state vectors are then only introduced as eigenvectors of some physically meaningful operator. This seems to impose a limitation on the superposition principle. This can be discussed, and should be discussed. On the good side, this version of the quantum theory leads to a simple understanding of so-called quantum paradoxes, like Schrödinger's cat, the two-slit experiment and Wigner's friend; see Helland (2023b).

6. Conclusions

'The discussion of quantum foundation and quantum interpretation will probably continue. I have presented my own views in several articles. This leads to a consistent theory, and a theory that also can be explained to outsiders. I see that as a great advantage. In particular, the theory can easily be explained in the discrete case, which has many applications, and is treated in very many textbooks. The continuous case can be approached by taking limits from a discrete construction; see again Helland (2021).

The introduction of quantum probabilities requires extra assumptions as described above. One of these assumptions is related to statistical theory. This opens for a possible communication between statisticians and quantum physicists, a communication that up to now has been very scarce. With the rapid progress of artificial intelligence, which is closely connected to statistics, and the fact that now several articles connecting artificial intelligence to quantum mechanics are appearing, see Dunjko and Briegel (2019), such communication should be treated as being of some importance.

The other assumption relates to the ideals of the relevant agent. These are assumed to be of a kind that can be modeled by a higher being, considered by the agent to be perfectly rational. In Helland (2023a), such ideals are discussed in connection to decision processes.

Finally, the theory can be taken as a basis for reviewing discussions within the quantum community. In the present article, I have considered the recent discussion between Khrennikov (2024) and a couple of QBists. From my point of view, I have stated arguments against the basic QBist assumptions.

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