

# Review of: "Zero-Divisor Graphs of $\mathbb{Z}_n$ , their products and $D_n$ "

S. Kalita

Potential competing interests: No potential competing interests to declare.

## Review on

### ZERO-DIVISOR GRAPHS OF $\mathbb{Z}_n$ , THEIR PRODUCTS AND $D_n$

AMRITA ACHARYYA AND ROBINSON CZAJKOWSKI

In this article authors have considered the zero divisor graphs of  $\mathbb{Z}_n$ ,  $\mathbb{Z}_1 \times \mathbb{Z}_2 \cdots \times \mathbb{Z}_n$  and po-set  $D_n$ .

The article is seeming to be interesting. Authors have tried to verify many results on the zero divisor graphs of  $\mathbb{Z}_n$ ,  $\mathbb{Z}_1 \times \mathbb{Z}_2 \cdots \times \mathbb{Z}_n$  and po-set  $D_n$  using the concept of type graph and strong type graph.

The authors have divided article into five sections.

Section 1 is introductory section, where authors tried to provide a brief description of past works on this topic.

In Section 2 authors tried to established some properties of zero divisor graphs of  $\mathbb{Z}_n \times \mathbb{Z}_2 \cdots \times \mathbb{Z}_n$  with the help of another graph called "Type graph" defined by B. Smith in his article Perfect zero divisor graphs of  $\mathbb{Z}_n$ . But In this Section, proof of most of the theorems are stated as analogous to the proofs of the results in Smith's article.

For example, for the proof of theorem 2.12, authors have mentioned that this result can be proof by using theorem 2.13, 2.14, and 2.15 but at the end final statement about the proof is that all the proofs are analogous to the results of the above-mentioned article. Here authors' contribution is not clear. It is better if the authors provide independent proofs for the results.

"To illustrate this, consider  $\Gamma(\mathbb{Z}_p^2 \times \mathbb{Z}_q)$  where  $p, q$  are prime. This type graph is isomorphic to  $\Gamma(\mathbb{Z}_r^2 \times \mathbb{Z}_t)$  where  $r, s, t$  are prime, even if the value of the primes are different."

In the above quoted statement, as  $\mathbb{Z}_p^2 \times \mathbb{Z}_q \not\cong \mathbb{Z}_p^2 \times \mathbb{Z}_{qp}$  but  $\mathbb{Z}_r^2 \times \mathbb{Z}_t \cong \mathbb{Z}_r^2 \times \mathbb{Z}_{st}$  if  $r, s, t$  are distinct primes, in this case authors should check the correctness of the statement again. Same may happen in Theorem 2.17. Both statements are contradicted by Theorem 2.18.

In Example 2.20, authors claimed  $\Gamma(\mathbb{Z}_p^2 \times \mathbb{Z}_q)$  is perfect because  $\Gamma(\mathbb{Z}_a^2 \times \mathbb{Z}_b)$  is perfect by the reference [3], but in [3]  $\Gamma(\mathbb{Z}_a^2 \times \mathbb{Z}_b)$  is perfect when  $a, b, c$  all are distinct primes.

"Theorem 5.3[3]. If  $n=p^aqr$  for distinct primes  $p, q, r$  and positive integer  $a$ , then  $\Gamma(\mathbb{Z}_n)$  is

perfect."

From Lemma 3.13 to Theorem 3.17, and wherever used, the authors did not mention clearly what  $n^*$  stand for.

The terms perfect graph and  $\beta$ — $\gamma$  perfect in Theorem 3.23 and Theorem 3.24 should be defined.

Result 4.1 can be presented as corollary of Theorem 4.2 as it is a general result.

In the last sections authors have just enlisted some properties of zero divisor graph of po-set  $\mathbb{D}_n$ , as remarks, some of which are already established by different authors. So, it can be removed or should present the properties as theorems with proofs.

At the end, this is a good article with some minor reviews.