Qeios

Peer Review

Review of: "Exact Purification Rates for Alternating Qubit Measurements and Their Blind Spot at Right Angles"

Eric Bittner[1]

1. Physics, University of Houston, United States

This manuscript presents an interesting analysis of alternating qubit measurements in which the relative Bloch angle θ between two measurement bases is treated as the principal control parameter. For sharp projective measurements, the antipodal symmetry makes the problem particularly transparent and leads to the closed-form result

$$r(\theta) = \frac{1}{2}\sin\theta\sqrt{1+\sin\theta}.$$

The comparison with the conventional noise-robustness measure of measurement incompatibility is useful: the two quantities have the same leading behavior near the compatible limit but differ substantially away from it. I also find the identification of the noise-insensitive point at $\theta = \pi/2$ interesting. At this angle, the trace distances associated with the two possible transitions are identical, so changes in their probabilities cannot affect the mean step distance. The manuscript is appropriately careful in emphasizing that $r(\theta)$ is an operational trajectory statistic rather than an incompatibility measure in the usual technical sense.

My main concern concerns the extension to unsharp measurements in Sec. IV. Proposition 1 establishes purity *preservation* for a pure initial state, but this should not be conflated with *purification* from a mixed initial state. For example, full-rank Kraus operators proportional to unitaries preserve every pure state while leaving the purity of every mixed state unchanged. Thus, full rank alone cannot establish asymptotic purification. Since the subsequent reduction to diffusion on the boundary circle requires the radial Bloch coordinate to have reached unity, the author should either explicitly restrict the analysis to initially pure conditioned trajectories or provide an argument that the particular alternating Lüders measurements considered here drive arbitrary mixed initial states asymptotically to the boundary.

 doi.org/10.32388/7ZPNLF

There is also an interesting physical interpretation of the weak-measurement result that I think would strengthen the paper. Equations (10) and (11) describe an effective stochastic angular dynamics,

$$b(\psi) = -\frac{1}{2}\cos\theta\,\sin(2\psi-\theta),$$

$$D(\psi) = 1 - \cos\theta\cos(2\psi-\theta),$$

which can naturally be viewed as competition between deterministic angular localization or phase locking and measurement-induced diffusion. At $\theta = \pi/2$, the deterministic drift vanishes, and the process becomes uniform diffusion around the circle, consistent with the uniform stationary distribution obtained in Eq. (12).

The weak-measurement limit also admits a useful interpretation in terms of stochastic phase dynamics. Equation (10) provides a sinusoidal angular drift that tends to localize the conditioned trajectory, while Eq. (11) supplies the corresponding state-dependent diffusion. This is mathematically analogous to noisy phase-locking models familiar from synchronization theory. In particular, at \theta=\pi/2, the drift vanishes identically, and the dynamics reduce to uniform angular diffusion, so the stationary state loses any preferred phase. A brief discussion of this interpretation would help place the result in the broader context of stochastic synchronization and measurement-induced phase dynamics.

Overall, I find the manuscript worthwhile. The sharp-measurement result is clean, treating the inter-basis angle as the continuously variable parameter provides an interesting perspective on a well-studied problem, and I particularly appreciate the author's effort to distinguish exact results from those obtained within the formal diffusion limit and from numerical or open claims. I recommend **revision**, principally to clarify or establish the purification argument underlying Sec. IV and the associated interpretation of $c(\theta)$. With that issue resolved, the manuscript would provide a useful contribution to the literature on sequential quantum measurement and stochastic quantum trajectories.

Declarations

Potential competing interests: No potential competing interests to declare.