

Peer Review

Review of: "On the Fractional Dynamics of Kinks in sine-Gordon Models"

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This article studies the kinks and kink-anti-kink pairs in fractional models of the sine-Gordon equation. The focus of this work is the comparison of models with different implementations of fractionality and the impact of fractional derivatives on the dynamics of these coherent structures. An interesting point of the paper is the discussion of stability and interactions.

The fractional models of sine-Gordon equations analyzed in this work are constructed using the Riesz and Caputo derivatives, and the analysis is based on numerical integration with very specific initial conditions.

The paper discusses three important models:

1. The first model contains a Caputo temporal derivative while preserving the standard second spatial derivative.
2. The second model maintains a fixed temporal second derivative and uses the spatial Riesz derivative.
3. The third scenario mixes both the Caputo temporal and Riesz spatial derivatives to investigate their combined effects.

The paper concludes that the kinks have different properties in each model, which depend on the parameters α and β of each derivative:

1. The application of the Caputo derivative introduces dissipative characteristics when the order $\beta < 2$, resulting in a deceleration and potential stop of kink movement. That could provoke the trapping and annihilation of kink-anti-kink pairs at low energies.
2. The Riesz derivative generates an attractive interaction between kinks and anti-kinks for $\beta < 2$, whereas for $\alpha > 2$, it introduces long-range repulsion. The resulting dynamics is complex, with attractive and repulsive forces that depend on the energy of the system.

3. The mixture of both derivatives can be used to describe potentially stationary models, with speed variations depending on the values of α and β .

This paper contributes to a better understanding of how various fractional derivatives influence the properties of kinks in the fractional sine-Gordon equation. More generally, it illustrates the range of phenomena that can be represented by fractional non-linear differential equations.

Declarations

Potential competing interests: No potential competing interests to declare.