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Peer Review

Review of: "Constructing Longulence in the Galerkin-Regularized Nonlinear Schrödinger and Complex Ginzburg-Landau Systems"

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The manuscript addresses specific solutions of the non-linear Schr\"odinger (NLS) equation within the Galerkin (Gr) approximation. In Section 2, the author derives and discusses a number of analytical solutions of the GrNLS equation containing a finite number of Fourier components. In Section 3, these results are numerically studied and later extended to the more general complex Ginzburg-Landau (CGL) equation, again within the Galerkin approximation. Global conclusions and expectations are provided in Section 4.

The main problem with the manuscript is its obscurity, which prevents its publication in any scientific journal. Terms like ``longulence'', ``longons'', ``whiskered tori'', ``persistence criteria'', ``final condensate wave profile'', and so on are continuously used throughout the manuscript, yet one does not know at any point what their exact meaning is. I strongly support the coining of novel terms that compactly encode precise ideas and concepts, but they should always be explicitly and clearly defined so the reader can understand the content of the article. Therefore, some space should be devoted to explaining in detail all the main concepts involved in the text, and the use of non-standard terms should be either avoided or explained in great detail.

Another major problem with the manuscript is the presentation of the numerical results. First of all, the specific equations that are integrated, the numerical methods employed, and the initial conditions imposed, along with the values of the parameters used (these are indeed specified), should be clearly stated in the manuscript for reproducibility. In particular, captions of figures should be as self-contained as possible. In addition, I would recommend enlarging the panels in Fig. 1 for clarity. Numerical results are central to this work and thus should be clearly presented. I really do not understand what is represented in the figures. How can the simulations in Fig. 1 become so unstable if the starting point is allegedly a plane wave solution with a single Fourier component? Even some possibility of blow-up is mentioned, which is certainly strange. Also, statements like ``stable (statistically, with respect to the small scale weaker dis-ordered component)'' are quite obscure. What is the specific meaning here of ``statistically'' and ``disorder''? The use of both terms should be clearly explained. Besides, what is meant by ``quasi-periodic''? In Figure 1, I do appreciate some patterns. Why not evaluating the Fourier transform in time for each point x, i.e., $\Psi(x,\omega)$? That would clearly reveal the periodic or quasi-periodic behavior of the system, as it has been previously done in PRA 105, 043302 (2022), where such a procedure revealed the underlying spontaneous Floquet structure of the corresponding NLS solution.

I also recommend improving the abstract, Introduction, and Expectation sections to further clarify the main results of the work and how they extend the present literature. On a more general note, although it is evident that the main target is the mathematical physics community, I miss some citations that would help to connect the results of the author to actual physics (for instance, some references should be included in the Introduction when speaking about NLS equations in hydrodynamics, optics, and condensates). On a general note, the style of the manuscript can be largely improved, and there are many typos that should be corrected.

Of course, it is a personal choice, and it is perfectly legitimate to just target the mathematical physics community. Nevertheless, I enumerate some interesting potential connections that may enrich the physical content of the paper:

-A similar equation to Eq. (2.9) was derived in the context of the generalized Gibbs ensemble in arXiv:2402.10784 (2024) when studying spontaneous symmetry breaking of spatial translation symmetry. As a result, the results of this work may have some potential connection with the rich field of supersolids, first experimentally observed in Nature 543, 87 (2017) and Nature 543, 91 (2017).

-Apart from atomic condensates, NLS equations (although with a source term and imaginary terms in the line of the CGL equation) also appear in polariton condensates (see Rev. Mod. Phys. 85, 299 (2013) for a seminal review), where they have also recently achieved supersolidity (arXiv:2407.02373, a result recently published in Nature: https://www.nature.com/articles/s41586-025-08616-9), and in magnonic condensates; see, for instance, Appl. Phys. Lett. 124, 100502 (2024).

-Similar exact NLS solutions with a discrete, finite number of Fourier components were derived in optical lattices in the seminal work New J. Phys. 5, 104 (2003), where, in addition, a finite Fourier truncation very similar to the Galerkin approximation used here was used to compute Bloch solutions.

Finally, I have a number of minor comments:

-I guess it is a matter of taste, but I recommend removing the hat from the magnitudes (hats are typically reserved in this context for second quantization magnitudes, although I am aware that in mathematics it is also used to signal Fourier transforms), especially from the complex number i.

-The Galerkin force term g should be explained, since it does not seem to appear anywhere else in the work and seems quite strange (it looks exactly like the non-linear interacting term in the NLS equation).

-In the Footnote, I guess it should be |m| (that is, only Fourier components -K≤m≤ K are considered) and $(\Psi^*)^2$.

-What is $\omega_K(k)$ on Page 6? It should be explained precisely.

-What is meant exactly by ``Diophantine condition'' on Page 10? Although it is understood that it is some commensurate relation between frequencies, it should be explicitly explained.

In summary, the manuscript should be largely polished and improved before publication in any scientific journal.

Declarations

Potential competing interests: No potential competing interests to declare.

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