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Peer Review

Review of: "The Non-Reflexive Formulation of Quantum Mechanics: the Whys and the Hows"

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This research article presents the latest work of **Prof. Krause** on **quasi-set theory**, a concept he originally introduced in his PhD thesis in 1990.

Key Features of Quasi-Set Theory:

- **Indistinguishable Elements** – Unlike classical set theory, quasi-set theory allows for elements that cannot be individually distinguished, reflecting the nature of quantum particles.
- **Non-Standard Identity** – The theory challenges classical identity principles, treating quantum entities as **non-individuals** rather than distinct objects.
- **Quasi-Cardinality** – Instead of counting elements in a traditional way, quasi-set theory assigns a quasi-cardinal to a collection, representing the number of indistinguishable entities without assuming they are distinct.
- **Mathematical Foundation for Quantum Mechanics** – It provides a formal structure that aligns with quantum statistics, avoiding the need for artificial labeling of particles.

The underlying logic of quasi-set theory is **non-reflexive**, as it does not preserve the **reflexive law of identity** $\forall x(x = x)$.

Criticisms of Quasi-Set Theory

Despite its theoretical appeal, quasi-set theory faces significant criticisms regarding both its foundational assumptions and practical applications:

1. **Lack of Empirical Necessity** – Some critics argue that quasi-set theory is not strictly necessary for quantum mechanics, as standard mathematical frameworks can often accommodate

indistinguishable particles without requiring a separate theory. For example, quantum field theory relies on classical logic and Zermelo–Fraenkel set theory to encode particle indistinguishability within the algebra of field operators, ensuring that swapping identical particles does not alter physical predictions.

2. **Identity and Ontology Debates** – Some critics question whether quasi-set theory truly solves the issue of identity in quantum mechanics or merely reframes the problem. Consider the Kochen-Specker theorem, which demonstrates that quantum mechanics is inherently contextual. While quasi-set theory offers an alternative ontological perspective that avoids classical identity constraints, it does not fully resolve the contextuality problem. Instead, it provides a framework better suited for discussing quantum properties without relying on traditional notions of individuality.

A Fundamental Obstacle to Non-Reflexive Quantum Mechanics

But perhaps the most serious critique is the following fundamental argument showing the impossibility of a non-reflexive formulation of quantum physics:

Metric spaces rely on the **identity of indiscernibles**, which states that if the distance between two points is zero, then they must be identical. However, non-reflexive QM challenges the classical notion of identity – particularly in the context of **indistinguishable quantum particles**, where identity is not well-defined.

This incompatibility arises because:

- **Metric axioms assume reflexivity** – The metric function $d(x, y)$ satisfies $d(x, x) = 0$, presupposing that every entity is identical to itself.
- **Non-reflexive QM rejects strict identity** – In quantum mechanics whose mathematical formalism utilizes quasi-set theory, identity is **not** a well-defined concept, making it difficult to apply standard metric structures. Indeed, if $\forall x(x = x)$ does not hold, then $d(x, x) = 0$ is not the case.

As a result, non-reflexive quantum mechanics is not compatible with the standard metric axioms.

Consequences of Rejecting Metric Axioms

If non-reflexive QM **eliminates metric axioms**, it has far-reaching consequences:

- **Loss of the real numbers** – The rejection of metric axioms eliminates a meaningful notion of distance, effectively discarding **real numbers** as the foundation of quantum formalism.

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- **Breakdown of infinitesimals** – Infinitesimal numbers rely on statements like: “One can choose two distinct numbers a and b as close to each other as one pleases.” However, if metric axioms do not hold, then the distance between a and b becomes meaningless.
- **Collapse of mathematical analysis** – Infinitesimals are fundamental to mathematical analysis, including calculus, differentiation, and integration, which are widely used across all branches of quantum physics. The rejection of the concept of infinitesimals makes mathematical analysis, as it is known today, impossible.

Comparison with Quantum Set Theory

It is instructive to compare **quantum set theory** with **quasi-set theory**:

- **Quantum set theory** is based on **quantum logic**, which—despite extensive research—fails to resolve fundamental quantum mechanical problems.
- **Quasi-set theory**, despite offering an intriguing alternative, may ultimately face the same fate as quantum set theory, struggling to overcome the deep mathematical and conceptual challenges in quantum physics.

Declarations

Potential competing interests: No potential competing interests to declare.