

RESEARCH ARTICLE

Possible Tetraquark Explanations for the B D-bar and B-star D-bar States Proposed by Lattice QCD Calculations

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Abstract

The recently proposed B D-bar and B^{*} D-bar structures are investigated using a first-order tetraquark mass formula. J^π values predicted by the first-order mass formula coupling structure for the proposed B D-bar and B^{*} D-bar structures are 0⁺ and 1⁺, respectively. These values are in agreement with the Lattice QCD model. The first-order tetraquark mass formula predictions for the B D-bar and B^{*} D-bar tetraquarks are 6.902 and 6.947 GeV/c², respectively. No tetraquark mass values were provided in the lattice QCD calculations.

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1. Introduction

Francis et al.^[1] and Alexandrou et al.^[2] investigated the energy dependence of the B + D-bar and B^{*} + D-bar isospin-0, S-wave scattering amplitudes using lattice QCD. These efforts investigated the possibility of the existence of mixed bottom-charm (b-bar c-bar u d) tetraquarks as either bound states or resonances. Possible J^π = 0⁺ B + D-bar and J^π = 1⁺ B^{*} + D-bar configurations are noted as possible bound states below or resonances above the respective thresholds^[2]. No masses were presented, but J^π values are addressed in Refs. 1 and 2.

In this paper, these states are investigated as possible tetraquarks using a first-order mass formula based on the semiempirical mass formula proposed by Zel'dovich and Sakharov^{[3][4]}. Masses as well as predicted J^π values are

provided. The first-order mass formula has been successfully used to describe other tetraquark systems^{[5][6][7][8][9][10][11][12][13][14][15][16][17][18][19][20][21][22]}.

2. Model and Formulation

The proposed tetraquark model utilizes the semiempirical mass formula proposed by Zel'dovich and Sakharov^{[3][4]}. This model assumes the two meson clusters are weakly bound and the angular momentum between the clusters is zero.

The Zel'dovich and Sakharov meson (m) mass (M) formula^{[3][4]} has the form:

$$M_m = \delta_m + m_1 + m_2 + b_m [m_0^2 / (m_1 m_2)] \sigma_1 \cdot \sigma_2 \quad (1)$$

where m_i is the mass of the quark comprising the meson cluster ($i = 1$ and 2). The average mass of a first generation quark (u and d) is m_0 ^{[23][24]}, and the spin vectors (σ) for the quarks complete the meson mass definition. The parameters δ_m and b_m are defined to have the values $40 \text{ MeV}/c^2$ and $615 \text{ MeV}/c^2$, respectively^[4]. The $\sigma_1 \cdot \sigma_2$ term represents the scalar product of the quark spin vectors, and has the value $-3/4$ and $+1/4$ for pseudoscalar and vector mesons, respectively^[4].

In Eq. 1, effective quark masses must be specified. The effective masses provided by Griffiths^[23] are utilized for d , u , s , c , b , and t quarks, and have the values 340 , 336 , 486 , 1550 , 4730 , and $177000 \text{ MeV}/c^2$, respectively.

Quarks are grouped into three generations: $[d(-1/3), u(+2/3)]$, $[s(-1/3), c(+2/3)]$, and $[b(-1/3), t(+2/3)]$ ^{[23][24]}. The quark charges are given within parentheses, and are expressed in terms of the unit charge e .

3. First-Order Mass Formula for the $B D$ -bar and $B^* D$ -bar States

Within the scope of the first-order mass formula, the spin of a tetraquark is derived from the angular momentum coupling of the two meson clusters

$$\mathbf{J}^\pi = \mathbf{J}^\pi(1) \times \mathbf{L} \times \mathbf{J}^\pi(2) \quad (2)$$

The first-order model summarized in Eq. 2 only provides a primitive $\mathbf{J}^\pi$ assignment. Detailed meson cluster structural information and strong coupling between the clusters are not included in the model formulation.

These aforementioned simplifications minimize model complexity, and permit the tetraquark mass formula to have the form^{[5][6][7][8][9][10][11][12][13][14][15][16][17][18][19][20][21][22]}

$$M = M_m(1) + M_m(2) + \Phi \quad (3)$$

where the 1 and 2 notation denotes the two meson clusters, and the cluster mass is given by Eq. 1. In Eq. 3 Φ is the interaction between the meson clusters. Given the negligible cluster coupling assumption, Eq. 3 represents a quasimolecular four quark system characterized by a weakly bound meson-meson system.

3.1. B D-bar Tetraquark

The B + D-bar tetraquark is modeled as a 0^- d b-bar cluster coupled to a 0^- u c-bar meson cluster. This tetraquark has a $0^- \times 0^- = 0^+$ assignment. The predicted first-order mass is based on Eq. 3

$$M(\mathbf{B} + \mathbf{D}\text{-bar}) = M(\mathbf{B}) + M(\mathbf{D}\text{-bar}) + \Phi(4)$$

with Φ assumed to be much smaller than the meson masses. Using Eq. 4 and the first-order mass formula of Eq. 1, a mass of $6.902 \text{ GeV}/c^2$ is predicted. The 0^+ spin assignment agrees with the Lattice QCD calculations^{[1][2]}. No experimental mass values are available^[24].

3.2. B* D-bar Tetraquark

The B* + D-bar tetraquark is modeled as an excited 1^- d b-bar cluster coupled to a 0^- u c-bar meson cluster. This tetraquark has a $1^- \times 0^- = 1^+$ assignment. The predicted first-order mass is based on Eq. 3, and is consistent with previous calculations^{[5][6][7][8][9][10][11][12][13][14][15][16][17][18][19][20][21][22]}

$$M(\mathbf{B}^* + \mathbf{D}\text{-bar}) = M(\mathbf{B}^*) + M(\mathbf{D}\text{-bar}) + \Phi(5)$$

with Φ being much smaller than the meson masses. Following the first-order model formulation approach for addressing an excited meson state^{[5][6][7][8][9][10][11][12][13][14][15][16][17][18][19][20][21][22]}, Eq. 4 is rewritten as:

$$M(\mathbf{B}^* + \mathbf{D}\text{-bar}) = M(\mathbf{B}) + M(\mathbf{D}\text{-bar}) + \Delta(\mathbf{B}^* - \mathbf{B}) + \Phi(6)$$

where $\Delta(\mathbf{B}^* - \mathbf{B})$ is the difference in masses of the B* and B mesons^[24]. Using Eq. 4 and 6 and the first-order mass formula of Eq. 1, a mass of $6.947 \text{ GeV}/c^2$ is predicted. The spin assignment agrees with the Lattice QCD calculations^{[1][2]}. No experimental mass values are available^[24]. Ref. 24 does provide a tetraquark listing for the $T_{cc\text{-bar}c\text{-bar}}(6900)^0$ with a mass of $6.899 \text{ GeV}/c^2$ that is close to the masses calculated in this paper.

4. Conclusions

B D-bar and B* D-bar structures proposed by Lattice QCD calculations are investigated using a first-order tetraquark mass formula. J^π values predicted by the first-order mass formula coupling structure for the proposed B D-bar and B* D-bar structures are 0^+ and 1^+ , respectively. These values are in agreement with the Lattice QCD model. The first-order mass formula predicts the B D-bar and B* D-bar tetraquarks have masses of 6.902 and $6.947 \text{ GeV}/c^2$, respectively. No experimental mass values are available for these states.

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